Model of a Conveyor-Type Transport System with Stochastic Input Material Flow

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Abstract

Belt conveyor systems play a key role in ensuring the continuous transportation of bulk materials in mining enterprises, both in open and underground conditions. One of the urgent tasks is to increase energy efficiency and material loading factor, which allows to reduce operating costs and meet modern environmental requirements. In this paper, the behavior of the transport system with a stochastic nature of the input material flow is investigated. A mathematical model describing the movement of material along the conveyor route is developed, taking into account the statistical characteristics of the input flow, modeled as a stationary random process. A review of modern approaches to transport system control is given, including belt speed regulation, flow control from accumulation bins, and the use of reversible conveyors for route optimization. Particular attention is paid to the influence of the randomness of the input flow on the system performance parameters. The obtained results allow us to identify dependencies between the stochastic characteristics of the input flow and key parameters of the system, which contributes to the development of more efficient and adaptive conveyor transport control strategies.

Keywords

Belt conveyor, input material flow, stochastic material flow, normal distribution, stochastic process realization, statistical characteristic, correlation function, ergodic process

1. Introduction

Belt conveyors play a key role in ensuring continuous transportation of bulk materials and are an integral part of technological processes in the mining industry [1, 2]. Their widespread use is due to their ability to effectively move large volumes of rock and minerals over significant distances in open pits and underground mines [3, 4]. In modern operating conditions, increased demands are placed on the reliability, durability, and energy efficiency of such systems [5]. A significant share of operating costs is electricity consumption, which is especially relevant against the current trend of rising energy prices and tightening environmental standards [6]. In addition, in modern production environments, there is increasing variability and instability in input material flows, which requires that stochastic factors be taken into account when designing and operating conveyor systems [7]. Ignoring these factors can lead to significant deviations in belt loading and a decrease in the overall efficiency of the transport process. One of the urgent problems is to increase the material loading factor of the transport conveyor, which contributes to a more rational use of power and resources [8 ,9]. In this regard, this study aims to model the movement of material along a transport conveyor with a stochastic nature of the incoming flow, to identify patterns affecting the load factor and develop approaches to optimizing the operation of the transport system.

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2. Literature review

An increase in the material loading coefficient of a belt conveyor can be achieved by eliminating the unevenness of the material flow along the conveying line [10]. For this purpose, various approaches to the control of the bulk material feeding and conveying system are considered in the literature [11].

The first approach is to control the input flow of material from the accumulation bunker [12]. At a fixed belt speed, a flow is formed that ensures an optimal value of the loading factor [13, 14]. This method allows achieving a uniform distribution of material along the length of the conveyor and reducing power losses [15, 16].

The second approach is related to the regulation of the belt speed [17, 18]. In these papers, the linear density of the material on the belt is achieved by proportionally changing the speed depending on the size of the input flow [19, 20]. This approach allows adaptive maintenance of target load with variable input material flow.

The third direction is the application of energy management methodology [21, 22], which involves operating the conveyor system during hours with the minimum cost of electricity. This allows for a reduction in total energy costs while maintaining the required productivity.

Another important area of research is the optimization of material transportation routes [23]. The use of reversible conveyors makes it possible to flexibly redirect flows, taking into account the current load and energy conditions, increasing the efficiency of the entire system. A characteristic feature of most of the listed studies is the assumption of determinism of the input material flow, which simplifies modelling but does not always reflect real conditions. At the same time, some works present experimental data indicating the stochastic nature of the input material flow [24, 25]. The analysis of statistical characteristics and modelling of a random input flow are considered [26, 27], in particular, in the papers [29, 30], where the influence of fluctuations and irregularities in input material flow values on the operating parameters of the transport system is shown. This study examines the movement of material under stochastic input flow and analyses the influence of its statistical characteristics on the flow parameters of the system.

Despite the wide range of models of material movement along a transport conveyor, existing models are focused on taking into account the stochastic nature of real input flows of conveyors or on taking into account the dynamics of flow parameters of a transport conveyor. Paper [9] proposes a belt speed control model that adapts to changes in material flow, but their deterministic assumptions limit applicability under conditions of stochastic input material flow. The study [28] focuses on modeling stochastic input material flow without interrelating with the dynamic characteristics of the conveyor. The assumption of unlimited size of the input bins implies the absence of input material spillage. The presence of spillage leads to the fact that for model [9] and model [28] the dynamics of the material flow along the conveyor can be significantly changed. Neural network-based models such as [20] require extensive training data, which assumes stable operating conditions of the conveyor system.

In contrast, the model proposed in this paper integrates a canonical decomposition of stochastic input, considers probability-based spillage estimation, and introduces dimensionless inter-conveyor dynamics. This enables a more accurate prediction of operational behavior under variability, supporting adaptive control strategies that are grounded in rigorous mathematical representation.

3. Problem statement

In the conditions of the modern mining industry, an important problem is to increase the efficiency of transport systems based on belt conveyors. A significant role in this is played by the nature of the input flow of material, which in real conditions is subject to stochastic fluctuations due to uneven extraction and supply from accumulating bunkers. This study is aimed at constructing a model of a conveyor transport system taking into account the stochastic nature of the input material flow. The purpose of the modelling is to analyze the influence of statistical characteristics of the input flow on the flow parameters of the transport system. The following assumptions were used in the modelling: a) the flow of material coming from the place of material extraction to the input of each of the conveyors is stationary and has the same statistical characteristics; b) the transport scheme considered in [23] is used as the basic configuration for the analysis; c) In the qualitative analysis of the movement of material along the transportation route, a zero approximation was used for the input flow of material; d) the speed of the belt for each conveyor is constant.

4. Deterministic model of a conveyor with constant belt speed

The model of a conveyor with a constant belt speed, studied in detail in [15], can be represented by a partial differential equation of the following form:

$$\frac{\partial \chi_0(t,S)}{\partial t}+\frac{\partial \chi_1(t,S)}{\partial S}=\delta(S)\lambda(\mathrm{t}),\ \ \chi_1(t,S)=a(t)\chi_0(t,S),\ \ \chi_0(0,S)=H(S)\Psi(S), \tag{1}$$

where $\chi_1(t,S)$, $\chi_0(t,S)$ are the flow of material and linear density of material flow for a conveyor of length S_d at a point $S\in[0,S_d]$ at time t; $\lambda(t)$ is the flow of material entering the conveyor input at point $S=0$; $\Psi(S)$ is linear density of the material at a point S at the initial moment of time $t=0$; $H(S)$, $\delta(S)$ are the Heaviside function and the Dirac function, respectively. The conveyor belt speed $a(t)$ determines the relationship between the flow of material and the density of the material.

