Identification of parameters of the mathematical model of a dynamic object of critical infrastructure

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Abstract
This paper addresses the problem of identifying the parameters of a mathematical model for a dynamic system based on its acceleration curve. The system is modeled using a high-order transfer function. An algorithm is developed to estimate the unknown parameters, and its performance is evaluated with respect to the number of data points in the original curve. The proposed identification method targets the unknown coefficients of the characteristic polynomial and employs a geometric approach that involves numerically integrating the area under the error function curve. The paper also examines the challenges associated with implementing the algorithm. A computational example is presented, and statistical regression analysis confirms both the adequacy of the model and the effectiveness of the proposed method.

Keywords
dynamic object, numerical integration, identification of parameters

1. Introduction

In the development of automatic control systems for critical infrastructure, having a precise representation of the plant is highly advantageous. Yet, this is not always feasible. Frequently, both the structure and the parameters of a dynamic system's model remain uncertain, and analytical derivation is impossible. Under such conditions, experimental identification techniques provide an effective alternative. Existing identification approaches are typically based on frequency response analysis [1], impulse transient characteristics [2], or regression analysis [3].

A key feature of using frequency characteristics is the need for specialized equipment to measure the amplitude ratio (output to input) and the phase shift between input and output sinusoidal signals. Such devices may not always be available or sufficiently accurate. The impulse transient method may also require knowledge of logarithmic characteristics, which can complicate its application. The third approach, regression analysis, relies on data processing using the least squares method.

An alternative approach presents the designer with a dilemma: either to synthesize the control system using learning methods that adapt to existing operating conditions, or to perform system identification, in which the structure and parameters of the dynamic object are clarified. In such cases, establishing the model's structure and parameters may be less complex. Consequently, approaches that provide reasonable accuracy while maintaining simplicity of implementation are particularly attractive. This study emphasizes the identification of the control object's mathematical model through a geometric method.

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2. Paper Review

Parametric optimization is a key area of modern research, widely used to address a variety of engineering problems. These methods typically rely on output measurements and simplified mathematical models. Examples of such studies can be found in [1–3]. For instance, Sharma et al. [1] proposed a method for identifying power plant parameters using output data measurements. Murotsu [2] identified a mathematical model of a robot using only the linear angular velocities and accelerations of a freely moving and rotating satellite platform. The results of identification are often used to tune controllers. For example, Osadchyy et al. [3] developed an identification method based on a simplified model of the control object to adjust the parameters of a PID controller.

Raskin [4] proposed an original method for identifying the state of an object, based on a non-productive inference mechanism that replaces traditional production rules with probability distributions of states.

Most developed approaches are based on the least squares method. To improve identification accuracy, researchers often introduce various enhancements. For example, using the least squares method, Dozein et al. [5] developed a technique for identifying dynamic models of photovoltaic distributed converters and entire systems, accounting for time delays. Tyuryukanov et al. [6] proposed a method to ensure slow coherence in a small group of generators through their evaluation, detection, and improved aggregation.

Identification methods based on linear regression models are presented in [7, 8]. Cardona and Serrano [7] carried out the identification of a quadcopter UAV's nonlinear dynamics through autoregressive modeling, employing GNU Octave software tools. In another study, Rubaiyat et al. [8] suggested an approach for parameter estimation in mathematical models formulated by partial differential equations. Their approach uses a cumulative distribution of the signed type, transforming the parameter estimation problem for a nonlinear system into a linear regression task.

A natural approach to solving such problems is to incorporate existing constraints on the system parameters. For example, the recursive identification procedure proposed by Ping et al. [9] takes into account restrictions on the transmission coefficient and is applied to systems with low-quality measurements. Zeng et al. [10] established a necessary and sufficient condition for the accuracy of identification systems based on sampled data, using the Koopman operator as the foundation of their analysis.

