Survival Models as Copulas for green risks prediction

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Abstract

This work provides a comprehensive overview of the current state of copulas and proposes to use survival models for newly introduced families. While the copula's appearance has seen widespread application in various fields, including environmental sciences (particularly hydrology), economics, and as enhancements to existing models to improve performance. The extensive range of potential copulas, which can be systematically expanded through rigorously defined procedures such as those for the Archimedean class, underscores their versatility. However, many existing approaches emphasize forms of copulas that focus on past values, providing insights into the likelihood of events occurring before the value of interest. This paper introduces an alternative approach aimed at assessing the likelihood of events occurring in the future. This forward-looking perspective leverages the strengths of copulas more effectively, particularly in risk-sensitive fields such as environmental management. To accomplish it, the newly developed copula is shown to be transformed in a survival form.

Keywords

CEUR-WS Copulas; Survival model; Archimedean class, Green risks

1. Introduction

Copulas or copulas models are newcomers in the model class. Their first introduction and formal definition was given in the fundamental work of Sklar [1] along with the proof of their existence. Since the introductions, copulas have been steadily improving with various new classes of copulas being added to the field and new members being added to the existing classes. The flexibility of models enabled precise change and in some cases introduction of new classes on the spot. On the contrary to other models, the copulas are inherently easy to change and extend. Further we will discuss several properties that make new copulas model creation straightforward.

The theory of copulas however includes several caveats that need to be addressed. Some procedures can be done only over specific classes of copulas – like Archimedean. However, after handling the underlying challenges, the possibilities to use copulas are truly vast.

2. Literature Overview

Copulas have steadily progressed over time, with their usage expanding significantly across various fields, each presenting unique implementation nuances.

We begin our overview with a comprehensive, seminal work aimed at formulating a rigorous procedure for copula usage [2] that investigates the most recent studies. This publication, supported by illustrative examples and cautionary notes, provides valuable insights into the general state of the field. It focuses on hydrology, where copulas are becoming an increasingly vital tool. With recent advances, the modeling of diverse water processes is flourishing [3-7].

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One key advantage of copulas lies in their ability to capture complex dependence structures, particularly under extreme events such as heavy rainfall, heat waves, and floods. These events affect critical environmental variables like temperature and water distribution in non-linear ways. Several hydrological studies exemplify the robustness of copula-based approaches in such contexts, making them valuable reference guides for researchers and practitioners alike [8].

A significant body of research addresses recent advancements in copula-based methodologies within practical hydrological settings. However, as documented in these reviews, most studies emphasize the theoretical underpinnings – focusing on statistical properties and foundational theory – while often overlooking implementation strategies and their inherent constraints. Only a small fraction of these works tackles real-world challenges or aims to alleviate the adverse impacts of extreme hydroclimatic factors.

Consequently, the field still lacks a standardized, systematic methodology. Studies show that researchers seeking applied results commonly adopt substantially varied approaches, not only in their methodological steps but also in their technical implementations, even when attempting to follow similar procedures. A recent effort has endeavored to systematize information about these procedures, motivations, and considerations, offering a comprehensive overview of both theoretical limitations and practical applications. This initiative lays the groundwork for a robust copula-modeling workflow that should encompass exploratory analysis, data preprocessing, copula selection, parameter estimation, and goodness-of-fit assessments.

Several notable publications further enhance our understanding of copula theory. A monograph [3] presents fundamental concepts alongside a systematic framework for modeling dependencies in hydrology and water resources – an indispensable resource for both beginners and experienced analysts. Another investigation [4] focuses on spatial data, examining hydrological droughts and leveraging copulas to integrate geospatial mapping methods. This provides an illustrative example of copula application to spatially explicit datasets, a useful approach in arid and semi-arid regions for improved water resource planning.

Innovative usages and extensions of existing copula families are common in hydrology, given the unique features and challenges each environment presents. For instance, the study [5] couples Gaussian mixtures with copulas to model multi-peak hydrological dependencies in the Yangtze River basin. Such work underscores the importance of applied research that not only refines theory but also addresses pressing real-world constraints. Another study [6] tackles forecast uncertainty by shifting from point predictions to predictive distributions, thereby offering a clearer understanding of accuracy and precision in hydrological forecasts.