To describe the flow of parameters, dimensionless parameters are introduced:

$$\tau=\frac{t}{T_d},\ \ \xi=\frac{S}{S_d},\ \ g(\tau)=a(t)\frac{T_d}{S_d},\ \ \delta(\xi)=S_d\delta(S),\ H(\xi S_d)=H(S), \tag{2}$$

$$\theta_0(\tau,\xi)=\frac{\chi_0(t,S)}{\chi_{0\max}},\ \ \psi(\xi)=\frac{\Psi(S)}{\chi_{0\max}},\ \ \theta_1(\tau,\xi)=\frac{\chi_1(t,S)}{\chi_{0\max}}\frac{T_d}{S_d}=\theta_0(\tau,\xi)g(\tau),$$
$$\gamma(\tau)=\frac{\lambda(\mathrm{t})}{\chi_{0\max}}\frac{T_d}{S_d}, \tag{3}$$

where T_d is the characteristic time for the material to transport the length of the route; $\chi_{0\max}$ is the maximum permissible linear density of the material for the conveyor. Considering the dimensionless parameters (2),(3), the equation for the flow parameters in dimensionless form is presented:

$$\frac{\partial \theta_0(\tau,\xi)}{\partial \tau}+g(\tau)\frac{\partial \theta_0(\tau,\xi)}{\partial \xi}=\delta(\xi)\gamma(\tau),\ \ \theta_0(0,\xi)=\mathrm{H}(\xi)\psi(\xi). \tag{4}$$

The equation (4) has a solution:

$$\theta_0(\tau,\xi)=\left[H(\xi)-H\left(\xi-\int_0^{\tau}g(z)dz\right)\right]\frac{\gamma(\tau-\Delta\tau_\xi(\tau))}{g(\tau-\Delta\tau_\xi(\tau))}+H\left(\xi-\int_0^{\tau}g(z)dz)\right)\psi\left(\xi-\int_0^{\tau}g(z)dz)\right), \tag{5}$$

$$G(\tau)=\int_0^{\tau}g(z)dz,\ \Delta\tau_\xi(\tau)=\tau-G^{-1}(G(\tau)-\xi), \tag{6}$$

where $\Delta\tau_\xi(\tau)$ is the transport delay at the moment of time τ for a point on the conveyor belt characterized by the coordinate ξ; $G^{-1}(y)$ is the inverse function for the function $y=G(\tau)$. Equation (5) allows us to calculate the value of the output flow parameters of the conveyor at a constant belt speed $g_0=g(\tau)$ for known values of the input flow parameters:

$$\theta_0(\tau,1)=[1-H(1-g_0\tau)]\frac{\gamma(\tau-1/g_0)}{g_0}+H(1-g_0\tau)\psi(1-g_0\tau)),\qquad G^{-1}(G(\tau)-1)=\tau-1/g_0, \quad (7)$$

$$\theta_1(\tau,1)=[1-H(1-g_0\tau)]\gamma(\tau-1/g_0)+g_0H(1-g_0\tau)\psi(1-g_0\tau)),\quad \Delta\tau_1(\tau)=1/g_0. \quad (8)$$

For the steady-state operation of the conveyor $g_0\tau\geq 1$, the expressions for the output flow parameters take the form:

$$\theta_0(\tau,1)=\frac{\gamma(\tau-1/g_0)}{g_0},\quad \theta_1(\tau,1)=\gamma(\tau-1/g_0),\quad g_0\tau\geq 1. \quad (9)$$

These expressions will be used to construct a stochastic model of a conveyor with constant belt speed.

5. Stochastic model of a transport system with a constant belt speed

To model a transport system consisting of m-separate conveyors, dimensionless parameters are introduced:

$$\tau=\frac{t}{T_d},\ \xi_m=\frac{S_m}{S_d},\ g_m(\tau)=a_m(t)\frac{T_d}{S_d},\quad \delta(\xi)=S_d\delta(S),\quad H(\xi S_d)=H(S),\quad m=1..M, \quad (10)$$

$$\theta_{0\mathrm{m}}(\tau,\xi)=\frac{\chi_{0\mathrm{m}}(t,S)}{\chi_{0\max}},\ \psi_m(\xi)=\frac{\Psi_m(S)}{\chi_{0\max}},\ \theta_{1\mathrm{m}}(\tau,\xi)=\theta_{0\mathrm{m}}(\tau,\xi)g_m(\tau),\ \gamma_m(\tau)=\frac{\lambda_m(\mathrm{t})}{\chi_{0\max}}\frac{T_d}{S_d}. \quad (11)$$

As characteristic values S_d, T_d, $\chi_{0\max}$ of these parameters of one of the conveyors (characteristic length of the conveyor; characteristic time of transportation of material along the conveyor) of the transport system are used. Another option for choosing values S_d, T_d is the maximum length of material transportation along the route of the transport system and the characteristic time of material transportation along this route. With this choice of parameters S_d, T_d average conveyor length can be estimated as $\max(\xi_m)\sim 1/M$. With a large number of conveyors in the transport system $(M>>1)$, the value $\max(\xi_m)$ becomes much less than 1, which affects the scale of the values of the flow parameters of the transport system. Taking these considerations into account, as characteristic values S_d, T_d, $\chi_{0\max}$, let's select the characteristic values of one of the basic conveyors of the transport system $S_d=S_b$, $T_d=T_b$, $\chi_{0\max}=\chi_{0\max b}$, $m=b$. When choosing such a basic conveyor, preferences may be given to the conveyor with the maximum material loading coefficient. Since the choice of S_d, T_d is arbitrary, then at a constant base conveyor belt speed $a_m(t)=a_b$, the values of S_d, T_d are determined from the condition

$$1=\frac{T_b}{S_b}a_b. \quad (12)$$

If the maximum permissible value of the linear density has the same value for each conveyor of the transport system

