Minimal and Credibility-Limited Iterated Revision of Deductively Closed Lockean Belief Sets

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Abstract

This short contribution presents some results on a characterization of those belief sets that are definable by the Lockean thesis and that are deductively closed. Furthermore, results on minimal revision and credibility-limited iterated revision are presented.

Keywords

Lockean thesis, deductive closure, belief revision, credibility-limited iterated belief revision

1. Introduction

The *Lockean thesis* is that philosophical principle according to which *beliefs* of rational agents are definable in terms of *confidence* and, more precisely, within the (subjective) probabilistic setting.

The principle was formulated by Foley in [1] as follows: fix a finite language $\mathcal{L}$ over the signature of classical propositional logic; it is rational for an agent to believe in a statement $\varphi \in \mathcal{L}$ provided that the agent's subjective probability $P(\varphi)$ of φ overcomes a certain threshold λ that is strictly greater than $1/2$.

Lockean thesis: For any $\varphi \in \mathcal{L}$, it is rational to believe in φ provided that its probability $P(\varphi)$ overcomes a certain threshold λ.

Given a threshold λ and a probability function P, one can hence define a *Lockean belief set*, as:

$$\mathscr{B}_{\lambda,P} = \{\varphi \in \mathcal{L} : P(\varphi) \geq \lambda\}. \tag{1}$$

Lockean belief sets can hence be used to represent the epistemic states of rational agents, and they have been already considered as a basis for theories of probabilistic belief change. The idea of grounding belief change on Lockean belief sets is in fact not new and it has been proposed, explored and investigated by others. Among them, it is worth recalling the following ones, whose ideas inspired the present paper: [2] by Shear and Fitelson; [3, 4] both by Hannson; [5] by Leitgeb and [6] by Cantwell and Rott who, similarly to what we also propose in the present paper, adopt (a variation of) Jeffrey conditionalization to deal with the belief revision process.

The elementary observation that if a rational agent believes φ then it is reasonable to assume that she does not believe its negation $\neg\varphi$ forces the threshold λ to be strictly above $1/2$. Indeed, this requirement implies that belief sets defined as in (1) do not contain pairs of contradictory formulas, in the sense that it is not the case that there exists a formula φ such that φ and its negation $\neg\varphi$ both belong to a Lockean belief set $\mathscr{B}_{\lambda,P}$.

Although they are not contradictory in the above sense, Lockean belief sets lack some properties that are usually required in the classical approaches to belief change as developed by Alchourrón, Gärdenfors,

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and Makinson [7]; Katsuno and Mendelzon [8] etc. One of them is *deductive closure*. Denoting by $\vdash$ the consequence relation of classical propositional logic, a belief set $\mathscr{B}$ is closed under logical deduction if $\mathscr{B} \vdash \varphi$ implies $\varphi \in \mathscr{B}$.

A characterization of those Lockean belief sets that are closed under logical deduction has been recently presented in the paper [9] of which the present abstract collects the main results. Those concerning the aforementioned characterization will be briefly recalled in the next Section 2. In Section 3 we will present results again from [9] on a way to change Lockean belief by a minimal revision operator. In Section 4 we will conclude presenting some ongoing research on the iteration of the revision operator in the spirit of Darwiche and Pearl [10].

2. Deductively closed Lockean belief sets

The basic ground of our investigation is classical propositional logic (CPL) and our language $\mathcal{L}$, up to redundancy, is built from a *finite* set of propositional variables, say $x_1, \ldots, x_n$, connectives $\wedge, \vee, \neg$ for *conjunction*, *disjunction*, and *negation* respectively, and constants $\top, \bot$ for *true* and *false*. Formulas in that language will be denoted by lower case Greek letters $\varphi, \psi, \ldots$ with possible subscripts. The connective of *implication* is defined as $\varphi \to \psi = \neg\varphi \vee \psi$, and *double implication* is $\varphi \leftrightarrow \psi = (\varphi \to \psi) \wedge (\psi \to \varphi)$. As anticipated above, the consequence relation of CPL is denoted by $\vdash$. From the semantic point of view, every formula φ can be identified with the set $[\![\varphi]\!]$ of its models, i.e., those logical valuations $\omega : \mathcal{L} \to \{0, 1\}$ such that $\omega(\varphi) = 1$, also written $\omega \models \varphi$. By Ω we will henceforth denote the finite set of logical valuations of $\mathcal{L}$. For all $\varphi \in \mathcal{L}$, $\uparrow[\![\varphi]\!]$ denotes the set of subsets of Ω that contains $[\![\varphi]\!]$, that is to say, $\uparrow[\![\varphi]\!]$ is the *filter* of 2^Ω, the algebra of parts of Ω, principally generated by $[\![\varphi]\!]$.