The robustness of the developed algorithms in reducing measurement errors is improved through the use of projective methods. In particular, the affine sign projection algorithm enhances the identification process under measurement noise conditions, as presented by Chu [11]. This approach incorporates both least squares and hybrid immune methods. Wang and Han [12] proposed an affine projection algorithm based on the least mean squares (LMS) algorithm and a higher-order error power criterion. Li et al. [13] introduced an adaptive algorithm for system identification in the presence of impulse noise affecting the input signal. Subsequently, Li et al. [14] developed an adaptive affine projection-type algorithm, which incorporates a cost function tailored to input signals distorted by impulse noise.

Camlibel et al. [15] proposed a linear system identification method based on an online experimental framework, where input signals are selected iteratively and guided by previously collected data samples. A key drawback of this approach is its inherent complexity, which exceeds that of methods relying on constant input data or simpler online experimental procedures.

Hao et al. [16] proposed a parameter identification method for dynamic models using a multi-strategy nonlinear RIME algorithm. Kadupitiya et al. [17] introduced a scoring system for solving problems presented in both numerical and textual formats. To reduce the computational complexity of system identification, Kang and Ahn [18] developed an iterative nonlinear identification scheme employing a moving window approach. Jin and Baek [19] achieved significant reductions in computational cost through their work on the indirect estimation of excitation forces in reduced-order system models.

Another challenging class of systems to identify are nonlinear systems, which include switching-type systems. Zheng et al. [20] proposed an iterative identification algorithm for a high-speed train model represented as a second-order switching dynamic system. This model combines two subsystems: a fast-switching nonlinear static component and a non-switching linear dynamic component. Cheng et al. [21] developed an algorithm for identifying the vibration load of a continuous system, based on numerical integration using the SSM-Newmark-β method. Ren et al. [22] addressed the recursive identification of a nonlinear, nonparametric system characterized by a finite impulse response, where measurements are taken using an event-triggered scheme. Their approach relies on binary identification and stochastic approximation techniques. Juraev et al. [23] proposed a method for identifying nonlinear systems using digital information about the system state, obtained in discrete form.

A modern approach to parametric identification leverages various learning algorithms, neural networks, and evolutionary strategies. For example, Lv et al. [24] proposed a recursive neural network for identifying system dynamics in cases where there is no feedback between input and output, particularly for systems with unknown order or delay. Liu et al. [25] introduced a deep learning-based fault detection scheme for industrial processes, which integrates multiple denoising autoencoders with a Softmax classifier. The identification process involves the use of a sparse denoising autoencoder, while the Softmax classifier is optimized using a state transition algorithm. Sun et al. [26] addressed identification challenges in dynamic switched networks (SDNs), modeled as switched linear dynamic systems, and proposed a sufficient condition for identification based on time shifts, formulated in the Lyapunov matrix form. Fabiani et al. [27] focused on designing a machine learning model for an unknown dynamic system using a finite set of state–input data points. Rukkaphan and Sompracha [28] tackled the optimal identification of a control system modeled by a fractional-order pressure process, employing the cuckoo search algorithm to find optimal parameters within a constrained space. Han et al. [29] proposed an approach combining dataset-enhanced learning with particle swarm optimization to identify nonlinear dynamic systems.

The analysis of the presented studies highlights the need for developing simple, accessible, and effective methods for identifying unknown parameters of dynamic systems.

3. Problem statement

Consider a set of points x_i, where $i = 1, \ldots, N$, obtained from an experiment. The experiment involves measuring the output of the system under study at fixed discrete time intervals T. These measurements reflect the system's response to a standard input stimulus, such as a unit step function. For research purposes, the collected values are normalized to fall within the range $x_i \in [0, 1]$.

The analysis is carried out under the assumption that the measurements are free of noise and that external disturbances can be disregarded. A fixed observation window $t \leq T_{max}$ is defined to guarantee the acquisition of a sufficient number of data samples. Furthermore, the system under investigation is presumed to be representable by a transfer function of the following structure:

$$W_{\text{M}}(s) = \frac{1+b_1 s+b_2 s^2+\cdots+b_m s^m}{1+a_1 s+a_2 s^2+\cdots+a_n s^n}, \quad (1)$$

where a_i and b_i are the coefficients to be determined, with $a_i \geq 0$, $b_i \geq 0$, and s is the Laplace transform operator.

