Transparency in Corporate Networks: A Graph-based Model to Reduce Computational Complexity in Identifying Total Ownership of Ultimate Beneficial Owners

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Abstract

Identifying Ultimate Beneficial Owners (UBOs) in complex corporate structures is critical for financial transparency and preventing economic crimes. Recursive cycles in ownership networks exacerbate this challenge by increasing computational complexity. This article proposes a model based on weighted directed graphs, where nodes represent individuals or legal entities and edges represent ownership percentages. Integrating graph theory and geometric series efficiently resolves ownership cycles, providing a mathematical framework for calculating effective ownership. Direct ownership is computed as the product of weights along paths, while cycles are addressed using recursive algorithms and convergence factors derived from geometric series. The methodology combines graph modeling, algorithmic design (including a DFS version), and experimental validation. Preliminary results demonstrate that the model significantly reduces computational complexity (from O(n!) to O(n+m)), transforming intricate corporate networks into compact UBO tables with their total ownership. While its effectiveness depends on data quality, this work lays the foundation for scalable corporate transparency systems, with applications in financial regulation and compliance.

Keywords

Ultimate Beneficial Owners, Algorithmic Efficiency, Corporate Transparency

1. Introduction

Identifying Ultimate Beneficial Owners (UBOs) in complex corporate structures is essential for financial transparency and combating economic crime worldwide. In recent years, international organizations such as FATF and the European Union have established strict guidelines for disclosing this information, recognizing its critical role in preventing money laundering, terrorism financing, and other illicit activities.

However, this task faces significant computational challenges due to complex ownership networks featuring multi-level indirect ownership, circular relationships, and exponential search space growth, especially in multinational structures [1]. These characteristics make UBO identification potentially NP-hard and traditional approaches computationally infeasible at scale. This work addresses questions such as: How can corporate structures be represented using graphs? Is it possible to reduce the complexity of corporate structures to determine the total ownership of each UBO? Can a DFS and geometric series model resolve recursivity in corporate structures?

The key contributions are: a weighted directed graph model for ownership representation, a DFS algorithm with geometric series for cycle resolution, a mathematical framework for ownership calculation, and experimental validation with real-world data, together enabling efficient UBO identification in complex corporate networks while reducing computational complexity from $O(n!)$ to $O(n+m)$.

ICAIW 2025: Workshops at the 8th International Conference on Applied Informatics 2025, October 8–11, 2025, Ben Guerir, Morocco
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2. Methodology

2.1. Ownership Structure and Ultimate Beneficial Owners

Ownership is defined as the share of a company held directly (via direct shareholding) or indirectly (through ownership chains). According to GAFI[1], UBOs are individuals deriving economic benefits or exercising control, as per FATF guidelines (see Figure 1).