Probability distributions will be defined on Ω as usual and hence the corresponding probability measures, that are denoted by the same symbol, will be defined additively on 2^Ω. For every formula $\varphi \in \mathcal{L}$ we adopt the convention of defining the probability of φ as the probability of the set of models of φ, i.e. we set $P(\varphi) = P([\![\varphi]\!]) = \sum_{\omega \in [\![\varphi]\!]} P(\omega)$. By a positive probability distribution (measure), we mean a distribution (resp. measure) P such that $P(\omega) > 0$ for all $\omega \in \Omega$ ($P(\varphi) > 0$ for all $\varphi \neq \bot$).

The first result introduces a notational convention that will be often used throughout the paper.

Lemma 1. *For a positive probability P on 2^Ω and $\lambda > 1/2$, there exist a minimal λ_m and a maximal λ_M such that: $\lambda_M = P(\psi)$ for some $\psi \in 2^\Omega$, $\lambda_m < \lambda \leq \lambda_M$, and $\mathscr{B}_{\lambda',P} = \mathscr{B}_{\lambda,P}$ for every $\lambda' \in (\lambda_m, \lambda_M]$.*

We say that a belief set $\mathscr{B}_{\lambda,P}$ is *trivial* when $\mathscr{B}_{\lambda,P} = \uparrow[\![\top]\!]$, that is to say, $\mathscr{B}_{\lambda,P}$ is trivial if it only contains logical theorems. Note that if P is a positive probability and $\mathscr{B}_{\lambda,P}$ is non-trivial, then $\lambda_M < 1$.

The following result is a characterization of those Lockean belief sets that are deductively closed.

Theorem 1. *For a positive probability $P : 2^\Omega \to [0, 1]$ and $\lambda > 1/2$, the following conditions are equivalent:*

1. *$\mathscr{B}_{\lambda,P}$ is deductively closed,*
2. *there exists $\psi \in \mathscr{B}_{\lambda,P}$ such that $P(\psi) = \lambda_M$ and $1 - \lambda_M < \min\{P(\omega) \mid \omega \in [\![\psi]\!]\}$. In such a case, $\mathscr{B}_{\lambda,P} = \uparrow[\![\psi]\!]$.*

Moreover, under the further hypothesis that $\lambda < 1$, the above conditions are also equivalent to

3. *there exists $\omega \in \Omega$ such that $P(\omega) > \sum_{\omega' : P(\omega') < P(\omega)} P(\omega') > 0$.*

Probabilities satisfying the above point 3 have been called to have an *ω-step* (or to have a *step at ω*) in [9]. The name "ω-step" is clearly inspired by *"big-stepped probabilities"* introduced in [11] and also considered in [5]. The next numerical example gives a hint on how these probabilities behave.

Example 1. *Consider P defined on $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ such that $P(\omega_1) = P(\omega_4) = 0.05$, $P(\omega_2) = 0.3$ and $P(\omega_3) = 0.6$. The probability P has the following steps:*

- *A step at ω_3 because $0.6 = P(\omega_3) > P(\omega_1) + P(\omega_2) + P(\omega_4) = 0.4$;*
- *A step at ω_2 because $0.3 = P(\omega_2) > P(\omega_1) + P(\omega_4) = 0.1$.*

A probability function might have more than one step as shown in the previous example.

