Evaluating Novel Arguments in Case Models: Lessons from Belief Change and Abstract Argumentation for Case-based Reasoning

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Abstract

In the intersection between case-based reasoning and non-monotonic reasoning, case models serve as frameworks to evaluate arguments with respect to cases. The evaluation is based on their coherence, presumptive validity, and conclusiveness, in which all valid arguments must be at least coherent, meaning that those arguments must be grounded in at least one case. This paper, on the other hand, attempts to explore new evaluations for *novel* arguments, which are not required to ground in any case. To develop the new evaluation, we introduce revision operators in case models and associated properties based on AGM postulates. We then define a *conclusively adherent* evaluation, in which novel arguments can be valid. After that, we relate their application to the understanding of abstract argumentation for case-based reasoning (AA-CBR). We demonstrate how the translation of AA-CBR case bases into case models can be described through revision sequences; and analyse the properties of conclusively adherent evaluation related to evaluation and attacks in AA-CBR.

Keywords

Case Models, Belief Change, AGM Postulates, Abstract Argumentation, Case-based Reasoning

1. Introduction

Case model [1] is a logical framework proposed to evaluate arguments based on their coherence, presumptive validity, and conclusiveness. Each case model consists of a set of consistent, mutually incompatible, and different propositional logical sentences representing cases, and a total and transitive preference ordering over the cases. Several applications of case models have been investigated, including ethical reasoning [1], evidential reasoning [2, 3], and legal reasoning [4].

One fundamental requirement in case models is that every valid argument must be at least coherent, meaning that the argument must be grounded in at least one existing case. However, in several applications, such as case-based reasoning, it requires to evaluate the validity of arguments even if they are not grounded in any existing cases, making the arguments always invalid in original evaluations.

Therefore, this paper introduces a crucial extension by defining new evaluations for *novel* arguments, which are not grounded in any existing cases. To facilitate this, this paper defines revision operators for case models, adapting the AGM postulates [5] from belief revision theory. Subsequently, the paper defines *conclusively adherent* evaluation, allowing to evaluate novel arguments by attempting to maximize the satisfaction of defined properties.

Furthermore, this paper explores the application of conclusively adherent evaluation to abstract argumentation for case-based reasoning (AA-CBR) [6]. AA-CBR is a case-based reasoning model based on presumptive reasoning and abstract argumentation [7]. This paper demonstrates that we can describe

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the translation of AA-CBR case bases into case models as revision sequences. This translation allows for an in-depth analysis of the properties of novel arguments within the AA-CBR case base. The analysis reveals the insights of how case-based reasoning inferences are made, linking conclusively adherent arguments to the AA-CBR inference and others to the AA-CBR attacks.

The contributions of this paper can be summarised as follows.

- The paper introduces families of revision operators for case models, including refinement-based revision operators and single revision operators, and specifically defines a top revision operator and a bottom revision operator, both of which add at most one case to a case model.
- The paper defines associated properties of these revision operators, including ℓ-success, ℓ-inclusion, vacuity, and extensionality.
- The paper investigates the satisfaction of these properties by different types of revision operators.
- The paper defines conclusively adherent arguments based on the satisfaction of the properties.
- The paper demonstrates how to describe the translation of AA-CBR case bases into case models through bottom revision sequences of novel arguments.
- The paper analyses the properties of novel arguments in the sequences related to AA-CBR concepts, stating that if a novel argument is conclusively adherent with respect to a translated case model, then the corresponding situation-outcome pair is valid with respect to the case base, and if a novel argument is not conclusively adherent with respect to a translated case model, the corresponding situation-outcome pair would attack some precedent in the AA framework if it is included in the case base, and vice versa.

The paper is structured as follows. Section 2 provides related work on the background of belief change and case-based reasoning research. Section 3 describes a formalism of case models, argument evaluations in case models, and applications of case models in case-based reasoning. Section 4 defines revision operators for case models with associated properties based on the AGM postulates and conclusively adherent evaluation. Section 5 presents an analysis of conclusively adherent arguments on the translation of AA-CBR case bases into case models. Section 6 discusses several implications of this paper and suggests future work. Finally, Section 7 concludes this paper.

2. Related Work

Nonmonotonic reasoning researchers have long been interested in making inferences that go beyond the data explicitly available to a system. This interest has derived the development of various reasoning paradigms, such as belief change and case-based reasoning, that aim to simulate the kind of flexible, context-sensitive reasoning that humans routinely perform. These paradigms are especially valuable when dealing with incomplete, evolving, or conflicting information. Integrating insights from these reasoning paradigms with modern machine learning models remains a key challenge to make AI systems more reliable and explainable.

Belief change considers two main entities: *beliefs* and *consequences*, usually represented in propositions. The main data structure in belief revision is *belief sets*, which is assumed to contain logical sentences considering some beliefs and all their consequences. To explain the change in belief sets, several operators have been formulated based on *AGM framework* [5], which becomes a fundamental foundation for the formal study of belief change. They establish rationality postulates that govern how a rational agent should incorporate new, possibly conflicting, information into an existing belief set while preserving consistency. The AGM postulates provide structure for operations such as expansion, contraction, and revision, focussing on maintaining coherence and minimal change. Beyond classical postulates, previous works have proposed new criteria to address limitations of the original framework, particularly concerning iterative belief revision [8] – how to consistently accommodate a sequence of belief updates rather than a single change. Following this, several works, such as [9], explore the relation between belief changes and rational inferences.

Meanwhile, case-based reasoning studies how to reason through *cases* from prior experience. Pioneered case-based reasoning systems, such as HYPO [10], relied mainly on analogical reasoning. Later works have developed logical frameworks for case-based reasoning, such as abstract argumentation for case-based reasoning [6] and precedential constraint [11]. Case models [1] can be considered as one of such developments, but in particular focus on representing cases as logical sentences and evaluating arguments. Previous works have investigated connections between case models and other case-based reasoning frameworks, such as the connection with abstract argumentation for case-based reasoning [12] and the connection with precedential constraint [13].

To our knowledge, this is the first study to connect case models with belief change. This connection is relevant to several aspects of nonmonotonic reasoning, such as cumulative reasoning and cautious monotonicity found in conditional logic [14] and case-based reasoning with cases represented as arguments [15]. However, our focus is on case models, which specifically address the logical properties of cases, such as logical consistency and mutual incompatibility, with cases represented as logical sentences rather than arguments, distinguishing case models from those works in conditional logic and case-based reasoning.

3. Case Models

In this paper, we use a classical logical language $\mathcal{L}$ generated from propositions in a standard way. We write $\neg$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, $\leftrightarrow$ for equivalence, $\top$ for tautology and $\bot$ for contradiction. The associated classical, deductive, and monotonic logical inference is denoted as $\vdash$. We say $L \subseteq \mathcal{L}$ is *logically consistent* iff each element of L is not a contradiction (formally, $\varphi \nvdash \bot$ for all $\varphi \in L$). We say $L \subseteq \mathcal{L}$ is *consistent* iff all elements of L can be true together (formally, $L \nvdash \bot$). We say $L \subseteq \mathcal{L}$ is *mutually incompatible* iff two elements of L cannot be true together (formally, for all $\varphi, \psi \in L$ such that $\varphi \neq \psi$: $\varphi \wedge \psi \vdash \bot$). Examples of mutually incompatible sets are $\{p \wedge q, \neg p \wedge r\}$ or even $\{p_1 \wedge q, p_2 \wedge r\}$ if we have a background knowledge that $p_1 \wedge p_2 \vdash \bot$.