The goal of the study is to determine the coefficients a_i and b_i of the transfer function such that its impulse response closely approximates the experimental measurement data. The accuracy of this approximation is assessed using the squared integral error criterion over the observation interval, defined as:

$$I(N) = \sum_{i=1}^{N} (h[i] - h^*[i])^2, \quad (2)$$

where h^* and h represent the numerical values of the impulse response obtained from the experiment and the transfer function, respectively. The adequacy of the model is further validated through statistical analysis.

4. Features of method implementation

The identification of the structure and parameters of dynamic systems is performed by analyzing the transition function, also known as the acceleration curve, obtained in response to a unit step input. The parameters estimated include the system's delay time, gain coefficient, and inertia – assuming the system can be described by a first-order differential equation. The presence of inflection points in the transition function indicates that the system is of higher order. In practical applications, it is typically assumed that the order of the system's characteristic equation does not exceed three.

In certain cases of automatic control system design, it is more practical to use a transfer function instead of a differential equation, as it can fully represent the system dynamics. In such cases, the area method can be employed to determine the structure and parameters of a transfer function of the form (1). This method enables the estimation of the unknown coefficients in transfer function (1) using the normalized acceleration curve $h(t)$ of the control object. A necessary condition for the physical realizability of the transfer function is the inequality $n \geq m$.

4.1. Fundamentals of the method

The method is based on the Taylor series expansion of the function inverse to (1) in the vicinity of the point $\xi = 0$, expressed as:

$$\frac{1}{W_{\text{M}}(s)} = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}{1 + b_1 s + b_2 s^2 + \cdots + b_m s^m} = 1 + S_1 s + S_2 s^2 + \cdots + S_k s^k + \cdots \tag{3}$$

At the same time $W_{\text{M}}^{-1}(0) = S_0 = 1$.

The coefficients S_k are determined by subtracting the right-hand side of equation (3) from both sides, bringing the expression to a common denominator, and equating the coefficients of like powers of s. This procedure yields the following expression:

$$a_k = b_k + S_k + \sum_{i=1}^{k-1} b_i S_{k-i} \tag{4}$$

Equation (4) involves $(n + m)$ unknown parameters. Consequently, in order to compute the coefficients a_k and b_k, it is required to form a system of independent equations by representing the variables S_k from equations (3) and (4) in terms of these coefficients, for $k = 1,2,3,\ldots$

To evaluate S_k, the deviation function $\varepsilon(t)$ is introduced, defined as the difference between the transient response $h(t)$ and the input signal $1(t)$. According to equation (3), this relation can be expressed in terms of their Laplace transforms, i.e., $L\{\varepsilon(t)\}$ [30, p. 8.4].

$$L\{\varepsilon(t)\} = L\{1(t) - h(t)\} = L\{1(t)\} - L\{h(t)\} = \frac{1}{s} - W_{\text{M}}(s)\frac{1}{s} = \frac{1 - W_{\text{M}}(s)}{s} = E(s). \tag{5}$$

Note that equation (5), by the definition of the Laplace transform [30], takes the form of an integral given by:

$$E(s) = \int_0^{\infty} \varepsilon(t) e^{-st} dt \tag{6}$$

If e^{-st} is expressed as an alternating series, then equation (6) can be rewritten as:

$$E(s) = \int_0^\infty \varepsilon(t)dt - s\int_0^\infty \frac{t\varepsilon(t)}{1!}dt + s^2\int_0^\infty \frac{t^2\varepsilon(t)}{2!}dt - \dots + s^i\int_0^\infty \frac{(-t)^i\varepsilon(t)}{i!}dt + \cdots \quad (7)$$

Series (7) can alternatively be expressed as an infinite power series, as shown in [31]:

$$E(s) = \mu_0 + \mu_1 s + \mu_2 s^2 + \cdots = \sum_{i=0}^{\infty} \mu_i s^i, \quad (8)$$

in which the weighting coefficients μ_i are written in the form

$$\mu_i = \int_0^\infty \frac{(-t)^i \varepsilon(t)}{i!} dt. \quad (9)$$

Stability of the system implies that all roots of the characteristic equation of model (1) have negative real parts, lying in the left half of the complex plane. Under this condition, series (8) is convergent, and therefore the coefficients μ_i take finite values.