3. Revision of Lockean belief via minimal change

The probabilistic setting of our approach suggests a natural and straightforward way to revise a Lockean belief set $\mathscr{B}_{\lambda,P}$ by a formula ψ. This is relying on the conditional probability $P(\cdot \mid \psi) = P_\psi(\cdot)$, and hence defining $\mathscr{B}_{\lambda,P} * \psi = \{\varphi \in \mathcal{L} \mid P_\psi(\varphi) \geq \lambda\}$. This method, which has been already considered by several authors, and notably in [12], does not generally fit with a basic intuitive principle of *minimal change* stipulating that, when revising beliefs, one should make the fewest possible changes to the existing epistemic state (the probability) while still accommodating the new information. Note that the conditional probability $P_\psi(\cdot)$ gives probability 1 to ψ while one only needs that the probability of ψ be λ. To accommodate such a minimal change principle, we propose a way to revise a generic Lockean belief set $\mathscr{B}_{\lambda,P}$ by a formula ψ, so that the obtained set $\mathscr{B}_{\lambda,P} * \psi$ meets the following desiderata:

1. If $\psi \in \mathscr{B}_{\lambda,P}$ then $\mathscr{B}_{\lambda,P} * \psi = \mathscr{B}_{\lambda,P}$, that is to say, no revision is produced by those formulas that are already believed.
2. If $\psi \notin \mathscr{B}_{\lambda,P}$, then we include ψ in $\mathscr{B}_{\lambda,P}$ by minimally changing its probability; in other words, if a priori $P(\psi) < \lambda$, then a posteriori $P_\psi(\psi) = \lambda$.

In order to fulfill the above requests, we will define the following method to update an initial positive probability function.

Definition 1. *Given a positive probability P on Ω and $\psi \in \mathcal{L}$ and $\lambda > 1/2$, the* revised probability of P by ψ and λ *is the probability function R^λ_ψ whose distribution on Ω is defined by:*

$$R^\lambda_\psi(\omega) = \begin{cases} P(\omega) \cdot \max\left\{1, \frac{\lambda}{P(\psi)}\right\} & \text{if } \omega \in [\![\psi]\!] \\ P(\omega) \cdot \min\left\{1, \frac{1-\lambda}{P(\neg\psi)}\right\} & \text{if } \omega \notin [\![\psi]\!] \end{cases} \tag{2}$$

Notice that the above definition accommodates the above desideratum 1 because, if $\psi \in \mathscr{B}_{\lambda,P}$, then $P(\psi) \geq \lambda$ and hence $\frac{\lambda}{P(\psi)} \leq 1$. Thus $R^\lambda_\psi(\omega) = P(\omega)$ for all $\omega \in [\![\psi]\!]$ and hence $R^\lambda_\psi(\psi) = P(\psi) \geq \lambda$. It also satisfies desideratum 2 because, if $P(\psi) < \lambda$, $R^\lambda_\psi(\psi) = \lambda$ as we required above.

One can easily check that $\sum_{\omega\in\Omega} R^\lambda_\psi(\omega) = 1$, therefore, R^λ_ψ is a probability distribution on Ω that extends to a probability function on $\mathcal{L}$ which we will indicate by the same symbol. Interestingly, the probability function R^λ_ψ is in relation with *Jeffrey conditionalization* [13]: if P is a positive probability on $\mathcal{L}$, $\lambda \in [0, 1]$, and $\psi \in \mathcal{L}$ is such that $0 < P(\psi) < 1$, Jeffrey's λ-update of P given ψ is defined as:

$$P^\lambda_\psi(\varphi) = \lambda P(\varphi \mid \psi) + (1-\lambda)P(\varphi \mid \neg\psi) = \lambda\frac{P(\varphi \wedge \psi)}{P(\psi)} + (1-\lambda)\frac{P(\varphi \wedge \neg\psi)}{P(\neg\psi)}.$$

Observe that, when $\lambda = 1$ the above expression recovers the usual definition of conditional probability. That is to say, $P^1_\psi(\varphi) = P(\varphi \mid \psi)$.