A **case model** [1] is a pair $\langle C, \geq\rangle$ where $C \subseteq \mathcal{L}$ is a finite, logically consistent, and mutually incompatible set of logical sentences, each of which is called a *case*. $\geq$ is a total preorder over C, commonly for modelling preference relations [16]. For $\varphi, \psi \in C$, we write $\varphi \sim \psi$ iff $\varphi \geq \psi$ and $\psi \geq \varphi$, and we write $\varphi > \psi$ iff $\varphi \geq \psi$ and $\psi \ngeq \varphi$. **Case model space** $\mathcal{M}$ is defined as as the set of all possible case models on the language $\mathcal{L}$. Hence, for all φ , ψ and $\chi \in C$, $\langle C, \geq\rangle \in \mathcal{M}$ has the following properties [1]:

1. logically consistent: $\nvdash \neg\varphi$ (i.e., $\varphi \nvdash \bot$);
2. mutually incompatible: If $\nvdash \varphi \leftrightarrow \psi$, then $\vdash \neg(\varphi \wedge \psi)$ (i.e., then $\varphi \wedge \psi \vdash \bot$);
3. different: If $\vdash \varphi \leftrightarrow \psi$, then $\varphi = \psi$;
4. total: $\varphi \geq \psi$ or $\psi \geq \varphi$;
5. transitive: If $\varphi \geq \psi$ and $\psi \geq \chi$, then $\varphi \geq \chi$.

In case models, an **argument** is considered as a pair of logical sentences. Given a case model $\langle C, \geq\rangle$ and an argument $(\varphi, \psi) \in \mathcal{L} \times \mathcal{L}$ where φ is called a *premise* and ψ is called a *conclusion*. An evaluation $\vDash$ is then considered as a relation from a case model to an argument. Three evaluations, called in this paper as *basic* evaluations, have been originally defined as follows [1]:

- $\langle C, \geq\rangle \vDash_{coh} (\varphi, \psi)$ iff (φ, ψ) is *coherent* with respect to $\langle C, \geq\rangle$ i.e., $\exists\omega \in C : \omega \vdash \varphi \wedge \psi$; we then say (φ, ψ) has a *grounding* in ω.
- $\langle C, \geq\rangle \vDash_{pres} (\varphi, \psi)$ iff (φ, ψ) is *presumptively valid* with respect to $\langle C, \geq\rangle$ i.e., $\exists\omega \in C : \omega \vdash \varphi \wedge \psi$, and $\forall\omega' \in C$: if $\omega' \vdash \varphi$; then $\omega \geq \omega'$.
- $\langle C, \geq\rangle \vDash_{conc} (\varphi, \psi)$ iff (φ, ψ) is *conclusive* with respect to $\langle C, \geq\rangle$ i.e., $\exists\omega \in C : \omega \vdash \varphi \wedge \psi$; and $\forall\omega \in C$: if $\omega \vdash \varphi$, then $\omega \vdash \varphi \wedge \psi$.

Consequently, basic evaluations are ranked from the strongest to the weakest as $\vDash_{conc}, \vDash_{pres}, \vDash_{coh}$ as $\langle C, \geq\rangle \vDash_{conc} (\varphi, \psi)$ implies $\langle C, \geq\rangle \vDash_{pres} (\varphi, \psi)$; and $\langle C, \geq\rangle \vDash_{pres} (\varphi, \psi)$ implies $\langle C, \geq\rangle \vDash_{coh} (\varphi, \psi)$.

Thus, every valid argument according to basic evaluations must be at least coherent, where the premise and the conclusion can be logically implied from at least one of the cases. This property was originally designated so that a conclusion is retracted if we extend the premise to the degree that there is no grounding in any case. We call an argument such that the premise has no grounding in any case as a **novel** argument, formally defined as follows.

Definition 1 (Novel argument)**.** *An argument* (φ, ψ) *is* novel *with respect to* $\langle C, \geq \rangle$ *iff* $\forall \omega \in C, \omega \not\vdash \varphi$.

Proposition 1. *A novel argument cannot be valid with respect to a basic evaluation.*

Proof. If $\models$ is a basic evaluation, then for every case model $\langle C, \geq \rangle$ and argument (φ, ψ) such that $\langle C, \geq \rangle \models (\varphi, \psi)$, $\langle C, \geq \rangle \models_{coh} (\varphi, \psi)$ and hence $\exists \omega \in C : \omega \vdash \varphi$. Therefore, (φ, ψ) cannot be a novel argument with respect to $\langle C, \geq \rangle$. □

Since preferences in case models are total and transitive, each case model $\langle C, \geq \rangle$ can be represented as a sequence of logical sentence sets $L_1, \ldots, L_n$ such that, for $1 \leq i, j \leq n$ (we call each L_i a *layer*):

1. $L_1, \ldots, L_n$ is a partition of C (i.e., $C = L_1 \cup \ldots \cup L_n$ and $L_i \cap L_j = \emptyset$); and
2. $\varphi \sim \psi$ iff $\varphi, \psi \in L_i$; and
3. $\varphi > \psi$ iff $\varphi \in L_i$ and $\psi \in L_j$ and $i < j$.

Hereafter, we write $L_1 > \ldots > L_n$ instead of $L_1, \ldots, L_n$ to distinguish it from a general sequence. Example 1 demonstrates a case model and some examples of basic evaluations.

Example 1. *Consider the case model* $\langle C, \geq \rangle = \{\neg \mathtt{big} \wedge \neg \mathtt{fly}\} > \{\mathtt{wing} \wedge \mathtt{fly}\} > \{\mathtt{big} \wedge \neg \mathtt{fly}\}$, *which represents a state of knowledge concerning the distribution of animals across three distinct groups: (1) the majority group of animals which are not big and cannot fly, (2) the significant group of animals which have wings and can fly, and (3) the minority group of animals which are big and cannot fly. Examples of basic evaluations and novel argument with respect to the case model are as follows:*

- $(\top, \mathtt{fly})$ *is* coherent *(which can be interpreted as some animals can fly) because the argument has a grounding in the case* $\mathtt{wing} \wedge \mathtt{fly}$.
- $(\top, \neg \mathtt{fly})$ *is* presumptively valid *(which can be interpreted as animals presumptively cannot fly) because the argument has a grounding in the case* $\neg \mathtt{big} \wedge \neg \mathtt{fly}$*, which is* $\geq$*-maximal within all the cases implying the premise.*
- $(\mathtt{wing}, \mathtt{fly})$ *is* conclusive *(which can be interpreted as all animals that have wings can fly, according to the current state of knowledge) because the argument has a grounding in the case* $\mathtt{wing} \wedge \mathtt{fly}$ *and all the cases that imply* $\mathtt{wing}$ *also imply* $\mathtt{wing} \wedge \mathtt{fly}$.
- $(\mathtt{wing} \wedge \mathtt{big}, \neg \mathtt{fly})$ *is* novel *(which can be interpreted as we cannot conclude whether big animals which have wings cannot fly, according to the current state of knowledge) because the argument has no grounding in any case.*