Risk management constitutes another significant domain for copula applications. Analyzing individual risk factors is comparatively straightforward; the real challenge emerges in accounting for joint dependencies and interactions. Numerous publications [8-15] explore how copulas support both theoretical and applied risk frameworks. Among these, one study [8] investigates various forms of risk modeling via copulas, illustrating how networks of states and conditions can be represented with high accuracy under certain assumptions. This systemic perspective is vital when linking local, regional, and global processes – for example, in supply chains or infrastructure networks – where compound risks can cascade.

Regarding copula families, risk management often benefits from vine copulas, renowned for their flexibility in high-dimensional settings. Paper [9] emphasizes a multi-dimensional risk approach that captures intricate dependencies and can be extended to hydrological data as well [16]. Risk can be quantified in several ways; Value-at-Risk (VaR), a common metric in finance, can also be modeled with copulas. A related study [10] addresses issues arising from non-normality of asset returns, highlighting scenarios in which specifying a joint distribution is challenging without copulas.

Copula models frequently serve as a bridge across various domains. For instance, agriculture benefits from a systematic copula-based approach to analyzing crop-related variables, as documented in [11] and [17]. Beyond these applications, several approaches exist for extending copulas themselves: via realized (time-varying) parameters [12], machine learning approaches [13,18–19], implicit copulas [18],[19], quasi-copulas [20], and Bayesian inference [21]. In engineering, copulas prove valuable in modeling temperature gradients of large-scale structures [22]. Copulas have also shown their use in handling errors and oscillations during approximation problems [23]. New classes of copula among trigonometric functions can also be viewed as promising instruments as a new way of dealing with existing problems [24].

In summary, over the past few decades, scientific research has been aiming to develop optimal models for a variety of applications. Copulas have emerged as a prominent new class of models, offering notable accuracy alongside relatively modest complexity. Advancing copula usage also depends on recognizing potential pitfalls and corner cases: even seemingly well-behaved data can mask complex dependencies. Real-world phenomena often exhibit intricate interactions among variables, making copulas an excellent choice for more sophisticated data scenarios.

Another significant work [25] explores generating a novel Archimedean copula family via a less restrictive approach than standard definitions, exemplifying how new parametric families (e.g., one based on half-logistic functions) can enter the modeling toolkit. Many such innovations stem from leveraging well-known distribution functions as linking mechanisms, allowing them to be seamlessly adapted into copula frameworks. These contributions illustrate the full cycle of copula development, from conceptual motivation to practical demonstration.

Current research shows that copulas are widely deployed in financial and environmental spheres. As emerging methods and novel copula classes continue to expand the field's capabilities, the toolkit for handling complex real-world problems grows accordingly. We expect that one of the most significant impacts of copula modeling may occur in the area of green risks − a topic thoroughly treated in [26]. This study reviews more than 40 references and provides conclusive perspectives and guidance. Given the rising emphasis on globalization and the human footprint on natural systems, effectively modeling ecological and financial data has become indispensable, underscoring the critical role copulas stand to play in shaping the future of risk analysis.

3. Definition of Copulas

Copulas are mathematical functions that bind marginal distributions to create a joint distribution, capturing dependencies between random variables. The most basic form can be obtained via definition that is given in the Sklar's theorem [1], the copulas is then function C that in the 2 dimensions satisfy the following condition:

$$H(x, y) = C(F(x), G(y)).$$

Here H is a joint distribution of the variables, and F, G are margins (marginal distributions) of the random variables.

According to the relevant theorem, for any multivariate distribution, there exists a function − under certain continuity assumptions about the marginals − that is unique in mapping these marginals to their joint distribution. Notably, the dependence structure itself is determined solely by this function, while each marginal distribution can take virtually any form. Consequently, one gains tremendous flexibility when selecting a linking function for real-world data, without needing to restrict or standardize the marginal distributions. This "modular" approach leverages the independence of each modelling component, allowing analysts to handle them separately and thereby conduct analyses in a more flexible and adaptive manner.