$$\chi_{0\max\mathrm{m}}=\chi_{0\max b}, \quad (13)$$

then the dimensionless function $\theta_{0\mathrm{m}}(\tau,\xi)$ is the ratio of the specific density of the material for $m-$th conveyor to the maximum permissible specific density for the conveyors of the transport system. To avoid spillage of material [12] for the conveyor, the following condition must be met:

$$\theta_{0\mathrm{m}}(\tau,\xi)\leq 1. \quad (14)$$

The dimensionless function $\gamma_m(\tau)$ is the ratio of the value of the input flow of material entering the input of the $m-$th conveyor to the maximum allowable value of the material flow for the base conveyor $\theta_{1\mathrm{b}\max}=\chi_{0\max}a_b$ for a dimensionless moment in time τ:

$$\frac{\lambda_m(\mathrm{t})}{\chi_{0\max} a_b} = \frac{\lambda_m(\mathrm{t})}{\chi_{0\max}} \frac{T_d}{S_d} = \gamma_m(\tau). \tag{15}$$

For a constant belt speed of each conveyor, the dimensionless quantity g_m is considered as the ratio of the belt speed a_m for $m-$th conveyor to the belt speed a_b for the base conveyor:

$$\frac{a_m}{a_b} = \left(a_m \frac{T_b}{S_b}\right) \Big/ \left(a_b \frac{T_b}{S_b}\right) = a_m \frac{T_b}{S_b} = g_m. \tag{16}$$

Thus, the introduced dimensionless parameters g_m, $\theta_{0\mathrm{m}}(\tau,\xi)$, $\gamma_m(\tau)$ have a strictly defined physical meaning and are convenient dimensionless parameters for modeling the movement of material in a transport system consisting of a large number of conveyors. The dimensionless parameter $\theta_{1\mathrm{m}}(\tau,\xi)$ is expressed through the introduced dimensionless parameters $\theta_{0\mathrm{m}}(\tau,\xi)$, $g_m(\tau)$. The physical meaning of the dimensionless parameter $\theta_{1\mathrm{m}}(\tau,\xi)$ is defined as the ratio of the material flow $\chi_{1\mathrm{m}}(t,S)$ at a certain point in time and point of the transportation route to the maximum permissible flow $\chi_{1\max\ \mathrm{b}} = \chi_{0\max} a_b$ in the base conveyor:

$$\frac{\chi_{1\mathrm{m}}(t,S)}{\chi_{0\max} a_b} = \frac{\chi_{0\mathrm{m}}(t,S) a_m}{\chi_{0\max} a_b} = \frac{\chi_{0\mathrm{m}}(t,S)}{\chi_{0\max}} \frac{a_m}{a_b} = \theta_{0\mathrm{m}}(\tau,\xi) g_m(\tau) = \theta_{1\mathrm{m}}(\tau,\xi). \tag{17}$$

Taking into account the introduced parameters and their physical meaning, dimensionless parameters $\theta_{1\mathrm{m}}(\tau,\xi)$, $g_m(\tau)$, $\gamma_m(\tau)$ for modeling the transport system are used, taking into account the constraint (14)

$$\frac{\theta_{1\mathrm{m}}(\tau,\xi)}{g_m(\tau)} \le 1. \tag{18}$$

The input and output flow of material is related by the ratio

$$\theta_{1\mathrm{m}}(\tau,\xi_{d\,m}) = \left[1 - H(\xi_{d\,m} - g_m\tau)\right]\gamma_m\left(\tau - \frac{\xi_{d\,m}}{g_m}\right) + g_m H(\xi_{d\,m} - g_m\tau)\psi(\xi_{d\,m} - g_m\tau), \quad \xi_{d\,m} = \frac{S_{d\,m}}{S_b}, \tag{19}$$

where $S_{d\,m}$ the length of $m-$th the conveyor. For the steady-state operating mode of the transport system, which is of both theoretical and practical interest, the output flow of material is determined by the following relationship

$$\theta_{1\mathrm{m}}(\tau,\xi_{d\,m}) = \gamma_m(\tau - \xi_{d\,m}/g_m), \quad \xi_{d\,m} - g_m\tau \le 0. \tag{20}$$

For two successively located $(m-1)-$th and $m-$th conveyors without an accumulation bunker between them, the output flow of the $(m-1)-$th conveyor is equal to the input flow of the $m-$th conveyor

$$\theta_{1(\mathrm{m-1})}(\tau,\xi_{d\,(m-1)}) = \theta_{1\mathrm{m}}(\tau,0) = \gamma_m(\tau). \tag{21}$$

For certainty, the parameter $\gamma_m(\tau)$ will denote only the input flow entering the transport system. For the flow of input material $m-$th conveyor, which is equal to the output flow of the $(m-1)-$th conveyor, the notation $\theta_{1\mathrm{m}}(\tau,0)$ is used, hereby emphasizing that the value of the material flow for the $m-$th conveyor is the result of the operation of the $(m-1)-$th conveyor. Expressions (20), (21) allow us to calculate the output flow of material from the transport system with known values of the input flow of material of the transport system.

The flow of material entering the input of the transport system can be represented on the considered interval $[0;\tau_{pr}]$ in the form of a canonical decomposition [29]

$$\gamma_m(\tau) = \gamma_{m0}(\tau) + \sum_{i=0}^{\infty} \Theta_{mi}\rho_i(\tau), \quad M[\Theta_{mi}] = 0, \quad \int_0^{\tau_{зк}} \rho_i(\tau)\rho_\alpha(\tau)d\tau = \delta_{i\alpha}, \quad \delta_{i\alpha} = \begin{cases} 1, & if \ \ i = \alpha, \\ 0, & if \ \ i \ne \alpha, \end{cases} \tag{22}$$

where Θ_{mi} are uncorrelated centered random variables with mathematical expectation equal to zero; $\rho_i(\tau)$ are non-random coordinate orthogonal functions of the expansion of a random process $\gamma_m(\tau)$ on the interval $[0;\tau_{pr}]$. The non-random function $\gamma_{m0}(\tau)$ is the mathematical expectation $m_m(\tau)=M[\gamma(\tau)]$ of the stochastic flow of material $\gamma_m(\tau)$. Centered random variables Θ_{mi} are the coefficients of the expansion of the stochastic flow of material $\gamma_m(\tau)$ into coordinate functions $\rho_i(\tau)$. The canonical decomposition of a stochastic material flow $\gamma_m(\tau)$ is in general, an infinite series that can be bounded with a given degree of accuracy by a finite sum of terms [29]. The statistical characteristics of the stochastic input flow of material $\gamma_m(\tau)$, entering the transport system are defined as follows