In this paper, we focus particularly on the application of case models in case-based reasoning [13, 17]. In this application, the arguments are constrained so that the premises and conclusions are from disjoint domains. Firstly, we assume an **outcome domain** $\mathcal{O}$, of which each element is called an **outcome**, as a finite mutually incompatible subset of $\mathcal{L}$. We call an argument (φ, ψ) an **outcome-inference argument** iff the premise φ does not imply any outcome (i.e., $\forall o \in \mathcal{O} : \varphi \not\vdash o$), and the conclusion ψ is an outcome or a special proposition nil, not included in $\mathcal{O}$ and not occurring in any case models. An outcome-inference argument of which the conclusion is an outcome is called an **outcome-based argument** while an outcome-inference argument of which the conclusion is nil is called an **outcome-free argument**, used when we could not find any outcome as a conclusion. We extend the basic evaluations by replacing " $\omega \vdash \varphi \wedge \psi$" in the original definition with "$\forall o \in \mathcal{O} : \omega \vdash \varphi$ and $\omega \not\vdash o$" to evaluate an outcome-free argument. That is, for a case model $\langle C, \geq \rangle$ and an outcome-free argument (φ, nil), the basic evaluations are extended as follows:

- $\langle C, \geq \rangle \vDash_{coh} (\varphi, nil)$ iff $\exists \omega \in C : \forall o \in \mathcal{O} : \omega \vdash \varphi$ and $\omega \nvdash o$; we then say (φ, nil) has a *grounding* in ω.
- $\langle C, \geq \rangle \vDash_{pres} (\varphi, nil)$ iff $\exists \omega \in C : \forall o \in \mathcal{O} : \omega \vdash \varphi$ and $\omega \nvdash o$, and $\forall \omega' \in C$: if $\omega' \vdash \varphi$; then $\omega \geq \omega'$.
- $\langle C, \geq \rangle \vDash_{conc} (\varphi, nil)$ iff $\exists \omega \in C : \forall o \in \mathcal{O} : \omega \vdash \varphi$ and $\omega \nvdash o$; and $\forall \omega \in C$: if $\omega \vdash \varphi$, then $\forall o \in \mathcal{O} :<< \omega \vdash \varphi$ and $>> \omega \nvdash o$. (The statement in «...» can be omitted).

Proposition 2. *Two outcome-inference arguments (φ, ψ) and (φ', ψ') with different conclusions $\psi \neq \psi'$ cannot have a grounding in the same case (i.e., a non-contradiction logical sentence).*

Proof. Following the extended definition of basic evaluations, if one conclusion is nil, then the proposition clearly holds. If two conclusions are included in $\mathcal{O}$, then the case implies $\psi \wedge \psi'$ which leads to contradiction as $\mathcal{O}$ are mutually incompatible but the case is non-contradiction. □

4. Evaluating Novel Arguments: Lessons from Belief Change

To develop a new evaluation for novel arguments, we first introduce revision operators and their associated properties based on AGM postulates [5]. A **revision operator** is defined as follows.

Definition 2 (Revision operator)**.** *Let $\mathcal{M}$ be the case model space and $\mathcal{I}$ be the set of all outcome-inference arguments with respect to outcome domain $\mathcal{O}$. A* revision operator *$\diamond$ is a binary operator from $\mathcal{M} \times \mathcal{I}$ to $\mathcal{M}$. The result (called a revised case model) of applying $\diamond$ to a case model $\langle C, \geq \rangle$ (called an original case model) and an outcome-inference argument (φ, ψ) (called a revision argument) is denoted as $\langle C, \geq \rangle \diamond (\varphi, \psi)$. A revision operator $\diamond$ can satisfy the following properties, for some $\ell \in \{coh, pres, conc\}$:*

1. ℓ-success *iff $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_\ell (\varphi, \psi)$.*
2. ℓ-inclusion *iff for every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$, if $\langle C, \geq \rangle \vDash_\ell (\tilde{\varphi}, \tilde{\psi})$ then $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_\ell (\tilde{\varphi}, \tilde{\psi})$.*
3. vacuity *iff for every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$, if $\langle C, \geq \rangle \nvDash_{coh} (\tilde{\varphi}, \tilde{\psi})$ and $\tilde{\psi} \neq \psi$ then $\langle C, \geq \rangle \diamond (\varphi, \psi) \nvDash_{coh} (\tilde{\varphi}, \tilde{\psi})$.*
4. extensionality *iff for every $\varphi' \in \mathcal{L}$ such that $\varphi \leftrightarrow \varphi'$ and every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$, if $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_\ell (\tilde{\varphi}, \tilde{\psi})$ then $\langle C, \geq \rangle \diamond (\varphi', \psi) \vDash_\ell (\tilde{\varphi}, \tilde{\psi})$, and vice versa.*

In Definition 2, we classify *success* and *inclusion* into three levels according to three basic evaluations using a label "ℓ-" . Intuitively, *ℓ-success* means that a revision argument should be valid in ℓ-evaluation with respect to the revised case model. *ℓ-inclusion* means that the valid arguments in ℓ-evaluation should be preserved their validity after revision. *Vacuity* means that every incoherent argument with different conclusion from a revision argument should remain incoherent with respect to the revised case model. Finally, *extensionality* means that if two revision arguments are logically equivalent, the revision should result two revised case models that evaluate any argument equivalently. We then define a family of revision operators based on *case model refinement* [18], defined as follows.

Definition 3 (Case Model Refinement [18])**.** *A case model refinement is an order $\preceq$ over the case model space $\mathcal{M}$ such that for $\langle C, \geq \rangle, \langle C', \geq' \rangle \in \mathcal{M}$, $\langle C, \geq \rangle \preceq \langle C', \geq' \rangle$ iff for all $\omega \in C$, there exists $\omega' \in C'$ such that $\omega' \vdash \omega$. We then say $\langle C', \geq' \rangle$ refines $\langle C, \geq \rangle$.*

It has been studied that $\preceq$ is a reflexive and transitive order [18], meaning that it is an order over the case model space, starting from the case model with the empty case set $\langle \emptyset, \emptyset \rangle$ to more refined case models. A *refinement-based* revision operator is defined as follows.

Definition 4 (Refinement-based revision operator)**.** *A revision operator $\diamond$ is* refinement-based *iff for every $\langle C, \geq \rangle \in \mathcal{M}$ and every outcome-inference argument (φ, ψ), $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_{coh} (\varphi, \psi)$ (coh-success); and $\langle C, \geq \rangle \preceq \langle C, \geq \rangle \diamond (\varphi, \psi)$ (revision-as-refinement).*

Consequently, refinement-based revision operators are assured to satisfy coh-inclusion.