In N-dimensional space, a copula is also a multivariate cumulative distribution function (CDF) that links marginal distributions to a joint distribution while preserving their dependence structure. For N random variables $X_1, X_2, \dots, X_N$, each with marginal CDF $F_i(x_i)$, the copula C satisfies:

$$C(u_1, u_2, \dots, u_N) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2), \dots, F_N^{-1}(u_N)\right),$$

where: $u_i = F_i(x_i)$ are the transformed marginal probabilities, $F(x_1, x_2, \dots, x_N)$ is the joint CDF of $(X_1, X_2, \dots, X_N)$, F_i^{-1} is the inverse of the marginal CDF for X_i, also called the quantile function. Sklar's theorem states that for any joint CDF $F(x_1, x_2, \dots, x_N)$ with marginals $F_1(x_1), F_2(x_2), \dots, F_N(x_N)$, there exists a unique copula C such that:

$$F(x_1, x_2, \dots, x_N) = C\big(F_1(x_1), F_2(x_2), \dots, F_N(x_N)\big).$$

The uniqueness still holds in the case of more than 2 dimensions and for the same conditions. If underlying marginal distributions is in density form the copula density $c(u_1, u_2, \dots, u_N)$ can be obtained as:

$$c(u_1, u_2, \dots, u_N) = \frac{\partial^N C(u_1, u_2, \dots, u_N)}{\partial u_1 \partial u_2 \cdots \partial u_N}.$$

Its form is not dependent on the marginals due to the fact that arguments are values of CDFs and so allow for any underlying distribution. As per the definition and properties are given in [1], for any multivariable distribution, it is possible to write down copulas that will catch the dependencies, in theory, and if proper copula model is used, semi-perfectly.

4. Classes of copulas

Copula models can be grouped into classes that exhibit particular properties. Several widely used classes and subclasses exist, each motivated by different theoretical or practical considerations. For instance, some copulas are generated using specialized methods, thereby acquiring distinct traits – for example, Archimedean copulas, which allow specific control over tail behavior. However, the Archimedean class also carries certain limitations, notably symmetry. To address this constraint, researchers often turn to vine copulas, which can be assembled from multiple copulas and thus offer greater flexibility in capturing asymmetric dependence. In both of these cases, the manner in which copulas are constructed engenders a new class, enabling researchers to introduce specialized configurations, parametric extensions, and refined modeling approaches.

5. Archimedean Copulas

Archimedean copulas are a family or class of copulas that are widely used due to their flexibility, simplicity, and ability to capture dependence structures. They are particularly useful for modeling symmetric dependence structures since both variables have identical impact and can be switched by places without change to the result of a function.

General way of writing an Archimedean copula in N dimensions is defined as:

$$C(u_1, u_2, \dots, u_N) = \psi^{-1}\big(\psi(u_1) + \psi(u_2) + \dots + \psi(u_N)\big).$$

The features of the class arise here, since special functions are used – generator functions ψ.

The definition of the generator is as follow:

$\psi: [0,1] \to [0, \infty]$ is a generator function that is decreasing, convex, and satisfies $\psi(1) = 0$, and ψ^{-1} is the pseudo-inverse of ψ, defined as $\psi^{-1}(t) = \psi^{-1}(t) = sup\{u \in [0,1]: \psi(u) \geq t\}$.

Using different generation functions, it is possible to obtain different types of copulas. The most common are listed below:

Clayton Copula:

Generator:

$$\psi(t) = \tfrac{1}{\theta}\big(t^{-\theta} - 1\big), \theta > 0.$$

Copula:

$$C(u_1, u_2, \dots, u_N) = \left(\sum_{i=1}^{N} u_i^{-\theta} - (N-1)\right)^{-1/\theta}.$$

Gumbel Copula:

Generator:

$$\psi(t) = (-ln\, t)^{\theta}, \theta \geq 1.$$

Copula:

$$C(u_1, u_2, \dots, u_N) = exp\, exp\left(-\left(\sum_{i=1}^{N}(-\ln \ln u_i\ )^{\theta}\right)^{\frac{1}{\theta}}\right).$$

Frank Copula:

Generator:

$$\psi(t) = -\ln\left(\frac{e^{-\theta t} - 1}{e^{-s} - 1}\right), \theta \neq 0.$$

Copula:

$$C(u_1, u_2, \dots, u_N) = -\frac{1}{\theta}\left(1 + \frac{\prod_{i=1}^{N}\left(e^{-\theta u_i} - 1\right)}{(e^{-\theta} - 1)^{N-1}}\right).$$

There are key features of them. Parameter is a part of the copula and is a meta-parameter. For Clayton and Gumbel, the parameter decides strengths of dependencies – how conditional distribution changes. The different behavior of tails is also present: for Clayton it is a lower tail dependence and for Gumbel it is an upper tail dependence.