To establish the relationship between the coefficients μ_i and S_i, we use equation (5), substituting $E(s)$ from equation (8) and the inverse model $W_{\text{M}}^{-1}(s)$ from equation (3). As a first step, we rewrite equation (5) in the following form:

$$\left(1 - sE(s)\right)W_{\text{M}}^{-1}(s) = 1. \quad (10)$$

After performing the specified substitutions, we obtain the following expression:

$$1 - \mu_0 s - \mu_1 s^2 - \mu_2 s^3 - \cdots + S_1 s - \mu_0 S_1 s^2 - \mu_1 S_1 s^3 - \cdots + S_2 s^2 - \mu_0 S_2 s^3 - \cdots + S_3 s^3 = 1. \quad (11)$$

After reducing similar terms and equating the coefficients at the same powers (11) of the variable s, we can obtain S_k from (3) in the form of a recurrence relation in the form

$$S_k = \mu_{k-1} + \textstyle\sum_{i=0}^{k-2} \mu_i S_{k-i-1}. \quad (12)$$

4.2. Identification algorithm

An algorithm for identifying the parameters of a mathematical model of type (1) is proposed:

1. From the measurements of the transition function $h(t)$ of the considered mathematical model, given as a set of discrete points over the interval $t \in [0, T_{max}]$, the values of $\varepsilon(t)$ are obtained according to expression (5).
2. The controlled object is represented by a mathematical model expressed as a transfer function of type (1), under the condition b_i=0.
3. The coefficients μ_i are determined through numerical integration of expression (9), for i=0, 1, 2,…. The coefficients S_k are then computed using the recursive relation (12), and the coefficients a_k are obtained from expression (4), for k = 1, 2, 3,….
4. The transition function of the obtained model is investigated, and its adequacy is determined.

5. Simulation

Consider a system whose behavior is described by a third-order transfer function expressed as follows:

$$W(s) = \frac{1}{a_3 s^3 + a_2 s^2 + a_1 s + 1} \quad (13)$$

It is assumed that the system is controllable, the roots of the corresponding characteristic polynomial have negative real parts, and the coefficients a_1, a_2, and a_3 take the following values: a_1 = 7, a_2=10.25, and a_3=1.25.

For the system under consideration, the transition function can be obtained in analytical form. This function will be used to determine the points of the acceleration characteristic. The normalized values of the transition function can be calculated using the following expression:

$$h(t) = 1 + 0.0095e^{-7.4641t} + 0.3157e^{-0.5359t} - 1.3252e^{-0.2t} \quad (14)$$

The graph of the transition functions (5) is shown in Figure 1.