Proposition 3. *Every refinement-based revision operator satisfies* coh*-inclusion.*

Proof. Let (φ, ψ) be an outcome-inference argument, $\diamond$ be a refinement-based revision operator, and $\langle C', \geq' \rangle = \langle C, \geq \rangle \diamond (\varphi, \psi)$. Since $\langle C, \geq \rangle \preceq \langle C', \geq' \rangle$, for every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$, if there is $\omega \in C$ such that $\omega \vdash \tilde{\varphi} \wedge \tilde{\psi}$ then there is $\omega' \in C'$ such that $\omega' \vdash \omega$ and hence $\omega' \vdash \tilde{\varphi} \wedge \tilde{\psi}$. Therefore, if $\langle C, \geq \rangle \vDash_{coh} (\tilde{\varphi}, \tilde{\psi})$ then $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_{coh} (\tilde{\varphi}, \tilde{\psi})$. □

Since every refinement-based revision operator satisfies coh-success and coh-inclusion, it cannot satisfy $conc$-success and $conc$-inclusion, showing that maintaining conclusiveness is challenging for refinement-based revision operators (see [18] for details on the hardness of the validity change).

Proposition 4. *Every refinement-based revision operator cannot satisfy* $conc$*-success.*

Proof. Let $\langle C, \geq \rangle$ be a case model with non-empty case set C (i.e., $C \neq \emptyset$). Consider an outcome-inference argument (φ, ψ) such that $\langle C, \geq \rangle \vDash_{coh} (\varphi, \psi)$, and (φ, ψ') be an outcome-inference argument such that $\psi' \neq \psi$. Since $\langle C, \geq \rangle \vDash_{coh} (\varphi, \psi)$, there is a case $\omega \in C$ that (φ, ψ) has a grounding in. Let $\diamond$ be a refinement-based revision operator and $\langle C', \geq' \rangle = \langle C, \geq \rangle \diamond (\varphi, \psi')$. Then, there is $\omega' \in C'$ such that $\omega' \vdash \omega$ (according to Definition 3) and hence (φ, ψ) has a grounding in ω', $\omega' \vdash \varphi$, and (φ, ψ') has no grounding in ω'. Thus, $\langle C, \geq \rangle \diamond (\varphi, \psi') \nvDash_{conc} (\varphi, \psi')$. Therefore, no refinement-based operator can satisfy $conc$-success in this setting. □

Proposition 5. *Every refinement-based revision operator cannot satisfy* $conc$*-inclusion.*

Proof. Let (φ, ψ) be an outcome-inference argument and $(\tilde{\varphi}, \tilde{\psi})$ be an outcome-inference argument with $\varphi \vdash \tilde{\varphi}$ and $\psi \neq \tilde{\psi}$. Consider a case model $\langle C, \geq \rangle$ such that $\langle C, \geq \rangle \vDash_{conc} (\tilde{\varphi}, \tilde{\psi})$. Let $\diamond$ be a refinement-based revision operator and $(C', \geq') = \langle C, \geq \rangle \diamond (\varphi, \psi)$. Since $\diamond$ satisfies coh-success and $\varphi \vdash \tilde{\varphi}$, there is $\omega' \in C'$ such that (φ, ψ) has a grounding in ω', $\omega' \vdash \tilde{\varphi}$, and $(\tilde{\varphi}, \tilde{\psi})$ has no grounding in ω'. Thus, $\langle C, \geq \rangle \vDash_{conc} (\tilde{\varphi}, \tilde{\psi})$ but $\langle C, \geq \rangle \diamond (\varphi, \psi) \nvDash_{conc} (\tilde{\varphi}, \tilde{\psi})$. Therefore, no refinement-based operator can satisfy $conc$-inclusion in this setting. □

Next, we define a sub-family of refinement-based revision operators, called **single** revision operators, which add at most one case to an original case model so that a revision argument has a grounding in the added case. To simplify the definition, we assume that, in a case model $\langle C, \geq \rangle$, each case $\omega_i \in C$ contains a unique proposition p_i not occurring in arguments such that $p_i \wedge p_j \vdash \bot$ for two distinct cases $\omega_i, \omega_j \in C$ (i.e. $\omega_i \neq \omega_j$). This p_i works as a *naming* proposition to ensure the requirement of mutual incompatibility in case models. We then say a single revision operator is **simple** if the added case is minimally constructed from the revision argument. Specifically, we define two simple revision operators. One is a **top** revision operator, where the new case is added to the new highest layer. The second is a **bottom** revision operator, where the new case is added to the new lowest layer. The formal definition is as follows.

Definition 5 (Single, top, and bottom revision operator)**.** *A revision operator* $\diamond$ *is* single *iff*

- *if* $\langle C, \geq \rangle \vDash_{coh} (\varphi, \psi)$, $\langle C, \geq \rangle \diamond (\varphi, \psi) = \langle C, \geq \rangle$*; and*
- *if* $\langle C, \geq \rangle \nvDash_{coh} (\varphi, \psi)$, $\langle C, \geq \rangle \diamond (\varphi, \psi) = \langle C \cup \{\omega_n\}, \geq' \rangle$

where (φ, ψ) *has a grounding in* ω_n *and the preference* $\geq$ *is preserved in* $\geq'$*, meaning that for every case* $\varphi, \psi \in C$, $\varphi \geq \psi$ *iff* $\varphi \geq' \psi$*.*

Definition 6 (Simple, top, and bottom revision operator)**.** *A single revision operator* $\diamond$ *is* simple *iff the new case* ω_n *in Definition 5 satisfies the following conditions:*

- *if* ψ *is an outcome (i.e.,* $\psi \in \mathcal{O}$*),* $\omega_n = p_n \wedge \varphi \wedge \psi$*; and*
- *if* ψ *is* nil*,* $\omega_n = p_n \wedge \varphi$*,*

where p_n is a new naming proposition. A top *revision operator* $\triangle$ *is a simple revision operator such that* $\forall \omega \in C : \omega_n >' \omega$ *and a* bottom *revision operator* ∇ *is a simple revision operator such that* $\forall \omega \in C : \omega >' \omega_n$.

Intuitively, the top revision operator gives preference to new cases, whereas the bottom one gives preference to existing cases. Example 2 demonstrates examples of revisions using the bottom revision operator.