6. Vine Copulas

In case dependencies between variables are pair-wise and known, the vine copulas are applied. They are constructed using blocks – bivariate copulas and a flexible class of copulas that allows for greater and faster incorporation of knowledge pairwise dependency [27]. On the contrary to the Archimedean, they are not symmetric and can be used for a wide usage. There are several Vine copula structures:

C-Vine which is the most common form of a copula.

Formula:

$$C(u_1, u_2, \dots, u_N) = \prod_{i=2}^{N} c_{1i}(u_1, u_i) \prod_{j=2}^{N-1} \prod_{i=j+1}^{N} c_{ij|1,\dots,j-1}\left(u_i, u_j \mid u_1, \dots, u_{j-1}\right),$$

where $c_{ij|1,\dots,j-1}$ are bivariate conditional copulas.

In a D-Vine structure, variables are connected sequentially, forming a chain. The dependencies between neighboring variables are described directly, and conditional dependencies are introduced later.

Formula:

$$C(u_1, u_2, \dots, u_N) = \prod_{i=1}^{N-1} c_{i,i+1}(u_i, u_{i+1}) \prod_{j=2}^{N-1} \prod_{i=1}^{N-j} c_{i,i+j|i+1,\dots,i+j-1}\left(u_i, u_{i+j} \mid u_{i+1}, \dots, u_{i+j-1}\right).$$

The R-Vine is the most flexible, as it allows arbitrary tree structures for dependencies. Each tree layer describes pairwise copulas, and subsequent trees describe conditional dependencies.

Formula:

$$C(u_1, u_2, \dots, u_N) = \prod_{k=1}^{N-1} \prod_{i=1}^{N-k} c_{i,i+k|S}(u_i, u_{i+k} \mid u_S),$$

where S is the set of conditioning variables determined by the structure of the vine.

7. Survival function as a copula

Survival Modelling is a statistical technique used to analyze the time until an event occurs. In the context of green risks, it can be applied to: predict outcomes of some events, to define important risk factors and to receive some non-seen before dependencies and insights. We can use survival functions for green projects effectiveness evaluation: estimating the time for receiving business success or aims. Also, survival functions help us to understand which factors contribute to the likelihood of adverse events.

Survival copula is a function of a special form which is used when the main interest is in the future events and that is based on the joint survival function of random variables, instead of their cumulative distribution function (CDF). The most common approach of the survival copulas is for stress-testing analysis (scenario analysis) since it allows for better interpretation of possible trajectories which the system can achieve in its evolution under particularly extreme conditions.

Given a copula $C(u_1, u_2, \dots, u_N)$, which links the marginal distributions $F_1(x_1), F_2(x_2), \dots, F_N(x_N)$ to the joint CDF $F(x_1, x_2, \dots, x_N)$, the survival copula $C\hat{}$ is the copula associated with the joint survival function $S(x_1, x_2, \dots, x_N)$, where:

$$S(x_1, x_2, \dots, x_N) = P(X_1 > x_1, X_2 > x_2, \dots, X_N > x_N).$$

The survival copula $C\hat{}(u_1, u_2, \dots, u_N)$ satisfies:

$$S(x_1, x_2, \dots, x_N) = C\hat{}\big(1 - F_1(x_1), 1 - F_2(x_2), \dots, 1 - F_N(x_N)\big),$$

where $1 - F_i(x_i)$ represents the survival function of the marginal distributions.

The survival copula $C\hat{}$ is related to the original copula C through the following formula:

$$C\hat{}(u_1, u_2, \dots, u_N) = u_1 + u_2 + \cdots + u_N - N + C(1 - u_1, 1 - u_2, \dots, 1 - u_N). \qquad (1)$$

This formula ensures that the survival copula is derived consistently from the original copula.

8. New Survival Copula Form

The approach with generation was used in the given work [25]. The idea was to use the half-logistic regression in order to create a model with only a positive range domain. Additionally, a new parameter was introduced to scale the model and improve its flexibility. The resulting copula is expressed as:

$$C_L(u, v; \theta) = \frac{1}{\theta} ln\left[\frac{e^{\theta(1+u+v)} - e^{\theta u} - e^{\theta v} + e^{\theta}}{e^{\theta(1+u)} + e^{\theta(1+v)} - e^{\theta(u+v)} - 1}\right].$$

The newly obtained copula has shown its advantage on the hydrology dataset. However, little to no attention to the survival form is given.