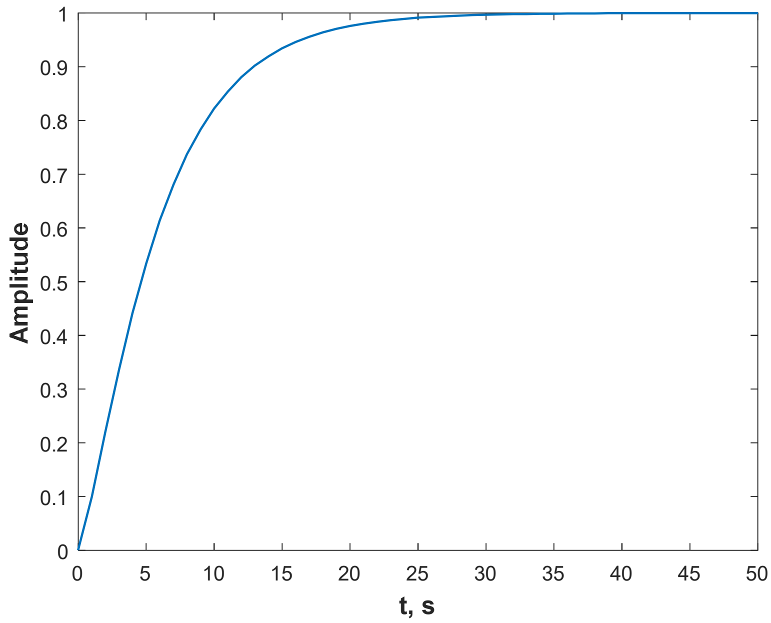

Figure 1: The system's acceleration characteristic is obtained according to equation (14).

Applying the geometric approach requires numerical integration of ε(t), the function describing the deviation from the system's steady state. Figure 2 illustrates the behavior of ε(t) for $t \in [0, 50]$.

Numerical integration was carried out using two methods: the trapezoidal method and the parabolic (Simpson's) method. The results of the transfer function coefficient estimation, based on the proposed approach, are presented in Table 1.

Table 1
Calculation of W(s) by numerical integration

Parameter	Real value	Trapezoid method		Parabola method		
		$\Delta t = 5$	$\Delta t = 2$	$\Delta t = 5$	$\Delta t = 2$	$\Delta t = 1$
μ_0	7	6.2388	6.0597	11.5120	8.0919	7.0465
μ_1	-39	-7.2016	-19.8914	-13.7408	-22.4131	-38.2203
μ_2	200	15.5514	59.5734	17.7082	60.3439	198.3902
a_1	7	6.2388	6.0597	11.51	8.0919	7.0465
a_2	10.25	31.7207	16.8284	118.8	43.0660	11.4324
a_3	1.25	168.5205	41.0126	1227	227.4658	9.6299

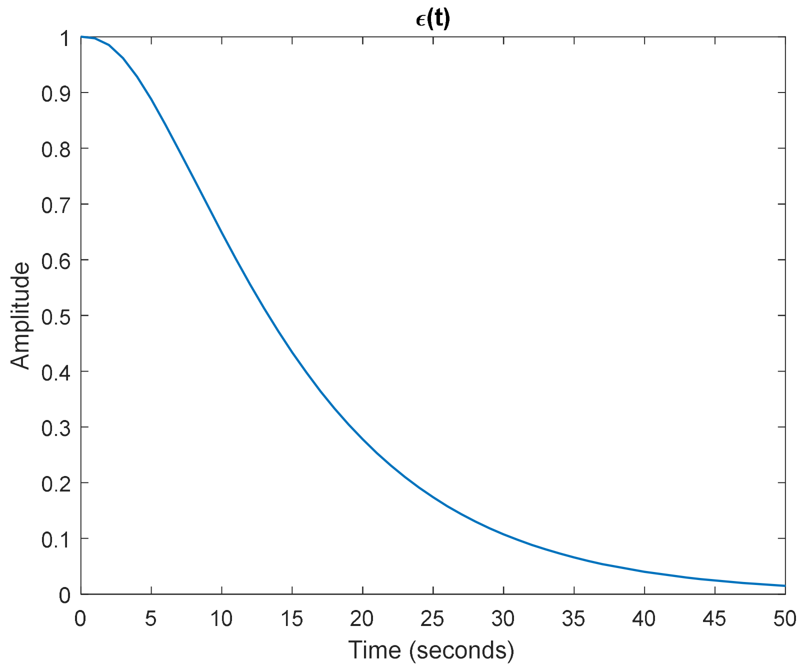

Figure 2: Function ε(t).

The data presented in Table 1 indicate that, for both the trapezoidal and parabolic integration methods, the accuracy of the results is influenced by the step size Δt; in particular, smaller steps lead to higher accuracy. Because a recursive procedure is employed to compute the coefficients a_k, the precision diminishes with increasing k due to the accumulation of numerical errors. Negative coefficient values should be discarded, which may justify reducing the model order.