Example 2. *Let* $\{p_0, p_1, p_2, p_3\}$ *be a mutually incompatible set of naming propositions. Consider an outcome domain* $\mathcal{O} = \{\texttt{fly}, \neg\texttt{fly}\}$ *and a case model* $\langle C, \geq \rangle = \{p_0 \wedge \neg\texttt{big} \wedge \neg\texttt{fly}\} > \{p_1 \wedge \texttt{wing} \wedge \texttt{fly}\} > \{p_2 \wedge \texttt{big} \wedge \neg\texttt{fly}\}$ *(similar to Example 1 but with naming propositions).*

- $\langle C, \geq \rangle \ \triangle \ (\texttt{wing} \wedge \texttt{big}, \neg\texttt{fly}) = \{p_3 \wedge \texttt{wing} \wedge \texttt{big} \wedge \neg\texttt{fly}\} > \{p_0 \wedge \neg\texttt{big} \wedge \neg\texttt{fly}\} > \{p_1 \wedge \texttt{wing} \wedge \texttt{fly}\} > \{p_2 \wedge \texttt{big} \wedge \neg\texttt{fly}\}$. *Since the top revision operator gives preference to the new case, the revision does not preserve the presumptive validity of some arguments, such as* $(\top, \neg\texttt{fly})$.
- $\langle C, \geq \rangle \nabla (\texttt{wing} \wedge \texttt{big}, \neg\texttt{fly}) = \{p_0 \wedge \neg\texttt{big} \wedge \neg\texttt{fly}\} > \{p_1 \wedge \texttt{wing} \wedge \texttt{fly}\} > \{p_2 \wedge \texttt{big} \wedge \neg\texttt{fly}\} > \{p_3 \wedge \texttt{wing} \wedge \texttt{big} \wedge \neg\texttt{fly}\}$. *Since the bottom revision operator gives preference to the existing cases, the revision preserves the validity of any presumptively valid arguments, such as* $(\top, \neg\texttt{fly})$.

Table 1 shows the satisfaction of the properties by refinement-based, single, top, and bottom, revision operators. ✓ means that the revision operator always satisfies the property while × means that the revision operator does not always satisfy the property. The following propositions show the proof of some non-trivial results in the table.

Table 1
Satisfaction of Properties by Revision Operators

Revision Operators	**Success**			**Inclusion**			**Vacuity**	**Extensionality**
	coh−	*pres−*	*conc−*	*coh−*	*pres−*	*conc−*		
Refinement-based	✓	×	×	✓	×	×	×	×
Single	✓	×	×	✓	×	×	✓	×
Top	✓	✓	×	✓	×	×	✓	✓
Bottom	✓	×	×	✓	✓	×	✓	✓

Proposition 6. *Every single revision operator is refinement-based.*

Proof. Let $\langle C, \geq \rangle$ be a case model, (φ, ψ) be an outcome-inference argument, $\diamond$ be a single revision operator, and $\langle C', \geq' \rangle = \langle C, \geq \rangle \diamond (\varphi, \psi)$. From Definition 6, $C \subseteq C'$. Therefore, for every $\omega \in C$, $\omega \in C'$ logically implies itself and hence a single revision operator is refinement-based. □

Proposition 7. *Every single revision operator satisfies vacuity.*

Proof. Let $\langle C, \geq \rangle$ be a case model, (φ, ψ) be an outcome-inference argument, $\diamond$ be a single revision operator, and $\langle C', \geq' \rangle = \langle C, \geq \rangle \diamond (\varphi, \psi)$. For every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$ such that $\psi \neq \tilde{\psi}$, if $\langle C, \geq \rangle \not\models_{coh} (\tilde{\varphi}, \tilde{\psi})$ and hence there is no $\omega \in C$ that $(\tilde{\varphi}, \tilde{\psi})$ has a grounding in, then there is no $\omega' \in C'$ such that $(\tilde{\varphi}, \tilde{\psi})$ has a grounding in either because ω_n is only a case that is added and (φ, ψ) has a grounding in ω_n. Therefore, for every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$ such that $\psi \neq \tilde{\psi}$, if $\langle C, \geq \rangle \not\models_{coh} (\tilde{\varphi}, \tilde{\psi})$ then $\langle C, \geq \rangle \diamond (\varphi, \psi) \not\models_{coh} (\tilde{\varphi}, \tilde{\psi})$. □

Proposition 8. *Every simple revision operator satisfies extensionality.*

Proof. Let $\langle C, \geq \rangle$ be a case model, (φ, ψ) be an outcome-inference argument, and $\diamond$ be a simple revision operator. Since it is trivial in the case $\langle C, \geq \rangle \models_{coh} (\varphi, \psi)$ then we prove in the case $\langle C, \geq \rangle \not\models_{coh} (\varphi, \psi)$. From Definition 6, if $\psi \in \mathcal{O}$, consider $\omega_n = p_n \wedge \varphi \wedge \psi$ and $\omega'_n = p'_n \wedge \varphi' \wedge \psi$; and if $\psi = nil$, consider

$\omega_n = p_n \wedge \varphi$ and $\omega'_n = p'_n \wedge \varphi'$. Then, every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$ that has a grounding in ω_n, also has a grounding in ω'_n, and vice versa (naming propositions do not occur in arguments). Therefore, $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_\ell (\tilde{\varphi}, \tilde{\psi})$ iff $\langle C, \geq \rangle \diamond (\varphi', \psi) \vDash_\ell (\tilde{\varphi}, \tilde{\psi})$ for every $\ell \in \{coh, pres, conc\}$. ☐

Proposition 9. *The top revision operator ($\triangle$) satisfies $pres$-success.*

Proof. Let $\langle C, \geq \rangle$ be a case model and (φ, ψ) be an outcome-inference argument. From Definition 6, $\langle C, \geq \rangle \triangle (\varphi, \psi) = \langle C \cup \{\omega_n\}, \geq' \rangle$ and $\forall \omega \in C : \omega_n >' \omega$. Then, $\forall \omega \in C \cup \{\omega_n\}$: if $\omega \vdash \varphi$ then $\omega_n \geq' \omega$. Therefore, $\langle C, \geq \rangle \triangle (\varphi, \psi) \vDash_{pres} (\varphi, \psi)$. ☐

Proposition 10. *The bottom revision operator (∇) satisfies $pres$-inclusion.*

Proof. Let $\langle C, \geq \rangle$ be a case model and (φ, ψ) be an outcome-inference argument. From Definition 6, $\langle C, \geq \rangle \nabla (\varphi, \psi) = \langle C \cup \{\omega_n\}, \geq' \rangle$ such that $\forall \omega \in C : \omega >' \omega_n$. Then, for every outcome-inference argument $(\tilde{\varphi}, \tilde{\psi})$, if $\langle C, \geq \rangle \vDash_{pres} (\tilde{\varphi}, \tilde{\psi})$ then there is $\omega \in C$ that $(\tilde{\varphi}, \tilde{\psi})$ has a grounding in, and for every $\omega' \in C$ if $\omega' \vdash \varphi$ then $\omega \geq \omega'$. Since $\omega >' \omega_n$ and $\omega >' \omega'$, for every $\omega'' \in C \cup \{\omega_n\}$ if $\omega'' \vdash \varphi$ then $\omega \geq \omega''$. Therefore, $\langle C, \geq \rangle \nabla (\varphi, \psi) \vDash_{pres} (\tilde{\varphi}, \tilde{\psi})$. ☐

Interestingly, if a revision argument is novel, every simple revision operator satisfies $conc$-success (and hence $pres$-success).