In practical applications, this form is often more relevant for assessing the probabilities of extreme future events. For instance, in most situation dealing with risks, one might be interested in predicting the likelihood of a variable (e.g., for our area of research it could be the probability of default or vice versa success of financing green projects) exceeding a certain threshold under specific conditions, such as factors that characterize different limits for pollutions for some industries (water, air, CO2). Applying this transformation to the newly proposed copula yields:

Substitute $1 - u$ and $1 - v$ into $C_L(u, v; \theta)$:

$$C_L(1 - u, 1 - v; \theta) = \frac{1}{\theta} ln\left[\frac{e^{\theta(1+(1-u)+(1-v))} - e^{\theta(1-u)} - e^{\theta(1-v)} + e^{\theta}}{e^{\theta(1+(1-u))} + e^{\theta(1+(1-v))} - e^{\theta((1-u)+(1-v))} - 1}\right].$$

Simplify the terms inside the exponents:

$$1 + (1 - u) + (1 - v) = 3 - u - v \; 1 + (1 - u) = 2 - u,$$

$$1 + (1 - v) = 2 - v \ (1 - u) + (1 - v) = 2 - (u + v)$$

Substituting these back, we get:

$$C_L(1-u, 1-v; \theta) = \frac{1}{\theta} ln \left[\frac{e^{\theta(3-u-v)} - e^{\theta(2-u)} - e^{\theta(2-v)} + e^{\theta}}{e^{\theta(2-u)} + e^{\theta(2-v)} - e^{\theta(2-(u+v))} - 1}\right].$$

Substituting $C_L(1-u, 1-v; \theta)$:

$$\underline{C}(u, v; \theta) = u + v - 1 + \frac{1}{\theta} ln \left[\frac{e^{\theta(3-u-v)} - e^{\theta(2-u)} - e^{\theta(2-v)} + e^{\theta}}{e^{\theta(2-u)} + e^{\theta(2-v)} - e^{\theta(2-(u+v))} - 1}\right].$$

Which after simplification:

$$\underline{C}(u, v; \theta) = \frac{\theta(u+v-1) + \theta + ln\left(\frac{e^{2\theta} - e^{\theta u} - e^{\theta} + e^{\theta(u+v)}}{-e^{2\theta} + e^{\theta(u+2)} - e^{\theta(u+v)} + e^{\theta(v+2)}}\right)}{\theta}. \quad (2)$$

That means that we can use this formula for modelling the probability of surviving (that means financial or environmental success) of the proposed green projects which are evaluated for priority finance as well as the industry or governmental support (interactions) for stimulating the green projects aimed to receive some sustainable metrics.

In our research, we decided to evaluate green risks as the probability in time which could be presented as a survival function. Thus, we also can determine our risk's probability as it was proposed in equation (2). It gives us the possibility to evaluate green risk as an environmental risk in the form of a copula [28] as well as present it as a probability in the time interval via the survival model. It means that we will evaluate our green investment project as a function of some environmental and financial parameters that can change and vary during the time of consideration. Overall, survival models are widespread and can be used in various domains to model complex behavior with relatively high levels of success [29].

9. Green Risks modelling via Copulas

Green Risks are environmental risks that can significantly affect each kind of business on the industrial level, each economy on the country level as well as ecosystems on a global level [30]. It means that financial losses will appear due to regulatory changes, natural disasters, or resource scarcity. We can observe the huge amounts of damage in different parts of the world which have affected a lot of countries. Economic downturns linked to environmental degradation or climate change totally increase the size and parts of the world (USA, Turkey, Spain). Today we observe different types and sources of green risks such as: climate change (extreme weather events, rising sea levels, and changing weather patterns), pollution (air, water, and soil contamination affecting health and productivity), resource depletion (overuse of natural resources leading to scarcity), biodiversity loss (extinction of species impacting ecosystem health and resilience).

In the context of green risks, copulas can be particularly useful for modeling the joint behavior of multiple environmental and financial factors, helping to assess the overall risk profile. Let's propose the first view on the main stages for implementing survival copula for the green risk's projects evaluation.

Stages for implementation survival copula form for predicting green risks

1st step. To define marginal survival functions for green risks.