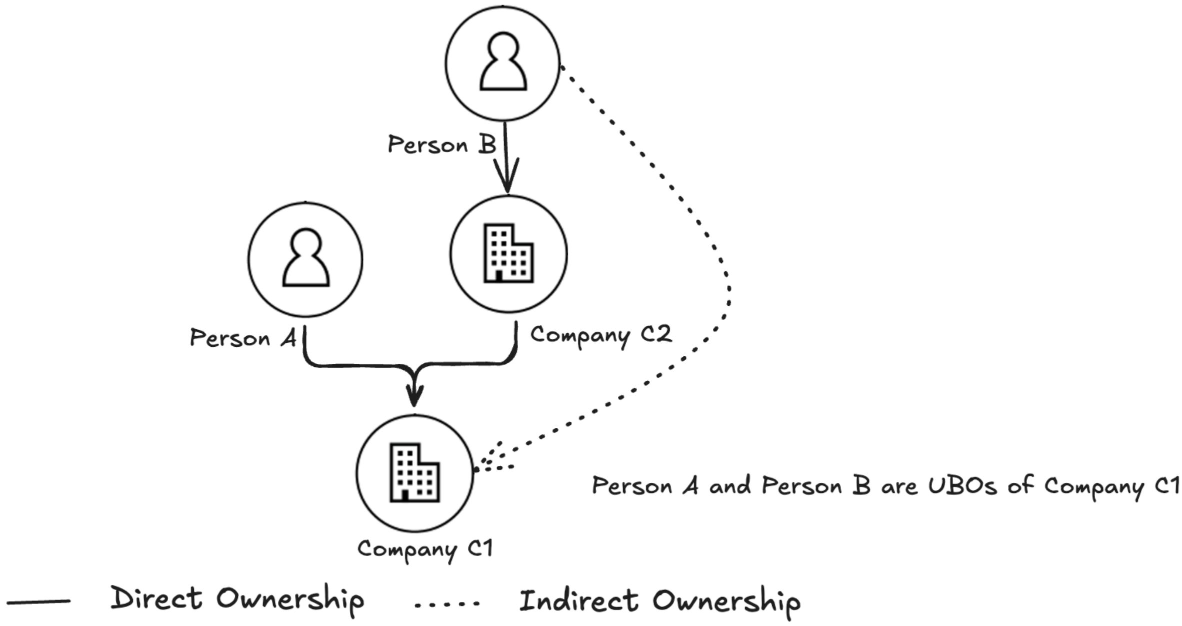

Figure 1: Definitions of UBO, Direct Ownership, and Indirect Ownership [2]

Ownership information for companies and trusts serves as a countermeasure to crime. Clear data is essential for combating illicit financial flows, and governments increasingly commit to beneficial ownership transparency as a key law enforcement tool [2, 3]. According to Open Ownership[2], beneficial ownership registries help prevent financial and economic crimes such as money laundering, terrorism financing, tax fraud, and corruption. These registries clarify where money is sent, preventing individuals from hiding potential financial crimes behind a corporation. All EU countries maintain UBO registries.

GAFI [2] emphasizes the importance of timely access to adequate, accurate, and up-to-date information. Key considerations include:

- Adequate information: Sufficient to unequivocally identify individuals who are UBOs, including the means and mechanisms through which they exercise ownership or control.
- Accurate information: Verified rigorously to confirm its correctness through reliable documentation, cross-checked data, or independently verifiable sources. Verification measures should be risk-based, and countries are encouraged to implement complementary measures, such as mandatory inconsistency reporting, to enhance accuracy.
- Up-to-date information: Reflects the current situation and must be updated within a reasonable timeframe (e.g., one month) after significant changes.

Beyond whether UBO registries should be public, a more pressing concern is ensuring their accuracy and integrity. The UK registry, maintained by Companies House, is a prime example of a well-intentioned but unreliable database. UBO registries aim to ensure that the ownership of assets held through legal structures, such as companies or trusts, is known at least to the registry administrator.

[1] https://biblioteca.gafilat.org/wp-content/uploads/2024/07/Recomendaciones-metodologia-actDIC2023.pdf
[2] https://www.openownership.org/en/about/what-is-beneficial-ownership-transparency/

For data organization, Open Ownership adopts the Beneficial Ownership Data Standard (BODS). Within this framework, a statement may refer to one of three core elements of a beneficial ownership network:

- Entities: Corporations, trusts, and various legal arrangements.
- Persons: Individuals who own, control, or benefit from entities.
- Relationships: Connections representing interests between an entity and a stakeholder.

2.2. Mathematical Modeling

The proposed approach transforms corporate networks into directed graphs, resolving cycles using nested functions and geometric series. This method reduces the complexity of ownership structures through mathematical modeling, algorithm design, and experimental validation (see Figure 2) [4].