Proposition 11. *If a revision argument is novel with respect to the original case model, every simple revision operator satisfies $conc$-success.*

Proof. Let $\langle C, \geq \rangle$ be a case model, (φ, ψ) be a novel argument with respect to $\langle C, \geq \rangle$, and $\diamond$ be a simple revision operator. From Definition 1 and Definition 6, $\langle C, \geq \rangle \diamond (\varphi, \psi) = \langle C \cup \{\omega_n\}, \geq' \rangle$ and for every $\omega \in C$, $\omega \nvdash \varphi$. Therefore, there is only $\omega_n \in C \cup \{\omega_n\}$ such that $\omega_n \vdash \varphi$ and (φ, ψ) has a grounding in ω_n. Therefore, $\langle C, \geq \rangle \diamond (\varphi, \psi) \vDash_{conc} (\varphi, \psi)$. ☐

Consequently, if a revision argument is novel with respect to the original case model, the bottom revision operator becomes satisfying all the properties defined except $conc$-inclusion. We then define **conclusively adherent** evaluation as follows.

Definition 7 (Conclusively adherent evaluation)**.** *Let $\langle C, \geq \rangle$ be a case model and (φ, ψ) be an outcome-inference argument. (φ, ψ) is* conclusively adherent *with respect to $\langle C, \geq \rangle$, denoted as $\langle C, \geq \rangle \vDash_{conc-ad} (\varphi, \psi)$, iff for every conclusive outcome-inference argument (φ', ψ') with $\varphi \vdash \varphi'$, $\psi' = \psi$.*

Example 3. *Continuing from Example 2, let $\mathcal{O} = \{\texttt{fly}, \neg\texttt{fly}\}$ and $\langle C, \geq \rangle = \{p_0 \wedge \neg\texttt{big} \wedge \neg\texttt{fly}\} > \{p_1 \wedge \texttt{wing} \wedge \texttt{fly}\} > \{p_2 \wedge \texttt{big} \wedge \neg\texttt{fly}\}$. Examples of conclusively adherent evaluations are as follows:*

- $\langle C, \geq \rangle \vDash_{conc-ad} (\texttt{wing} \wedge \texttt{australia}, \texttt{fly})$ *because its conclusion is consistent with $(\texttt{wing}, \texttt{fly})$, which is the only conclusive outcome-inference argument with the premise that can be implied from $\texttt{wing} \wedge \texttt{australia}$.*
- $\langle C, \geq \rangle \nvDash_{conc-ad} (\texttt{wing} \wedge \texttt{big}, \texttt{fly})$ *because its conclusion is not consistent with $(\texttt{big}, \neg\texttt{fly})$, which is a conclusive outcome-inference argument with the premise that can be implied from $\texttt{wing} \wedge \texttt{big}$.*

Following Definition 7, it is impossible to have two conclusively adherent arguments with the same premise but different conclusions. Interestingly, conclusively adherent arguments can be either coherent arguments or novel arguments. For example, $(\texttt{wing} \wedge \texttt{australia}, \texttt{fly})$ in Example 3 demonstrates a conclusively adherent argument which is novel. Following the proof in Proposition 5, if a revision argument is conclusively adherent, the bottom revision operator becomes satisfying $conc$-inclusion, making it satisfy all properties defined in Definition 2.

Corollary 1. *If a revision argument is conclusively adherent with respect to the original case model, the bottom revision operator (∇) satisfies $conc$-inclusion.*

5. Evaluating Novel Arguments: Lessons from AA-CBR

In this section, we apply conclusively adherent evaluation studied in the previous section to understand **abstract argumentation for case-based reasoning** (AA-CBR) [6]. AA-CBR is one case-based reasoning model based on presumptive reasoning and abstract argumentation. It assumes a binary distribution of outcomes, that is, assumes an outcome domain $\mathcal{O} = \{o_d, \neg o_d\}$ where o_d is a *default* outcome. Let $\mathcal{F}$ be a finite consistent set of propositions, of which each subset is called a *situation*. We define the conjunction of all the propositions in a situation X as $\mathrm{con}(X)$. That is, $\mathrm{con}(X) = \bigwedge_{\varphi_i \in X} \varphi_i$ if $X \neq \emptyset$, and $\mathrm{con}(\emptyset) = \top$ (i.e., tautology) if $X = \emptyset$.

A *case base* $\Gamma \subseteq 2^{\mathcal{F}} \times \mathcal{O}$ is a finite set of situation-outcome pairs representing precedents. In AA-CBR, it assumes that a case base Γ is default-consistent (that is, $(\emptyset, \neg o_d) \notin \Gamma$) and outcome-consistent (that is, $(X, o) \in \Gamma$ implies $(X, o') \notin \Gamma$ for any $o' \neq o$).

A *case-based evaluation* $\mid\!\sim$ is a relation from a case base to a situation-outcome pair. Let Γ be a case base and (X, o) be a situation-outcome pair. $\Gamma \mid\!\sim (X, o)$ intuitively determines whether a situation-outcome pair (X, o) is valid with respect to Γ. Analogous to what we have defined, if every $(X', o') \in \Gamma$, $X \nsubseteq X'$, then we say (X, o) is a *novel* situation-outcome pair with respect to Γ. As it is named, an AA-CBR evaluation is based on *abstract argumentation* framework [7], recapped as follows.

Definition 8 (abstract argumentation framework)**.** *An abstract argumentation framework (AA framework) is a pair $(\mathcal{A}, \rightsquigarrow)$. Each element of $\mathcal{A}$ represents an argument and $\rightsquigarrow$ is a binary relation over $\mathcal{A}$. For $x, y \in \mathcal{A}$, if $x \rightsquigarrow y$ then we say x attacks y. For a set of arguments $E \subseteq \mathcal{A}$ and an argument $x \in \mathcal{A}$, E defends x if, for every $y \in \mathcal{A}$ that attacks x, there is an argument $z \in E$ attacks y. Then, the* grounded extension *of $(\mathcal{A}, \rightsquigarrow)$ can be constructed inductively as $G = \bigcup_{i \geq 0} G_i$, where G_0 is the set of unattacked arguments, and for $i \geq 0$, G_{i+1} is the set of arguments that G_i defends.*

The AA-CBR evaluation is determined by the default argument $(\emptyset, o_d)$ and relevant precedents, forming an AA-framework in which precedents with different outcomes and concisely specific situations attack the others. We denote the AA-CBR evaluation as $\mid\!\sim_{aacbr}$, formally defined as follows.

Definition 9 (AA-CBR evaluation)**.** *An* AA framework *corresponding to a case base Γ, a default outcome o_d, and a situation N is $(\mathcal{A}, \rightsquigarrow)$ satisfying the following conditions [6]:*

- *$\mathcal{A} = \Gamma \cup \{(N, ?)\} \cup \{(\emptyset, o_d)\}$;*
- *(precedent attacks) for $(X, o_x), (Y, o_y) \in \Gamma \cup \{(\emptyset, o_d)\}$, it holds that $(X, o_x) \rightsquigarrow (Y, o_y)$ iff*
  1. *(different outcomes) $o_x \neq o_y$, and*
  2. *(specificity) $Y \subsetneq X$, and*
  3. *(concision) $\nexists (Z, o_x) \in \Gamma$ with $Y \subsetneq Z \subsetneq X$;*
- *(irrelevant attacks) for $(Y, o_y) \in \Gamma$, $(N, ?) \rightsquigarrow (Y, o_y)$ holds iff $Y \nsubseteq N$.*

If $(\emptyset, o_d)$ is in the grounded extension of $(\mathcal{A}, \rightsquigarrow)$ then $\Gamma \mid\!\sim_{aacbr} (N, o_d)$. Otherwise, $\Gamma \mid\!\sim_{aacbr} (N, \neg o_d)$.