Each type of green risk group factors should have its own survival function. We can firstly simplify and define that only environmental and financial group factors [31] influence green risk. They have different nature and that's why should be modelled by different type of survival functions:

$S_1(t) = 1 - P(x_1, x_2, \dots, x_n)$ – survival function for environmental risk, where $x_1, x_2, \dots, x_n$ - factors which characterise environmental indicators after implementing a green project (for example CO_2, water, air pollution), P – is a probability of environmental risk.

$S_2(t) = 1 - P(y_1, y_2, \dots, y_k)$ – survival function for financial risk of the green project, where $y_1, y_2, \dots, y_k$ - factors which characterise financial risk of the implementation of proposed green projects (it could be financial stability, credit factors, liquidity), P — is a probability for financial risk.
2nd step. To choose the type of copula for modelling dependencies between survival functions defined above.
It could be Gaussian, Archimedean, Vine copula, which have been described earlier.
3rd step. Construct the joint survival function for both risks of green projects.
The joint survival function could be defined as follow:

$$S(t_1, t_2) = C(S_1(t_1), S_2(t_2)), \quad (3)$$

where $S_1(t_1), S_2(t_2)$ — are the marginal survival functions, C — is the copula function that captures the dependency between the two survival functions.
4th step. Based on the historical data which characterize environmental and financial risks we evaluate the parameters of the copula.
5th step. Implement developed survival copula models for evaluating new green projects and environmental and financial risks for them.
In our earlier work [28], we described the perspective and possibility of using copulas for different types of financial risks. We can build and try to use them on real data for financial risks prediction, changing the type of copula depending on circumstances. Now we can present the possibility of green risks prediction with a usage of surviving functions as a model which includes time-variable and can evaluate effectiveness of proposed green projects in time independently for environmental and financial aspects. It means that we can forecast in how many months we achieve limits for environmental indicators and aims (quotes) as well as how many months we receive profitability of green projects. For example, for the green projects which focus on green solar energy we can evaluate some metrics (sustainable development indicators) that we expect to exceed in 1-2 years as well as to calculate in how many months financial profit will be received and given credit will be paid back.
These environmental and financial groups of factors could be modelling the different types of survival functions. For the environmental group of factors which exponential increase the threat of environmental changes and risks we will use exponential type of survival model such as:

$$S_1(t) = e^{-\lambda_1 t_1},$$

and for financial group of factors we will use the survival function as Weibull model:

$$S_2(t) = e^{-\lambda_2 t_2^2}.$$

The joint survival function for green risks can be expressed as: $S(t_1, t_2) = C(S_1(t_1), S_2(t_2))$. If we determine copula as Gaussian copula:
$C(u_1, u_2) = \Phi_\Sigma(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$,
where Φ — cumulative distribution function of the standard normal distribution, Φ_Σ — the joint cumulative distribution function of a multivariate normal distribution with a specified correlation structure.

So, survival function for green risks in such definitions will be:

$$S(t_1, t_2) = C\big(S_1(t_1), S_2(t_2)\big) = \Phi_\Sigma\left(\Phi^{-1}\big(S_1(t_1)\big), \Phi^{-1}\big(S_2(t_2)\big)\right).$$

Using mathematical copulas for green risks modelling gives us the possibility for deeper understanding of complex dependencies between various environmental and financial factors.

10. Conclusions

Overall, the advancements in the field of development and application copulas are promising, both in terms of theoretical developments and practical applications. Existing tools have enabled researchers to generate copulas for a wide variety of scenarios, and real-world case studies have demonstrated impressive results. Fields such as environmental science and economics have particularly benefited from the implementation of copulas, where they have been instrumental systems in modelling complex dependencies and improving predictive accuracy.

The proposed copulas hold significant value, facilitating decision-making and enhancing risk management processes. By refining these approaches further, it is possible to make copulas more applicable to real-world scenarios while reducing the technical expertise required by users. Additionally, employing the survival form of copulas simplifies computations, allowing for more straightforward risk calculations and making these models more accessible for practical use.

Thus, the approach to use new forms of copulas for green risks looks really promising while it gives the possibility not only to define the nature and factors of risks as well as to predict the time-period and as follows to mitigate risks. In future developing this approach and implementing for evaluation the perceptiveness and financial capacity of the green projects ultimately support increasing and appearance of more projects for sustainable development and resilience against environmental challenges. Proposed in the paper, the general method and main stages for implementation survival copula form for predicting green risks will be further developed and practically tested on real data which will include both environmental and financial data.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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