$$m_m(\tau)=M[\gamma_m(\tau)]=M\left[\gamma_{m0}(\tau)+\sum_{i=0}^{\infty}\Theta_{mi}\rho_i(\tau)\right]=M[\gamma_{m0}(\tau)]+M\left[\sum_{i=0}^{\infty}\Theta_{mi}\rho_i(\tau)\right]=\gamma_{m0}(\tau), \quad (23)$$

$$\sigma_m^2(\tau)=M\left[\gamma_m^2(\tau)\right]=M\left[\sum_{i=0}^{\infty}\Theta_{mi}\rho_i(\tau)\sum_{\alpha=0}^{\infty}\Theta_{m\alpha}\rho_\alpha(\tau)\right]=\sum_{i=0}^{\infty}\rho_i^2(\tau)M\left[\Theta_{mi}^2\right]=\sum_{i=0}^{\infty}\rho_i^2(\tau)\sigma_{mi}^2, \quad (24)$$

$$k_m(\vartheta)=M[\gamma_m(\tau)\gamma_m(\tau+\vartheta)]=M\left[\sum_{i=0}^{\infty}\Theta_{mi}\rho_i(\tau)\sum_{\alpha=0}^{\infty}\Theta_{m\alpha}\rho_\alpha(\tau+\vartheta)\right]= \quad (25)$$

$$=\sum_{i=0}^{\infty}\rho_i(\tau)\rho_i(\tau+\vartheta)M\left[\Theta_{mi}^2\right]=\sum_{i=0}^{\infty}\rho_i(\tau)\rho_i(\tau+\vartheta)\sigma,$$

where $m_m(\tau)$, $\sigma_m(\tau)$, $k_m(\vartheta)$ are the mathematical expectation, the mean square deviation and the correlation function of the stochastic input flow for the $m-$th conveyor, respectively. Using expressions (23),(24),(25) for calculating the characteristics of the stochastic input flow of material $\gamma_m(\tau)$, entering the transport system, as well as coefficients (20), (21), it is possible to calculate the statistical characteristics of the input flow of material for the subsequent conveyor of the transport system.

In the absence of accumulation bunker between conveyors in the transport system, material spillage occurs [12] due to non-fulfilment of the restriction (27). Material spillage will occur if the value of the material flow entering the conveyor input exceeds a certain critical value $\theta_{1\mathrm{m}}(\tau,0)>\theta_{1\mathrm{m\,cr}}(\tau)=g_m(\tau)$. This critical value $\theta_{1\mathrm{m\,cr}}(\tau)$ determines the minimum conveyor belt speed at which the amount of spilt material per unit of time is limited to specified values. The probability that the material flow value will exceed the critical value is

$$P(\theta_{1\mathrm{m}}(\tau,0)>\theta_{1\mathrm{m\,cr}}(\tau))=1.0-F_m(\tau,\theta_{1\mathrm{m\,cr}}(\tau)), \quad F_m(\tau,x)=\int_0^x f_m(\tau,\theta)d\theta, \quad (26)$$

where the function $F_m(\tau,x)=P(X(\tau)<x)$ is a one-dimensional distribution law of a random process $X(\tau)=\theta_{1\mathrm{m}}(\tau,0)$, characterizing the value of the input flow of material $\theta_{1\mathrm{m}}(\tau,0)$ at the moment of time τ. The function $F_m(\tau,x)$ depends on two arguments: firstly, on the value τ, for which the cross-section is taken; secondly, on the critical value $x=\theta_{1\mathrm{m\,cr}}(\tau)$, when exceeded by the value of the stochastic material flow $X(\tau)=\theta_{1\mathrm{m}}(\tau,0)$, material spillage occurs past the conveyor belt. The statistical characteristics of the flow of material $\theta_{\mathrm{fall\ 1m}}(\tau)=\max(\theta_{1\mathrm{m}}(\tau,0)-\theta_{1\mathrm{m\,cr}}(\tau),0)$, spilling past the conveyor, are as follows:

$$M[\theta_{\mathrm{fall\ 1m}}(\tau)]=\int_0^{\theta_{1\mathrm{m\,cr}}(\tau)}0\cdot f_m(\tau,x)dx+\int_{\theta_{1\mathrm{m\,cr}}(\tau)}^{\infty}(x-\theta_{1\mathrm{m\,cr}}(\tau))f_m(\tau,x)dx=\int_{\theta_{1\mathrm{m\,cr}}(\tau)}^{\infty}(x-\theta_{1\mathrm{m\,cr}}(\tau))f_m(\tau,x)dx, \quad (27)$$

$$M\left[\theta^2_{\text{fall 1m}}(\tau)\right]=\int\limits_{\theta_{\text{1m cr}}(\tau)}^{\infty}(x-\theta_{\text{1m cr}}(\tau))^2 f_m(\tau,x)dx-(M[\theta_{\text{fall 1m}}(\tau)])^2. \quad (28)$$

Expressions (27), (28) make it possible to calculate the critical value $\theta_{\text{1m cr}}(\tau)$ for the stochastic flow of material $\theta_{\text{1m}}(\tau,0)$. For a conveyor with a constant belt speed, the critical value $\theta_{\text{1m cr 0}}$ is a constant value for $m-$th conveyor. As one of the methods for calculating the critical value $\theta_{\text{1m cr 0}}$ the condition can be adopted under which the average flow of material that spills at the conveyor inlet is limited by the maximum permissible value $\theta_{\text{fall 1m 0}}$ (27). Then the constant speed of the conveyor belt is calculated from condition (18):

$$\frac{\theta_{\text{1m cr 0}}}{g_{m0}}\leq 1. \quad (29)$$

Equations (28), (29) together with equations (20), (21), (22) form a model of a transport system with a stochastic input flow of material and a constant conveyor belt speed.