Hence, the AA-CBR evaluation is consistent and complete, meaning that for every situation X, either $\Gamma \mid\!\sim_{aacbr} (X, o_d)$ or $\Gamma \mid\!\sim_{aacbr} (X, \neg o_d)$.

Example 4. *Let $\mathcal{O} = \{\texttt{fly}, \neg\texttt{fly}\}$ where $\neg\texttt{fly}$ is a default outcome and a case base $\Gamma = \{(\{\texttt{wing}, \texttt{thin}\}, \texttt{fly}), (\{\texttt{big}\}, \neg\texttt{fly})\}$. Figure 1 depicts the AA framework corresponding to Γ, $\neg\texttt{fly}$, and $\{\texttt{wing}, \texttt{big}\}$. Consequently, $\Gamma \mid\!\sim_{aacbr} (\{\texttt{wing}, \texttt{big}\}, \neg\texttt{fly})$ as $(\emptyset, \neg\texttt{fly})$ is in the grounded extension of the AA framework.*

The previous work [13] shows that we can translate any AA-CBR case bases into case models. Given a case base Γ and a case-based evaluation $\mid\!\sim$, a *translated* case model $\langle C, \geq \rangle$ is defined as any case model that evaluates every non-novel argument as presumptively valid iff the corresponding situation-outcome pair is valid with respect to Γ. That is, for every non-novel situation-outcome pair (X, o) with respect to Γ, $\Gamma \mid\!\sim (X, o)$ iff $\langle C, \geq \rangle \models_{pres} (\text{con}(X), o)$.

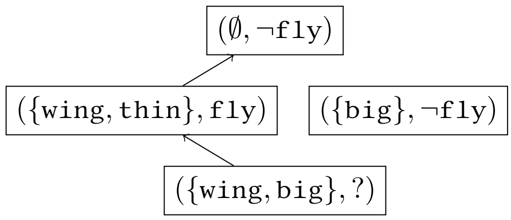

Figure 1: Corresponding AA framework

Again, we cannot evaluate novel arguments in the translated case model using $\models_{pres}$. For instance, given Γ in Example 4, since $(\{\texttt{wing}, \texttt{australia}\}, \neg\texttt{fly})$ is a novel situation-outcome pair with respect to Γ, the argument $(\texttt{wing} \wedge \texttt{australia}, \neg\texttt{fly})$ can be novel with respect to the translated case model and hence it cannot be presumptively valid. To evaluate novel arguments, we describe a translated case model discussed in [12, 13], as a bottom revision sequence in which each argument in a revision sequence is novel with respect to the prior revised case model.

Proposition 12 (adapted from [12, 13]). *Given a AA-CBR case base* Γ*, there exists a translated case model* $\langle C, \geq \rangle$ *which can be written as a revision sequence:* $\langle C, \geq \rangle = \langle \emptyset, \emptyset \rangle \triangledown (con(X_1), o_1) \triangledown \ldots \triangledown (con(X_n), o_n)$ *such that if* $X_i \subsetneq X_j$ *then* $i < j$ *for every* $1 \leq i, j \leq n$.

A revision sequence in Proposition 12 can be obtained by enumerating all non-novel situation-outcome pairs that are valid with respect to the case base. Example 5 demonstrates a translated case model from a bottom revision sequence, given the case base Γ in Example 4.

Example 5. *Given* Γ *in Example 4,* $\langle C, \geq \rangle = \langle \emptyset, \emptyset \rangle \triangledown (\top, \neg \mathit{fly}) \triangledown (\mathit{wing}, \neg \mathit{fly}) \triangledown (\mathit{thin}, \neg \mathit{fly}) \triangledown (\mathit{wing} \wedge \mathit{thin}, \mathit{fly}) \triangledown (\mathit{big}, \neg \mathit{fly})$*, which is equivalent to* $\{p_0 \wedge \neg \mathit{fly}\} > \{p_1 \wedge \mathit{wing} \wedge \neg \mathit{fly}\} > \{p_2 \wedge \mathit{thin} \wedge \neg \mathit{fly}\} > \{p_3 \wedge \mathit{wing} \wedge \mathit{thin} \wedge \mathit{fly}\} > \{p_4 \wedge \mathit{big} \wedge \neg \mathit{fly}\}$*, is a translated case model.*

In Example 5, each revision argument is also novel with respect to the prior revised case model in the sequence. Now, we consider whether each revision argument is conclusively adherent with respect to the prior revised case model as follows.

- $\langle \emptyset, \emptyset \rangle \models_{conc-ad} (\top, \neg\texttt{fly})$
- $\langle \emptyset, \emptyset \rangle \triangledown (\top, \neg\texttt{fly}) \models_{conc-ad} (\texttt{wing}, \neg\texttt{fly})$
- $\langle \emptyset, \emptyset \rangle \triangledown (\top, \neg\texttt{fly}) \triangledown (\texttt{wing}, \neg\texttt{fly}) \models_{conc-ad} (\texttt{thin}, \neg\texttt{fly})$
- $\langle \emptyset, \emptyset \rangle \triangledown (\top, \neg\texttt{fly}) \triangledown (\texttt{wing}, \neg\texttt{fly}) \triangledown (\texttt{thin}, \neg\texttt{fly}) \not\models_{conc-ad} (\texttt{wing} \wedge \texttt{thin}, \texttt{fly})$
- $\langle \emptyset, \emptyset \rangle \triangledown (\top, \neg\texttt{fly}) \triangledown (\texttt{wing}, \neg\texttt{fly}) \triangledown (\texttt{thin}, \neg\texttt{fly}) \triangledown (\texttt{wing} \wedge \texttt{thin}, \texttt{fly}) \models_{conc-ad} (\texttt{big}, \neg\texttt{fly})$

We have that if a novel revision argument is not conclusively adherent with respect to the prior revised case model, such as $(\texttt{wing} \wedge \texttt{thin}, \texttt{fly})$ in the example, then it contributes a precedent attack in AA-CBR in the corresponding AA-framework.

Proposition 13. *Let* Γ *be an AA-CBR case base with default outcome* o_d *and* $\langle C, \geq \rangle$ *be a translated case model. For every novel situation-outcome pair* (X, o) *with respect to* Γ*,* $\langle C, \geq \rangle \not\models_{conc-ad} (con(X), o)$ *iff* (X, o) *attacks an argument in the AA-framework corresponding to* $\Gamma \cup \{(X, o)\}$*,* o_d *and any situation* N*.*

Proof. (Left-to-right) If $\langle C, \geq \rangle \not\models_{conc-ad} (\text{con}(X), o)$, then there exists a conclusive argument $(\varphi, \neg o)$ with $\text{con}(X) \vdash \varphi$. It implies that (1) there is $(Y, \neg o) \in \Gamma \cup \{(\emptyset, o_d)\}$ such that $(\text{con}(Y), \neg o)$ is conclusive with respect to $\langle C, \geq \rangle$ and $\varphi \vdash \text{con}(Y)$ (2) there is no $(Z, o) \in \Gamma \cup \{(\emptyset, o_d)\}$ such that $(\text{con}(Z), o)$ is coherent and $\text{con}(Z) \vdash \varphi$; otherwise, $(\text{con}(Y), \neg o)$ cannot be conclusive. It implies that there is no $(Z, o) \in \Gamma \cup \{(\emptyset, o_d)\}$ such that $Y \subsetneq Z \subsetneq X$. Therefore, (X, o) attacks $(Y, \neg o)$ in the corresponding AA framework.