The results obtained using the trapezoidal method with $\Delta t = 2$ and the parabolic method with $\Delta t = 1$ are the closest to the actual values. Therefore, two identification models can be used, with their corresponding transfer functions having the following forms. The first mathematical model, derived using the trapezoidal method, is given by the transfer function:

$$W_1(s) = \frac{1}{41.0126s^3 + 16.8284s^2 + 6.0597s + 1} \tag{15}$$

and the second, with the transfer function

$$W_2(s) = \frac{1}{9.6299s^3 + 11.4324s^2 + 7.0465s + 1}. \tag{16}$$

The transition functions are constructed based on the points of the acceleration function (14) and the identified models (15) and (16). The graphs of these step responses are shown in Figure 3.

Table 2 presents the evaluation of identification accuracy based on criterion (2).

Table 2
Calculation of W(s) by numerical integration

Method	Δt	Criterion $I(N)$
trapezium	5	0.1218
	2	0.0773
Parabola	5	1.8691
	2	1.7011
	1	0.0047

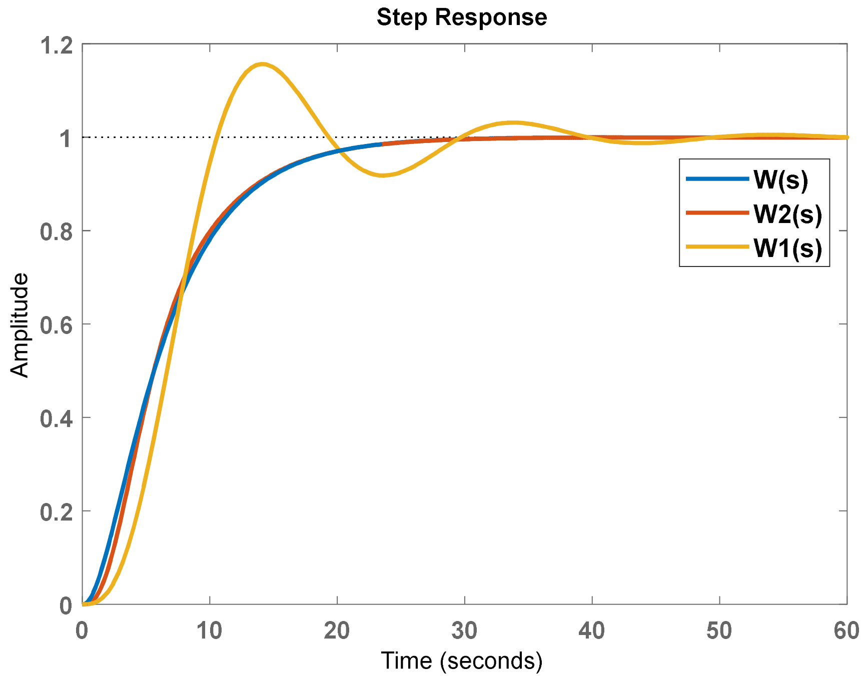

Figure 3: Transient characteristics of the models, represented by transfer functions (13), (15), and (16).

The analysis of Figure 3, supported by the results in Table 2, indicates that the most accurate identification was achieved using the parabolic method with a data sampling interval of Δt = 1.

6. Assessing the adequacy of the model to the object

We will evaluate the adequacy of the $W_2(s)$ model based on the type of regression line, the equation of which is written in the form

$$y_{\text{M}} = a + by_0. \tag{17}$$

In this scenario, the unknown coefficients a and b in equation (17) are determined using the least squares method, and then their statistical analysis is performed using the confidence probability α = 0.95.

First, the estimates of the coefficients a and b are computed using the following formulas [32]

$$\hat{b} = \frac{N\sum y_o y_{\text{M}} - \sum y_o y_{\text{M}}}{N\sum y_o^2 - (\sum y_o)^2}, \tag{18}$$

$$\hat{a} = \frac{\sum y_{\text{M}} - \hat{b}\sum y_o}{N} \tag{19}$$

where N denotes the total number of points along the curve, N=51.