6. Analysis of results

When modelling the transport system, a typical material transportation route is considered, similar to a separate part of the schema of the "Rudna" copper mine conveyor system of the "Rudna" copper [23]. The test transport system is represented by 7 conveyors, in which conveyor 3(8) is a reversible conveyor, Figure 1. This scheme was chosen as a test one because the present study is the initial one for subsequent works related to the design of algorithms for optimal control of transport systems with a variable structure of conveyors along the material transportation route (transport systems containing reversible conveyors). Designations 3 and 8 for the same conveyor are introduced to simplify the demonstration of the material transportation route. In order to simplify the calculation, it is assumed that at the input of the transport system, namely conveyors 1 and 2, a stochastic flow of material and equally good characteristics is created (23), (24), (25). This assumption allows us to simplify the qualitative analysis by demonstrating the main features of the operation of a transport system with a stochastic flow of material entering the input conveyors 1 and 2 of the system. The test transport system allows two transport routes based on the direction of movement of the reversible conveyor belt (Figure 1b, Figure 1c). The two transportation routes are similar in the structure of the conveyor arrangement. The transportation route Figure 1b is taken as the basic option when analyzing the features of the functioning of the transport system. In future research, the design of the material flow control system using the reversible conveyor will include a detailed analysis of the conveying system, allowing two conveying routes. In order to draw an analogy with a separate part of the transport system of the "Rudna" copper [23], each of the conveyors of the test transport system can be supplied with a similar conveyor of the "Rudna" copper: 1: T229 L1;2:T321 S-1;3: S-10; 4: S-12; 5: S11; 6: S-5; 7: S-3. When modelling the transport system, the statistical characteristics of the material flow studied in the work [24] are used, Figure 2. To model the transport system, dimensionless parameters (10), (11) are defined.

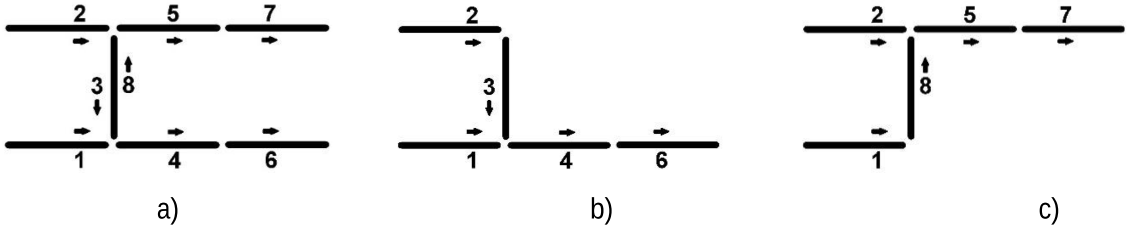
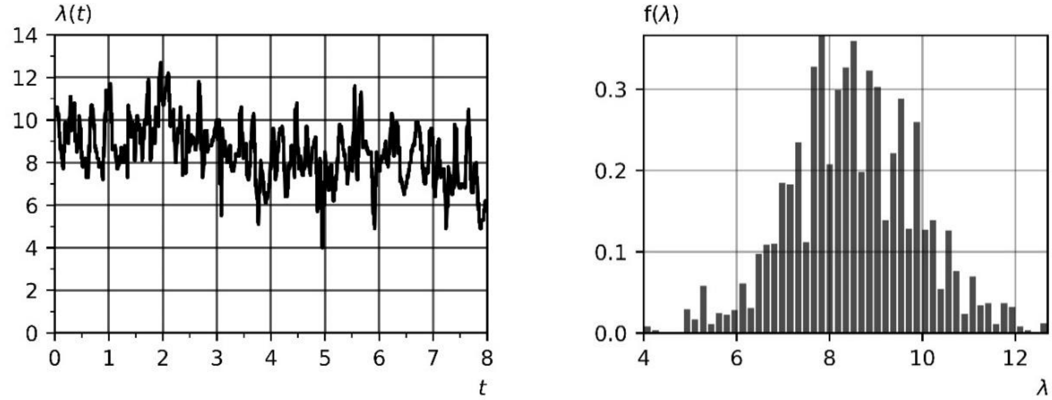


Figure 1: Structural diagram of a transport route: a) general route; b) main route; c) auxiliary route.

For the input flow of material [31] c with statistical characteristics $m_\lambda = M[\lambda(t)] = 8.911$ kg/s, $\sigma_\lambda^2 = M\left[(\lambda(t) - m_\lambda)^2\right] = 1.367$ kg/s, incoming at the input of a conveyor of length $S_d = 7.0$ m at a belt speed of $a = 1.0$ m/s and a planned material loading factor of the belt equal to 0.795, expressions for calculating the dimensionless parameters are obtained:

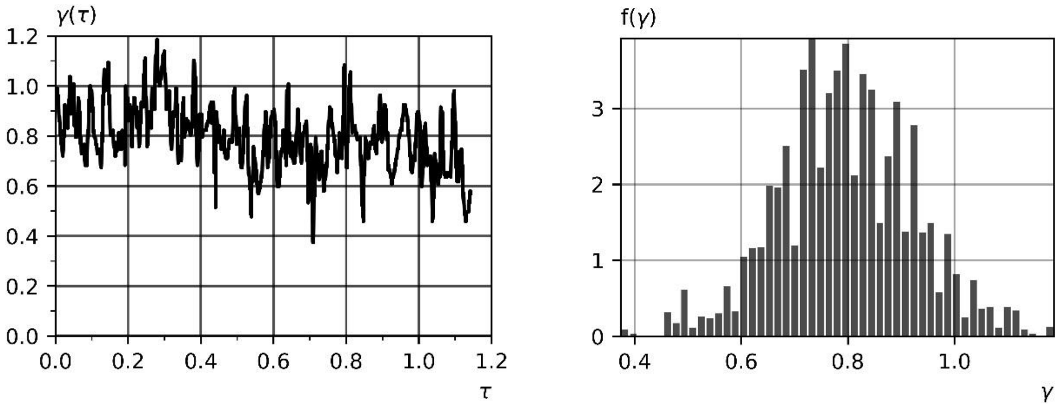
$$\gamma(\tau) = \frac{\lambda(t)}{[\chi]_{0\max} a_b} = \frac{\lambda(t)}{8.511 \cdot 1.0 / 0.795} \approx \frac{\lambda(t)}{10.7}, \quad \tau = \frac{t}{T_d} = \frac{t}{S_d / a} = \frac{t}{7.0/1.0} = \frac{t}{7.0}. \qquad (30)$$



a) b)

Figure 2: The measured data of instantaneous non-uniform iron ore powder distribution with a flow of 8.511 kg/s at a stable speed of 1.0 m/s [31]: a) realization of the input material flow; b) histogram of the distribution of values λ of the input material flow.