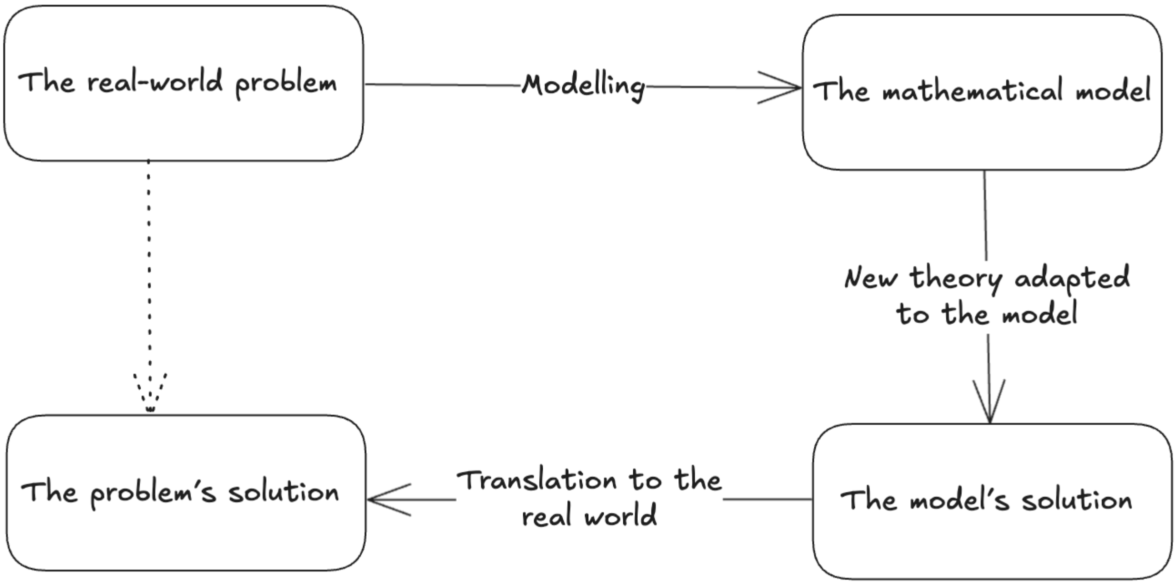

Figure 2: Flow of Mathematical Modeling for a Real-World Problem [4]

2.3. Graph Modeling

Based on BODS representation, the model uses graph theory to represent corporate structures:

- Nodes: Individuals or legal entities.
- Edges: Ownership relationships, weighted by ownership percentages, forming a graph $G = (V, E, w)$, where $V = \{individuals \cup entities\}$, $E \subseteq V \times V$, and $w : E \to [0, 1]$ are normalized weights. For each company node C, the sum of incoming weights equals 1.
- Open Path: A directed graph $G = (V, A)$ is a graph where edges represent a connection from vertex v_1 to v_2 [5].
- Cycles: Closed path $C = \{v_1 \to v_2 \to \cdots \to v_k \to v_1\}$ among entities, introducing recursion. To handle these structures:
  - Nested functional equations model recursive ownership by propagating contributions through the network. This involves a recursive definition where a function depends on itself, such as $g(x) = a + f(g(x))$ [4].
  - Geometric series resolves cycles by summing infinite recursive ownership interactions compactly. In such a series, each term grows by a constant ratio r, where $|r| < 1$ and $n \to \infty$, then $S_n = a(1 - r)^{(-1)}$.

2.4. Applying the Depth-First Search (DFS) Algorithm

As the name suggests, explore as deeply as possible in a graph. In our model, DFS traverses weighted edges to identify ownership paths to UBOs [6]. After exploring all edges from a vertex, the search backtracks to the vertex where it was discovered. This process continues until all vertices reachable from the source vertex are discovered. If undiscovered vertices remain, DFS selects one as a new source and repeats the search. The algorithm continues until all vertices are discovered.

When applying a DFS algorithm to a corporate structure graph to determine the real ownership of individuals as shareholders in a reference company, we find that open paths represent ownership links between a shareholder and an entity. Direct ownership is a direct connection, calculated by multiplying ownership percentages. Indirect ownership involves intermediaries and is the total of multiplied percentages along each indirect path (see Figure 3).

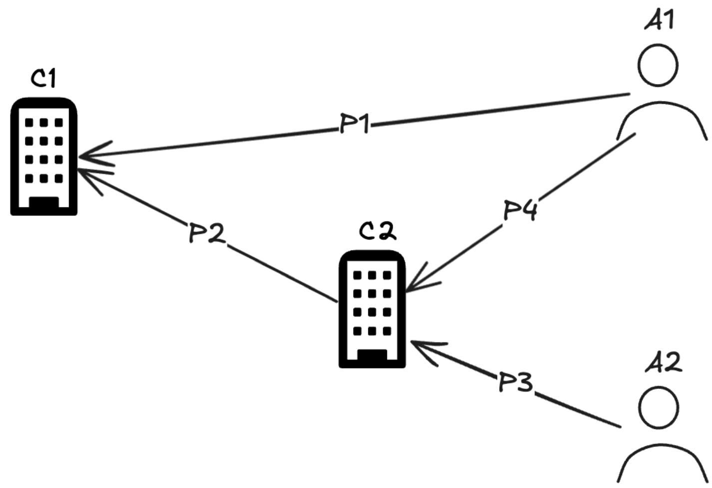

Figure 3: Representation of Two Companies with Two Individual Shareholders

Consider two companies $(C1, C2)$ and UBOs $A1, A2$ (see Figure 3). Applying DFS for $C1$:

- UBO $A1$:
  - Open path $A1 \to C1$, with direct ownership of $P1$
  - Open path $A1 \to C2 \to C1$, with indirect ownership of $P4P2$
- UBO $A2$:
  - Open path $A2 \to C2 \to C1$, with indirect ownership of $P3P2$
- Total Ownership:
  - UBO $A1 : P1 + P4P2$
  - UBO $A2 : P3P2$

Closed paths (cycles) occur exclusively among entities, introducing mathematical complexity through recursivity. This manifests in two key effects: (1) individual-to-entity edges maintain fixed ownership percentages, while (2) entity-to-entity edges generate converging sums via geometric series, with values distributed to individuals according to defined edge weights.

Consider two companies $(C1, C2)$ with a Cycle $C1 \to C2 \to C1$ and UBOs $A1, A2$ (Figure 4). Applying DFS for $C1$:

- UBO $A1$:
  - Open path $A1 \to C1$, with direct ownership of $P1$
  - Open path $A1 \to C2 \to C1$, with indirect ownership of $P4P2$

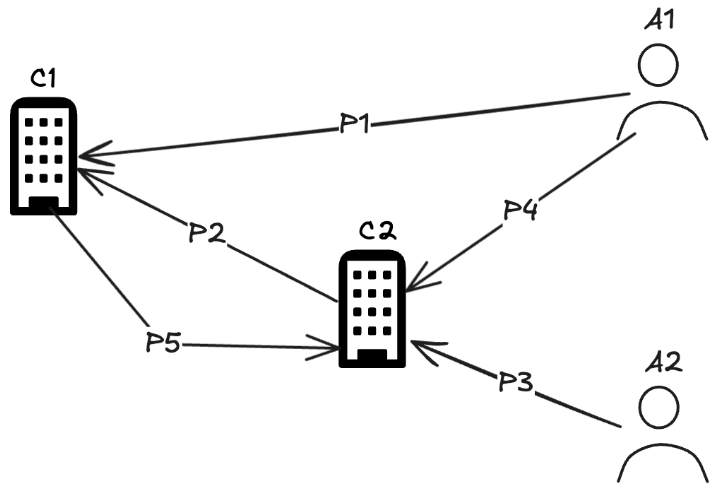

Figure 4: Representation of Two Companies with Two Individual Shareholders, Including a Cycle

- UBO $A2$:
  - Open path $A2 \rightarrow C2 \rightarrow C1$, with indirect ownership of $P3P2$

A closed path $C1 \rightarrow C2 \rightarrow C1$, introduces recursivity, requiring P_2^c and P_5^c, to be computed as convergent sums via geometric series, forming nested functions:

$$P_2^c = P_2(P_3 + P_4 + P_5^c) \tag{1}$$

$$P_5^c = P_5(P_1 + P_2^c) \tag{2}$$

Using the sum of ownership percentages for C:

$$1 = P_1 + P_2^c \tag{3}$$

Substituting P_5^c from (2) and rearranging from (2) to (3):

$$1 = P_1 + P_2(P_3 + P_4) + P_2 P_5^c \tag{4}$$

Substituting P_5^c:

$$1 = P_1 + P_2(P_3 + P_4) + P_2 P_5(P1 + P_2^c) \tag{5}$$

Since $(P_1 + P_2^c)$ repeats in (4), it can be substituted as a nested function:

$$1 = P_1 + P_2(P_3 + P_4) + P_2 P_5(P1 + P_2(P_3 + P_4) + P_2 P_5(P_1 + P_2^c)) \tag{6}$$

This extends to n, where $n \rightarrow \infty$:

$$1 = (P_1 + P_2(P_3 + P_4))(1 + P_2 P_5 + (P_2 P_5)^2) + \cdots + (P_2 P_5)^n + (P_2 P_5)^{n+1}(P_1 + P_2^c) \tag{7}$$

This equation has a complexity order of $O(n!)$.
The last term converges to zero because $(P_2 P_5)^{n+1} < 1$

$$(P_2 P_5)^{n+1}(P_1 + P_2^c) = 0 \tag{8}$$

The remaining terms form a geometric series:

$$(1 + P_2P_5 + (P_2P_5)^2 + \cdots + (P_2P_5)^n) = (1 - P_2P_5)^{-1} \tag{9}$$

Finally:

$$1 = (P_1 + P_2(P_3 + P_4))(1 - P_2P_5)^{-1} \tag{10}$$

$$(P_1 + P_2(P_3 + P_4))^{-1} = (1 - P_2P_5)^{-1} \tag{11}$$

This equality is significant (11), as the coefficient can be expressed in two forms. Having calculated open paths, the Total Ownership for each UBO is:

- UBO $A1$: $(P_1 + P_2P_4)(P_1 + P_2(P_3 + P_4))^(-1)$
- UBO $A2$: $(P_2P_3)(P_1 + P_2(P_3 + P_4))^(-1)$

The Total Ownership of each UBO is the sum of the product of ownership percentages along each independent path, divided by the sum of all independent paths for all UBOs multiplied by their ownership percentages. This reduces the complexity order to $O(n + m)$.

3. Results

3.1. Algorithm Design

The proposed DFS algorithm is:

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Algorithm Total Ownership UBOs(G):
Input: Weighted directed graph G = (V, E, w)
Output: Table T of UBOs and their total ownership
  1. Initialize T as empty
  2. For each node v ∈ V:
    3. Run DFS from v to identify paths and cycles
    4. For each acyclic path P from v to u:
      5. Compute Pd = ∏ x(e_i) for e_i ∈ P
        6. Compute the cyclic coefficient as the sum of Direct and Indirect
           Ownership
  7. Compute Total Ownership for each UBO
  8. Return T
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3.2. Case 1: Recursivity with Three Companies Forming a Cycle

Consider three companies $(C1, C2, C3)$ with a Cycle $C1 \to C3 \to C2 \to C1$ and UBOs $A1, A2, A3$ (Figure 5). Applying DFS for $C1$:

- UBO A_1
  - Open path $A1 \to C1$, with direct ownership of $P1$.
- UBO A_2
  - Open path $A2 \to C2 \to C1$, with direct ownership of $P4P2$.
- UBO A_3
  - Open path $A3 \to C3 \to C1$, with direct ownership of $P6P5$.

Total ownership accounts for the cycle using geometric series, yielding:

- UBO $A1$: $P_1(P_1 + P_4P_2 + P_6P_5)^{-1}$
- UBO $A2$: $P_4P_2(P_1 + P_4P_2 + P_6P_5)^{-1}$
- UBO $A3$: $P_6P_5(P_1 + P_4P_2 + P_6P_5)^{-1}$

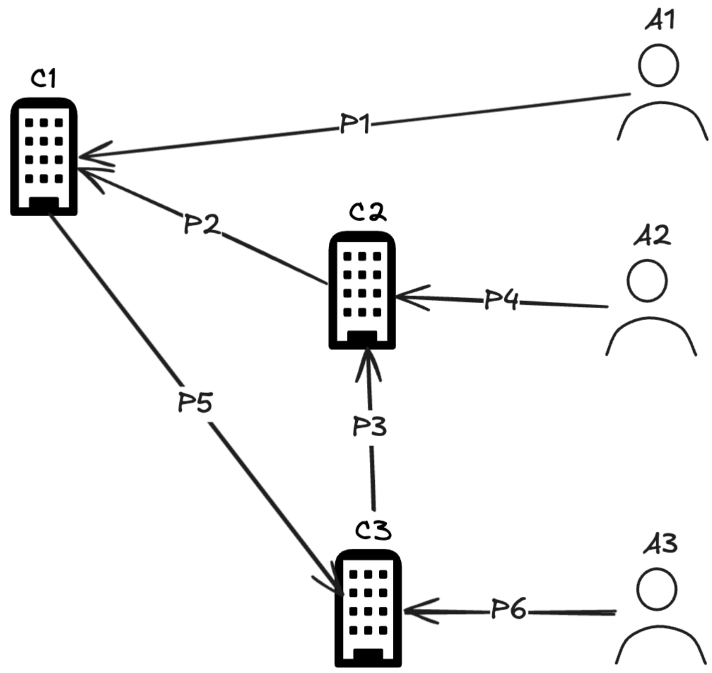

Figure 5: Recursivity of Three Companies with a Cycle $C1 \rightarrow C3 \rightarrow C2 \rightarrow C1$

3.3. Case 2: Multiple Recursivities in Three Companies

Consider three companies $(C1, C2, C3)$ with a Cycle $C1 \rightarrow C2 \rightarrow C1$, $C1 \rightarrow C3 \rightarrow C1$ and UBOs $A1, A2, A3$ (Figure 6). Applying DFS for $C1$:

- UBO A_1
  - Open path $A1 \rightarrow C1$, with direct ownership of $P1$.
- UBO A_2
  - Open path $A2 \rightarrow C2 \rightarrow C1$, with direct ownership of $P4P2$.
- UBO A_3
  - Open path $A3 \rightarrow C3 \rightarrow C2 \rightarrow C1$, with direct ownership of $P6P3P2$.
  - Open path $A3 \rightarrow C3 \rightarrow C1$, with direct ownership of $P6P5$.