The following data were obtained under experimental conditions: $\hat{b} = 0.4072$, $\hat{a} = 0.5168$.

Let us now check the significance of the obtained coefficients. To do this, we estimate the sample variance y_o using the formula

$$S_{y_o}^2 = \frac{1}{N-1}\sum(y_o - \overline{y}_o)^2 = 0.0621, \tag{20}$$

where $\overline{y}$ is the mean, $S_{y_o} = 0.2492$, and the variance y_{M}

$$S_{y_м}^2 = \frac{1}{N-2}\sum(y_м - \hat{y}_м)^2 = 0.0216, \tag{21}$$

where $\hat{y}_м = \hat{a} + \hat{b}y_o$, $S_{y_м} = 0.147$. Next, we calculate

$$S_b = \frac{S_{y_м}}{S_{y_o}\sqrt{N-1}} = \frac{0.147}{0.2492\sqrt{50}} = 0.0835, \tag{22}$$

$$S_a = S_{y_м}\sqrt{\frac{1}{N} + \frac{\overline{y}_o^2}{(N-1)S_{y_o}^2}} = 0.0756. \tag{23}$$

For the confidence level α=0.95, we have $t_{\frac{1-0.95}{2}}(N-2) = t_{0.975}(49) = 2$.

We check the significance of the coefficient $|b|$=0.4072 > $t_{0.975}(49) \cdot S_b = 0.167$.

Therefore, with a reliability of α = 0.95, we conclude that the regression coefficient is significant.

We test the hypothesis about the equality of the regression coefficient b_0 = 0.4:

$$|b - b_0| = 0.0072 < t_{0.975}(49)S_b = 0.167$$

Therefore, the hypothesis is not rejected. Then, the confidence interval for b is

$$0.2402 \leq b \leq 0.5742.$$

We will solve this problem for the coefficient a in a similar way. We check the hypothesis a=0.5: $|a|$=0.5168 > $t_{0.975}(49) \cdot S_a = 0.0432$.

Therefore, the coefficient a with a probability of 0.95 does not differ significantly from 0.5. Thus, its value can be equated to 0.5. The two-sided confidence interval for a has the form

$$\hat{a} - t_{0.975}(49) \cdot S_a \leq a \leq \hat{a} + t_{0.975}(49) \cdot S_a. \tag{24}$$

$$0.3656 \leq a \leq 0.668.$$

Thus, the regression equation $y_м$ by y_o is adequately represented by the equation $y_м = 0.5 + 0.4072 y_o$.

According to Fisher's criterion, $F = \frac{S_{y_м}^2}{S_{y_o}^2} = 0.348$.

The tabular value of F_T is obtained using the FINV function of the Excel package. In this case, we have $F_T(0.05;49;49)$=1.61. Since $F < F_T$, we conclude that the compared variances are indistinguishable and, therefore, that the regression equation is adequate. This conclusion allows us to increase confidence in the constructed model.

7. Conclusion

The paper deals with the problem of determining the parameters of a dynamic system using a model of a specified form via an identification procedure. The proposed method employs a geometric approach, where the area under the curve of the derivative of the acceleration deviation function, with respect to a given state, is obtained using numerical integration. The analysis shows that, among the tested methods, the parabolic (Simpson's) method achieves the highest identification accuracy when the number of acceleration data points is held constant.

The proposed algorithm may be suitable for determining the initial values of the tunable coefficients of a PID controller, aligning with one of the tuning approaches described in [33–35]. This can be particularly beneficial for control systems used in critical infrastructure applications.

Future research should focus on improving the accuracy of the method, with the goal of ensuring that the roots of the characteristic polynomial of the identified model closely approximate those of the original dynamic system.

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Declaration on Generative AI

In the course of preparing this work, the authors utilized Grammarly to check for grammatical and spelling errors. The content was subsequently reviewed and revised where needed, with the authors taking full responsibility for its accuracy.

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