Proposition 1. *For every probability P, $\lambda > 1/2$ and ψ such that $P(\psi) > 0$, we have for all $\varphi \in \mathcal{L}$,*

$$R^\lambda_\psi(\varphi) = \begin{cases} P^\lambda_\psi(\varphi), & \text{if } P(\psi) \leq \lambda \\ P(\varphi), & \text{otherwise.} \end{cases}$$

Thus, when $P(\psi) \leq \lambda$, R^λ_ψ is the Jeffrey conditionalization of P by ψ.

We now formally introduce our revision operator $*_{ml}$ for Lockean belief sets.

Definition 2. *Given a Lockean belief set $\mathscr{B}_{\lambda,P}$, and a proposition ψ, the* minimal Lockean *revision of $\mathscr{B}_{\lambda,P}$ by ψ is defined as:*

$$\mathscr{B}_{\lambda,P} *_{ml} \psi = \{\varphi \mid R^\lambda_\psi(\varphi) \geq \lambda\} = \mathscr{B}_{\lambda,R^\lambda_\psi}$$

Next result fully describes the situations in which $\mathscr{B}_{\lambda,P} *_{ml} \psi$ is deductively closed.

Theorem 2. *Let $\mathscr{B}_{\lambda,P}$ be a Lockean belief set, ψ a proposition and $\tau_\psi = \min\{P(\omega) \mid \omega \in [\![\psi]\!]\}$. Then:*

(i) $\mathscr{B}_{\lambda,P} *_{ml} \psi$ is deductively closed whenever $\tau_\psi > \frac{1-\lambda}{\lambda} P(\psi)$. In that case, $\mathscr{B}_{\lambda,P} *_{ml} \psi = \uparrow[\![\psi]\!]$.

(ii) Conversely, assuming $\psi \notin \mathscr{B}_{\lambda,P}$, if $\mathscr{B}_{\lambda,P} *_{ml} \psi$ is deductively closed then $\tau_\psi > \frac{1-\lambda}{\lambda} P(\psi)$.

Actually, the present setting may be read within the framework introduced in [14] by defining an epistemic space $\mathcal{E}_\lambda = (\mathscr{P}, B_\lambda)$ where $\frac{1}{2} < \lambda < 1$, $\mathscr{P}$ are the positive probabilities on 2^Ω and for every $P \in \mathscr{P}$ we put $B_\lambda(P) = \mathscr{B}_{\lambda,P}$. Then, on this epistemic space, we define an operator $\circ : \mathscr{P} \times \mathcal{L}^* \to \mathscr{P}$ by putting $P \circ \psi = R^\lambda_\psi$. Note that $\mathscr{B}_{\lambda,P} *_{ml} \psi = B_\lambda(P \circ \psi)$.

Some of the basic AGM postulates for revision (K*1-K*6) from [7] are satisfied in our framework. More detailed proofs can be found in [9].

K*1 (Closure): *$\mathscr{B}_{\lambda,P} * \psi$ is logically closed.* This is not generally satisfied. However, it is satisfied if either $\psi \in \mathscr{B}_{\lambda,P}$ and $\mathscr{B}_{\lambda,P}$ is closed or $\psi \notin \mathscr{B}_{\lambda,P}$ and ψ satisfies the condition (i) of Theorem 2.

K*2 (Success): $\psi \in \mathscr{B}_{\lambda,P} * \psi$. This holds by definition of R^λ_ψ.

K*3 (Inclusion): $\mathscr{B}_{\lambda,P} * \psi \subseteq Cn(\mathscr{B}_{\lambda,P} \cup \{\psi\})$, where Cn denotes the deductive closure operator. This postulate holds. Essentially, this is due to the fact that if $\varphi \in \mathscr{B}_{\lambda,P} * \psi$ then $(\psi \to \varphi) \in \mathscr{B}_{\lambda,P}$ (see details in [9]).

K*4 (Preservation): *If $\mathscr{B}_{\lambda,P} \not\vdash \neg\psi$ then $\mathscr{B}_{\lambda,P} \subseteq \mathscr{B}_{\lambda,P} * \psi$.* This postulate is not satisfied in general (see [9] for a counterexample).