Total ownership accounts for the cycle using geometric series, yielding:

- UBO $A1$: $P_1(P_1 + P_4P_2 + P_6P_3P_2 + P_6P_5)^{-1}$
- UBO $A2$: $P_4P_2(P_1 + P_4P_2 + P_6P_3P_2 + P_6P_5)^{-1}$
- UBO $A3$: $(P_6P_3P_2 + P_6P_5)(P_1 + P_4P_2 + P_6P_3P_2 + P_6P_5)^{-1}$

4. Discussion

The proposed graph-based model tackles the tricky computational challenges of identifying Ultimate Beneficial Owners (UBOs) with impressive efficiency. By mapping out corporate structures as weighted directed graphs and cleverly handling recursive cycles through geometric series, this approach turns what could be an NP-hard problem into one that operates with linear complexity $(O(n + m))$. Utilizing Depth-First Search (DFS) allows for a thorough exploration of ownership paths, while the underlying mathematical framework ensures accurate calculations of total ownership. Plus, the model's alignment with the Beneficial Ownership Data Standard (BODS) makes it even more relevant for regulatory and compliance purposes.

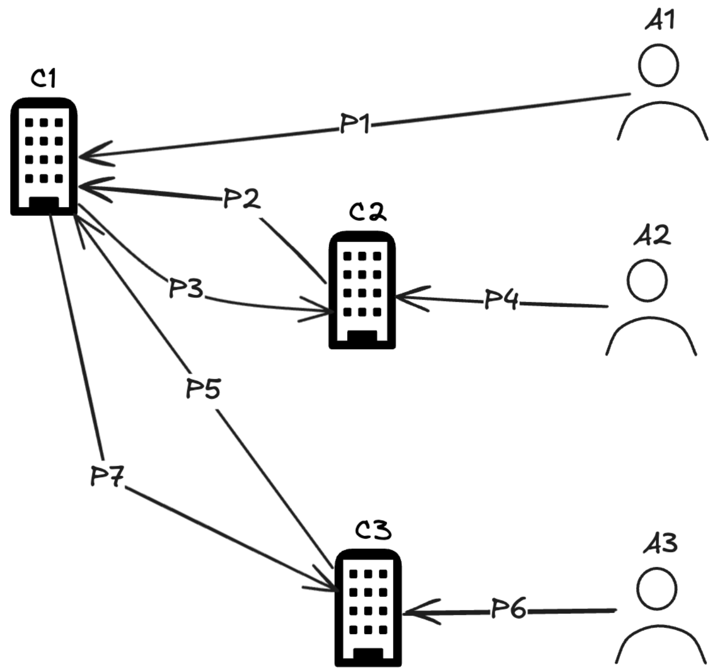

Figure 6: Recursivity of Three Companies with a Cycle $C1 \rightarrow C3 \rightarrow C2 \rightarrow C1$ and $C1 \rightarrow C3 \rightarrow C1$

5. Conclusions

Accurately identifying Ultimate Beneficial Owners (UBOs) in complex corporate structures is essential for financial transparency and combating economic crime.

Using a weighted directed graph model combined with geometric series, this approach efficiently addresses recursive ownership cycles—transforming a potentially NP-hard problem into one with linear complexity, reducing complexity from O(n!) to O(n+m).

The methodology generates concise UBO tables with precise total ownership figures and is compatible with international standards such as the Beneficial Ownership Data Standard (BODS), making it suitable for regulatory and compliance applications.

When it comes to future research, there are some exciting areas to explore. We could look into how to integrate financial risk detection, tackle the challenges of cross-border data inconsistencies, and fine-tune algorithms for real-time analysis. It would also be beneficial to validate findings across various economic sectors and examine how legal representatives, proxies, and other types of beneficiaries influence the graph model.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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