(Right-to-left) If (X, o) attacks $(Y, \neg o) \in \Gamma \cup \{(\emptyset, o_d)\}$ in the corresponding AA framework, then $Y \subsetneq X$ and there is no $(Z, o) \in \Gamma \cup \{(\emptyset, o_d)\}$ such that $Y \subsetneq Z \subsetneq X$. Thus, there is a conclusive argument $(\varphi, \neg o)$ such that $\text{con}(X) \vdash \varphi$ and $\varphi \vdash \text{con}(Y)$. Therefore, $\langle C, \geq \rangle \not\models_{conc-ad} (\text{con}(X), o)$. $\square$

On the other hand, if a novel revision argument is conclusively adherent, then the corresponding situation-outcome pair is valid with respect to the case base.

Proposition 14. *Let* Γ *be an AA-CBR case base and* $\langle C, \geq \rangle$ *be a translated case model. For every novel situation-outcome pair* (X, o) *with respect to* Γ*, if* $\langle C, \geq \rangle \vDash_{conc-ad} (con(X), o)$ *then* $\Gamma \mathrel{|\!\sim}_{aacbr} (X, o)$*.*

Proof. If $\langle C, \geq \rangle \vDash_{conc-ad} (\mathrm{con}(X), o)$, then for every conclusive argument (φ, ψ) with $\mathrm{con}(X) \vdash \varphi$, $\psi = o$. It implies that (1) there is $(X_n, o) \in \Gamma \cup \{(\emptyset, o_d)\}$ with $\varphi \vdash \mathrm{con}(X_n)$ and hence $X_n \subseteq X$ (2) there does not exist $(Y, \neg o)$ such that $X \subsetneq Y$; otherwise, such (φ, ψ) cannot be conclusive. Therefore, all such (X_n, o) are nearest cases [6] to X and share the same outcome o and hence $\Gamma \mathrel{|\!\sim}_{aacbr} (X, o)$. □

The converse of Proposition 14 is not true. That is, $\Gamma \mathrel{|\!\sim}_{aacbr} (X, o)$ does not imply that $\langle C, \geq \rangle \vDash_{conc-ad} (\mathrm{con}(X), o)$. The reason is that, for any novel situation outcome pair (X, o), there are possibly two conclusive arguments with opposite conclusions (φ, o) and $(\varphi', \neg o)$ such that $\mathrm{con}(X) \vdash \varphi$ and $\mathrm{con}(X) \vdash \varphi'$. For instance, from Example 5, we have $\Gamma \mathrel{|\!\sim}_{aacbr} (\{\mathtt{wing}, \mathtt{thin}, \mathtt{big}\}, \neg \mathtt{fly})$ but the translated case model $\langle C, \geq \rangle \nvDash_{conc-ad} (\mathtt{wing} \wedge \mathtt{thin} \wedge \mathtt{big}, \neg \mathtt{fly})$ because there are two conclusive arguments with opposite conclusions, namely $(\mathtt{wing} \wedge \mathtt{thin}, \mathtt{fly})$ and $(\mathtt{big}, \neg \mathtt{fly})$.

6. Discussion and Future Work

The analysis in the previous section highlights two key observations. First, we showed that novel arguments which are not conclusively adherent are related to precedent attacks in AA-CBR. Second, we showed that novel arguments which are conclusively adherent are related to the AA-CBR evaluation. Interestingly, these observations are parallel to *precedential constraint* [11]. In precedential constraint, certain cases are constrained – i.e., the outcome must be decided in a specific way – otherwise, they cannot be consistently added as new precedents. In contrast, unconstrained cases can be consistently added as new precedents regardless of their outcome. This similarity suggests a promising direction for future work of translating precedential constraint case bases into case models through revision sequences.

Since AA-CBR employs argumentation to infer an outcome for every situation, it allows arguments that are not conclusively adherent to be considered valid. When such valid arguments are added as new precedents, they introduce new attacks that alter the grounded extensions of the corresponding AA frameworks. This observation is consistent with previous findings on AA-CBR [15], which show that the addition of valid arguments can change the validity of other arguments. Formally, it is possible that $\Gamma \mathrel{|\!\sim}_{aacbr} (X_1, o_1)$ and $\Gamma \mathrel{|\!\sim}_{aacbr} (X_2, o_2)$ do not imply $\Gamma \cup \{(X_1, o_1)\} \mathrel{|\!\sim}_{aacbr} (X_2, o_2)$. This implies the same conclusion to [15] that AA-CBR lacks cautious monotonicity and does not satisfy all the postulates of iterated belief change.

Cumulative AA-CBR (cAA-CBR) [15] has been developed as a cautious monotonic version of AA-CBR that includes a new argument into its case base only if the argument is invalid with respect to the existing case base. Thus, translated case models from the cAA-CBR case bases can be described as revision sequences of novel arguments that are not conclusively adherent with respect to the preceding revised case model. This motivates further research to identify more evaluations for novel arguments in order to examine other CBR models, such as precedential constraint and cAA-CBR.

7. Conclusion

This paper investigated new evaluations in case models for novel arguments, which are not required to have a grounding in any existing cases. To facilitate this, we introduced **revision operators** for case models and proposed a set of associated properties –adapted from the AGM postulates – including ℓ-success, ℓ-inclusion, vacuity, and extensionality. We then systematically examined whether these properties are satisfied by various types of revision operators.

A key contribution of this work is the introduction of the **conclusively adherent** evaluation. An outcome-inference argument is *conclusively adherent* iff every conclusive arguments with more general premises shares the same outcome. We showed that the bottom revision operator satisfies all the defined properties only if the revision argument is conclusively adherent. Furthermore, we investigated translated case models from abstract argumentation for case-based reasoning (AA-CBR) case bases as revision sequences of novel arguments. The investigation revealed two findings:

1. Novel arguments that are not conclusively adherent correspond to precedent attacks in AA-CBR. This results from the presence of a conclusive argument with a differing outcome, triggering an attack in the corresponding AA framework.
2. Novel arguments that are conclusively adherent correspond to the AA-CBR evaluation. That is, if a novel argument is conclusively adherent with respect to the translated case model, then the corresponding situation-outcome pair is valid with respect to the AA-CBR case base.

Future research can explore how to generalise the translation of case bases into case models via revision sequences for other case-based reasoning models and explore suitable evaluations for them.

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Declaration on Generative AI

During the preparation of this work, the first author used AnswerThis for the literature review, Google NotebookLM for summarisation, and ChatGPT/Writefull for the grammar and spelling check. After using these tools and services, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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