K*5 (Consistency): *If ψ is consistent then $\mathscr{B}_{\lambda,P} * \psi$ is consistent.* This postulate is not satisfied in general (see [9] for a counterexample). However, if ψ satisfies the conditions of Theorem 2.(i), the postulate holds.

K*6 (Extensionality): *If $\vdash \psi \leftrightarrow \varphi$ then $\mathscr{B}_{\lambda,P} * \psi = \mathscr{B}_{\lambda,P} * \varphi$.* This postulate clearly holds because the revision is defined semantically.

It is worth noticing that the distribution R^λ_ψ used to define our revision operator $*_{ml}$ is the one minimally differing from P, according to the *Kullback-Leibler divergence*, among all the probability distributions satisfying that the probability of ψ is equal to λ, see also [9] for the proof. It is in this sense that $*_{ml}$ can be considered as a *minimal* revision operator.

4. Credibility limited iterated revision

Regarding the updated probability R^λ_ψ introduced above, notice that, unless $\lambda = 1$, we have $R^\lambda_\psi(\omega) > 0$ for $\omega \in [\![\psi]\!]$ and this fact makes the resulting revision operator $*_{ml}$ amenable to be iterated because the above expression for Jeffrey conditionalization is definable and hence so is the further revision, by say γ, of the positive probability function R^λ_ψ. Therefore, if ψ and γ are two formulas, it makes sense to consider the iterated revision $(\mathscr{B}_{\lambda,P} *_{ml} \psi) *_{ml} \gamma$ of a Lockean belief set $\mathscr{B}_{\lambda,P}$. The following result on iterated revision and deductive closure holds.

Proposition 2. *If $\mathscr{B}_{\lambda,P} *_{ml} \psi$, with $\psi \notin \mathscr{B}_{\lambda,P}$, is deductively closed and γ is such that $\gamma \vdash \psi$, then $(\mathscr{B}_{\lambda,P} *_{ml} \psi) *_{ml} \gamma$ is deductively closed and moreover*

$$(\mathscr{B}_{\lambda,P} *_{ml} \psi) *_{ml} \gamma = \mathscr{B}_{\lambda,P} *_{ml} \gamma.$$

The last claim of the above proposition ensures that the first postulate of iterated revision, as proposed by Darwiche and Pearl in [10], holds. Namely:

DP1 *If $\gamma \vdash \psi$ then $(\mathscr{B}_{\lambda,P} * \psi) * \gamma = \mathscr{B}_{\lambda,P} * \gamma$.*

Now, let us consider the other postulates, namely

DP2 *If* $\gamma \vdash \neg\psi$ *then* $(\mathscr{B}_{\lambda,P} * \psi) * \gamma = \mathscr{B}_{\lambda,P} * \gamma$,

DP3 If $\mathscr{B}_{\lambda,P} * \gamma \vdash \psi$ then $(\mathscr{B}_{\lambda,P} * \psi) * \gamma \vdash \psi$,

DP4 If $\mathscr{B}_{\lambda,P} * \gamma \nvdash \neg\psi$ then $(\mathscr{B}_{\lambda,P} * \psi) * \gamma \nvdash \neg\psi$.

In order for them to hold we assume a further special condition for the formulas ψ and γ. To properly express this extra hypothesis, let us consider a set $\mathscr{C}_{\lambda,P}$ of *credible* formulas, where P and λ are as above and that is defined as follows:

$$\mathscr{C}_{\lambda,P} = \mathscr{B}_{\lambda,P} \cup \left\{\varphi \mid \tau_\psi > \frac{1-\lambda}{\lambda} P(\psi)\right\}$$

where, as in the above section, $\tau_\varphi = \min\{P(\omega) \mid \omega \in [\![\varphi]\!]\}$. Notice that, as it is reasonable and intuitive to assume, $\mathscr{B}_{\lambda,P} \subseteq \mathscr{C}_{\lambda,P}$, i.e., formulas that are believed according to the Lockean thesis, are credible according to the present definition.