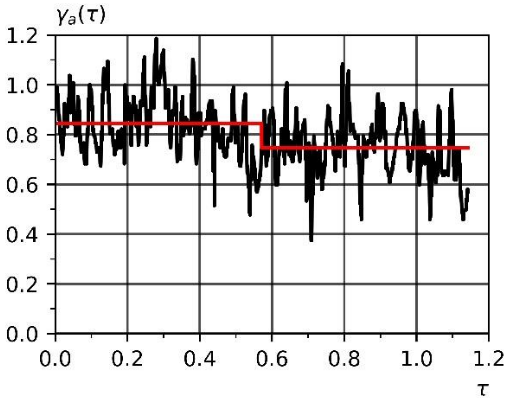
Figure 3 shows the dimensionless realization of the material flow and the corresponding density distribution of the dimensionless values of the material flow.



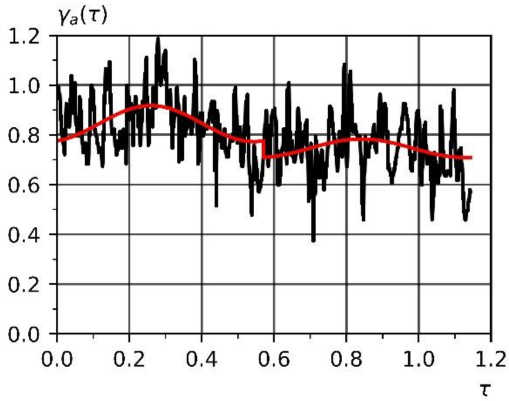
a) b)

Figure 3: Dimensionless input material flow $\gamma(\tau)$: a) realization of the input material flow; b) histogram of the distribution of values γ of the input material flow.

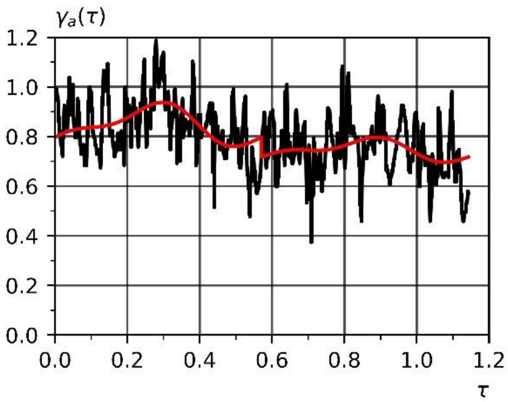
The dimensionless input material flow with statistical characteristics given in Table 1 is the ratio of the input material flow to the maximum allowable material flow at the belt speed characteristic of the conveyor system a_b. The characteristic time T_d is selected from condition (12).



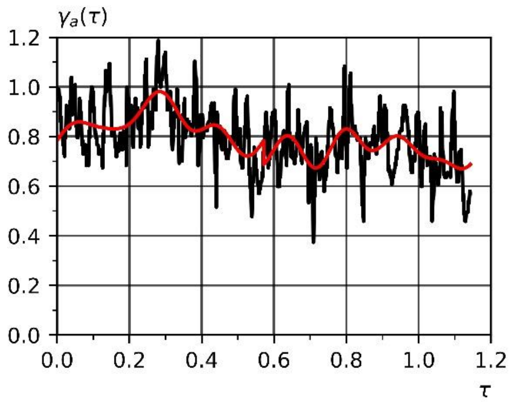
a) $N = 0$;



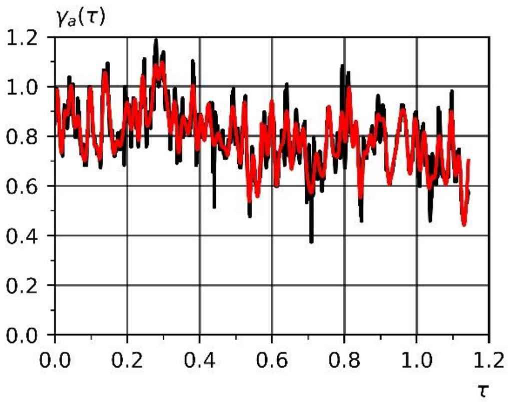
b) $N = 1$;



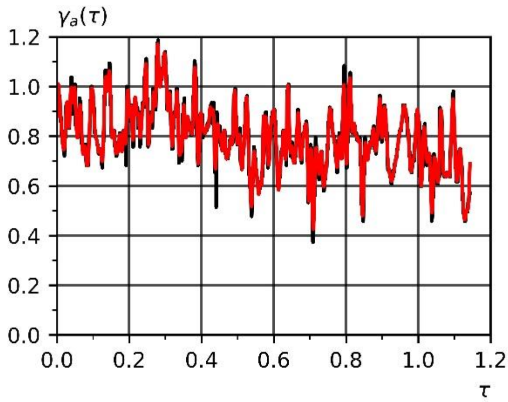
c) $N = 2$;



d) $N = 4$;



e) $N = 32$;



f) $N = 64$;

Figure 4: Approximated dimensionless input material flow $\gamma_a(\tau)$ for different numbers N of terms in the canonical representation of the original material flow.

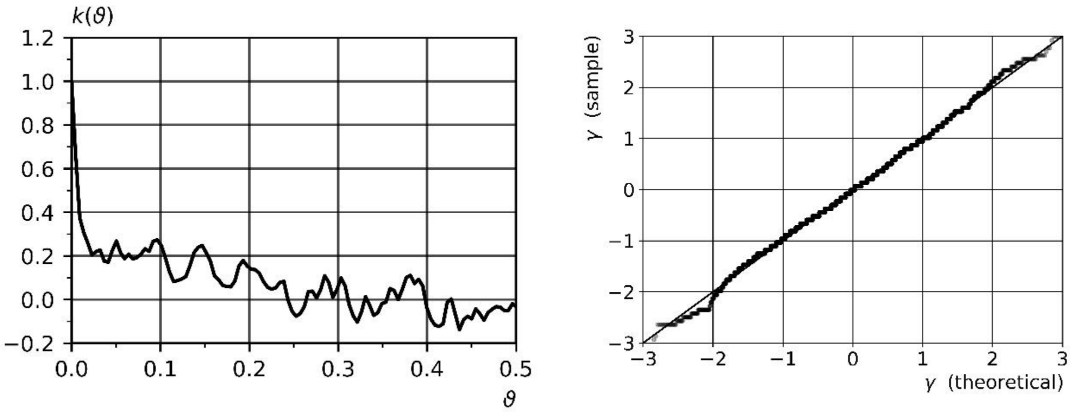
Table 1
Comparative analysis of experimental and approximation implementation of the input flow

Parameter	Experimental implementation	Dimensionless experimental implementation	Dimensionless approximated implementation
Mathematical expectation	8.51	0.7956	0.7954
Standard deviation	1.23	0.1276	0.1207

When modeling the transport system, the input flow of material is approximated by a canonical representation:

$$\gamma_a(\tau) = \gamma_{a0} + \sum_{j=0}^{N} \Theta_{cj} \cos(\omega_j \tau) + \Theta_{sj} \sin(\omega_j \tau), \;\; \omega_j = \pi j, \;\; \gamma_{a0} = const\,. \qquad (31)$$

The dimensionless realization of the approximated material flow $\gamma_a(\tau)$ for different numbers of terms of the approximating expression (31) is shown in Figure 4. The statistical characteristics of the input material flow $\gamma(\tau)$ are reflected in the graphs presented in Figure 5. The correlation function $k(\vartheta)$ decreases exponentially with increasing correlation time ϑ, and asymptotically tends to zero with a characteristic correlation time ϑ_{cor}, much less than the characteristic time of material transportation along the conveyor T_d, $\vartheta_{cor} \approx 0.01 << T_d$, Figure 5a. The presented time dependence between the values of the stochastic material flow in different sections of the random process is typical for stationary processes. The relationship between material flow values separated by a time interval ϑ weakens with increasing correlation time ϑ. The density distribution of the dimensionless input material flow values (Figure 3b) corresponds to a distribution law close to the one-dimensional normal distribution law, which is confirmed by the Q-Q plot of the material flow values $\gamma(\tau)$ (Figure 5b).



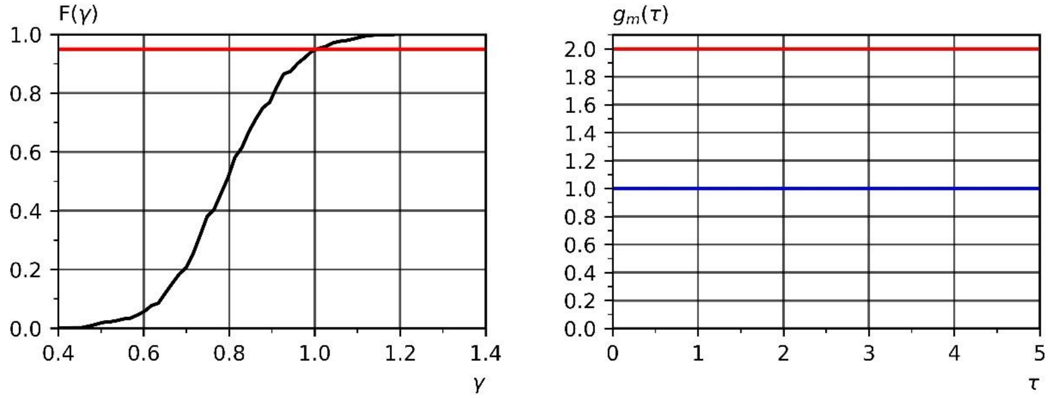
a) correlation function $k(\vartheta)$; b) Q-Q plots (Quantile-to-Quantile);

Figure 5: Statistical characteristics of the input material flow $\gamma(\tau)$.

The fact that a normal distribution law can approximate the distribution of material flow values $\gamma(\tau)$ makes it possible to fairly easily calculate the critical value $\theta_{1\mathrm{m\,cr}}(\tau)$, above which the probability of no material spillage is no more than a given value, for example, 0.05. For the standard normal distribution at $\Phi(z) = 0.95$, z~1.65 the critical value is given by

$$\theta_{11=\mathrm{cr}} = M[\gamma(\tau)] + z\sigma_\gamma = 0.7956 + 1.6449 \cdot 0.1276 = 1.0055 \approx 1. \qquad (32)$$

The numerical solution for the dimensionless material flow $\gamma(\tau)$ is presented in Figure 6 and gives a close result $\theta_{11=\mathrm{cr}} = 1.0088$, which confirms the assumption about the distribution law of the values of the dimensionless material flow $\gamma(\tau)$. For the steady-state operating mode of the transport system, the statistical characteristics of the material flow of each conveyor of the transport system can be expressed through the known statistical characteristics of the input material flow for conveyors 1 and 2 (Table 2): a) at the input to the conveyor $m_{\gamma 1} = m_{\gamma 2}$, $\sigma_{\gamma 1} = \sigma_{\gamma 2}$; б) at the output from the conveyor $m_{\gamma 10} = m_{\gamma 20}$, $\sigma_{\gamma 10} = \sigma_{\gamma 20}$ taking into account the effect of material spillage.



a) b)

Figure 6: a) Critical value of material flow $\theta_{1\mathrm{m\,cr}}(\tau)$; speed modes of belt g_{m0} for the $m-$th conveyor of the transport system.