More formally, in order to consider iterated revision, we consider now the epistemic space $\mathcal{E}^c_\lambda = (\mathscr{P}_\lambda, B_\lambda)$ in which a *credibility-limited revision operator* [15] will be defined where now $\mathscr{P}_\lambda$ are the positive probabilities on 2^Ω such that $\mathscr{B}_{\lambda,P}$ is closed and, as before, for every $P \in \mathscr{P}_\lambda$ we put $B_\lambda(P) = \mathscr{B}_{\lambda,P}$. Then, on this epistemic space, we define an operator $\circ : \mathscr{P}_\lambda \times \mathcal{L}^* \to \mathscr{P}_\lambda$ by putting

$$P \circ \psi = \begin{cases} R^\lambda_\psi, & \text{if } \psi \in \mathscr{C}_{\lambda,P}, \\ P \circ \psi = P, & \text{otherwise} \end{cases}$$

Note that, by Theorem 2, this operator is well-defined, and it can be proved that the following postulates for iterated revision proposed in [15] (in a slightly different form), hold in our framework.

CLDP1 If $\gamma \vdash \psi$ and $\gamma \in \mathscr{C}_{\lambda,P}$, then $B_\lambda((P \circ \psi) \circ \gamma) = B_\lambda(P \circ \gamma)$,

CLDP2 If $\gamma \vdash \neg\psi$, $\gamma \in \mathscr{C}_{\lambda,P}$ and $\gamma \notin B_\lambda(P)$, then $B_\lambda((P \circ \psi) \circ \gamma) = B_\lambda(P \circ \gamma)$,

CLDP3 If $B_\lambda(P \circ \gamma) \vdash \psi$, $\gamma, \psi \in \mathscr{C}_{\lambda,P}$ and $\gamma \notin B_\lambda(P)$, then $B_\lambda((P \circ \psi) \circ \gamma) \vdash \psi$,

CLDP4 If $B_\lambda(P \circ \gamma) \nvdash \neg\psi$, $\gamma, \psi \in \mathscr{C}_{\lambda,P}$ and $\gamma \notin B_\lambda(P)$, then $B_\lambda((P \circ \psi) \circ \gamma) \nvdash \neg\psi$.

As a matter of fact, we have that if we define $\mathscr{B}_{\lambda,P} * \psi = B_\lambda(P \circ \psi)$ where $\circ$ is the above operator defined on $\mathcal{E}^c_\lambda$, the postulates (K*1), (K*3), (K*5), (K*6) are satisfied, although the postulate (K*4) is not satisfied (the same counterexample in [9] works). Moreover, the following postulates of *relative success*, *strong coherence* and *success monotonicity* are satisfied:

(Relative success) $\mathscr{B}_{\lambda,P} * \psi \vdash \psi$ or $\mathscr{B}_{\lambda,P} * \psi = \mathscr{B}_{\lambda,P}$,
(Strong coherence) $\mathscr{B}_{\lambda,P} * \psi \nvdash \bot$,
(Success monotonicity) If $\mathscr{B}_{\lambda,P} * \gamma \vdash \gamma$ and $\gamma \vdash \psi$ then $\mathscr{B}_{\lambda,P} * \psi \vdash \psi$.

In summary, the operator $\circ$ satisfies the majority of the basic postulates of credibility-limited revision plus the above postulates, and thus showing a good iterative behavior.

5. Concluding remarks

We have provided two characterizations of those probability functions P on formulas ensuring the existence of suitable thresholds λ for which the corresponding Lockean set of beliefs $\mathscr{B}_{\lambda,P}$ is closed under logical deduction. Then we have discussed how to revise a Lockean belief set in a way that is compatible with an intuitive principle of minimal change. We have shown that this principle univocally leads to a revision operator $*_{ml}$ for Lockean belief sets closely related (but not equal) to Jeffrey conditionalization, satisfying several AGM postulates. Finally, we have introduced a credibility-limited revision operator satisfying adapted versions of all Darwiche and Pearl postulates for iterated revision.

As for future work, we plan to further investigate and characterize our revision operators and compare them to the ones proposed in the related literature. In particular, we plan to study in more detail the links of our approach with that of Leitgeb based on P-stable$^\lambda$ sets [5].

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Declaration on Generative AI

The authors have not employed any Generative AI tools.

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