Table 2
Statistical characteristics of material flow for conveyor systems

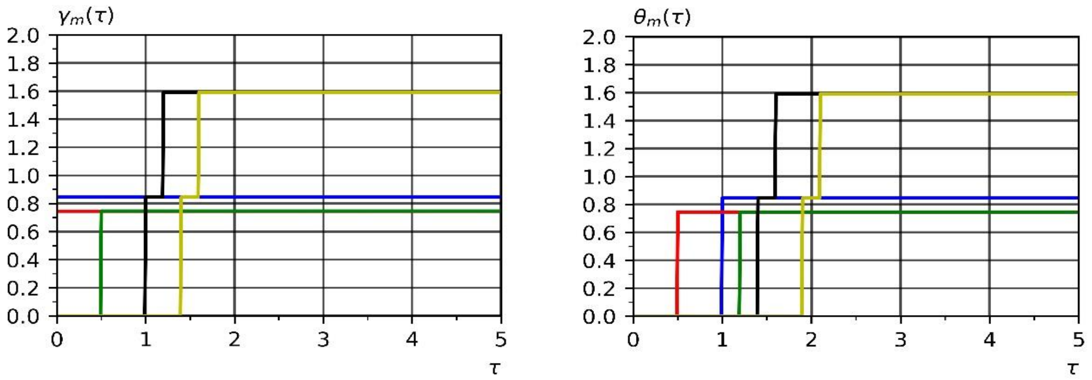
№№ conveyor, m	Mathematical expectation, m_γ	Standard deviation, σ_γ	critical value $\theta_{1\mathrm{m\,cr}}(\tau)$	$\max(\gamma_m)$	min speed, g_{m0}	material loading factor
1	$m_{\gamma 1}=0.7956$	$\sigma_{\gamma 1}=0.1276$	1.0	1.1869	1.0	0.7956
2	$m_{\gamma 2}=0.7956$	$\sigma_{\gamma 2}=0.1276$	1.0	1.1869	1.0	0.7956
3	$m_{\gamma 3}=m_{\gamma 20}=0.7922$	$\sigma_{\gamma 3}=\sigma_{\gamma 20}=0.121$	-	1.0	1.0	0.7922
4	$m_{\gamma 4}=m_{\gamma 10}+m_{\gamma 3}=1.5844$	$\sigma_{\gamma 4}=\sqrt{\sigma_{\gamma 3}^2+\sigma_{\gamma 10}^2}=0.1711$	-	2.0	2.0	0.7922
6	$m_{\gamma 6}=m_{\gamma 4}=1.5844$	$\sigma_{\gamma 6}=\sigma_{\gamma 4}=0.171$	-	2.0	2.0	0.7956

For a qualitative analysis of the features of material movement in the transport system, this paper examines the material flow presented by the approximation of Figure 4-a. For a more in-depth analysis of the transport system in subsequent papers, the type of approximating expression will be determined by the required accuracy specified when setting the problem. In the current paper, the approximation used is sufficient to demonstrate the general patterns of movement of the material in the transport system. The length S_{d1} of the first conveyor is taken as a characteristic value. Each of the conveyors of the transport system has a limitation on the belt speed $a_m \le a_{m\max}$, which is specified by the condition of no spillage of the material (29). For a constant value of material flow, the output material flow for the $m-$th conveyor can be calculated according to expression (20). The calculation results are presented in Table 3.

Table 3
Dimensionless flows parameters of the transport system

№№ conveyor, m	material flow value γ_m at the conveyor input	$\max(\gamma_m)$	density value critical	initial	length $\xi_{d\,m}$	transport output $\xi_{d\,m}/g_m$	delay of the material flow cumulative
1	0.8450	1.0	1.0	0.0	1.0	1.0	1.0
2	0.7459	1.0	1.0	0.0	0.5	0.5	0.5
3	0.7459	1.0	1.0	0.0	0.7	0.7	1.2
4	1.5909	2.0	1.0	0.0	0.8	0.4	1.4; 1.6
6	1.5909	2.0	1.0	0.0	1.0	0.5	1.9; 2.1

The transport delay $\Delta\tau_m = \xi_{d\,m} / g_m$ for each conveyor is determined by the ratio of the conveyor length $\xi_{d\,m}$ to the conveyor belt speed g_m . Dynamics of material flow in a transport system with time-varying material flow values at the input and output of the $m-$th conveyor, Figure 7a,b.



a) b)

Figure 7: The material flow value of the $m-$th conveyor : a) at the input; b) at the output.

Thus, the conducted numerical analysis confirms the significant influence of the stochastic nature of the input flow on the flow parameters of the transport system. The developed model makes it possible to obtain dependencies between the input parameters and output flow parameters of the transport system, to justify the value of the material loading coefficient of each of the conveyors. The obtained results are the basis for improving the control systems of the flow parameters of the transport conveyor. The mathematical model developed in this study is a further step in the significant improvement of the model [10] and serves as a basis for adaptive control strategies, allowing real-time estimation of material flow parameters and critical threshold values for each conveyor segment. Since the stochastic input flow is represented by a canonical decomposition, the model allows continuous estimation of the probability of the expected input flow exceeding the permissible limit at an arbitrary time step, which allows making predictive control decisions and preventing unplanned conveyor stops due to material spillage.

7. Conclusions

This study examines the impact of stochastic input flow on belt conveyor performance, which is particularly relevant under variable load conditions typical of the mining industry. Analysis of existing scientific and technical developments shows that uneven loading significantly affects energy consumption, equipment wear and tear, and overall efficiency of the conveyor system. Modeling the movement of material along the transport line made it possible to identify key patterns of flow distribution and determine the parameters that have the greatest impact on the load factor and energy efficiency. The use of approaches that take into account the random nature of cargo receipt opens up opportunities for more precise adjustment of control systems, as well as the development of adaptive algorithms for regulating the speed and power of drives. The results obtained can be used to optimize the operation of existing transport systems, as well as when designing new lines, especially in conditions of unstable production and limited energy resources. Promising areas for further research include the development of flow parameter control systems for a transport system with a stochastic input flow of material. Of particular interest will be transport systems with material transportation routes that have a variable conveyor structure (reversible conveyor-type transport systems). Numerical simulation showed that the average value of the dimensionless input material flow is $\gamma_m = 0.7956$ with a standard deviation of $\sigma_\gamma = 0.1276$. The critical flow value, above which material spillage occurs, is $\theta_{11=\mathrm{cr}} = 1.0088$, which indicates the need to increase the belt speed by approximately 26% to prevent losses during peak loads. In the conditions of the considered transport system with a reversible conveyor, maintaining a constant belt speed $g_m = 1.0$ ensures stable operation with a load factor close to 0.8. Numerical estimates can be used as a basis for qualitative analysis in the design of adaptive control systems and for the verification of boundary conditions in real time.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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