Deontic Argumentation*

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Abstract
We address the issue of defining a semantics for deontic argumentation that supports weak permission. Some recent results show that grounded semantics do not support weak permission when there is a conflict between two obligations. We provide a definition of Deontic Argumentation Theory accounting for weak permission, and we recall the result about grounded semantics. Then, we propose a new semantics that supports weak permission.

Keywords
deontic argumentation argumentation semantics weak permission grounded semantics stable semantics

1. Introduction

Suppose there is a norm n_1 prescribing C under condition A, with a sanction S_1 for not doing C. Concurrently, there is a second norm n_2, independent of the first, forbidding C when B holds, and the violation of this norm is sanctioned by S_2. Both norms are at the same hierarchical level in the normative system, meaning there are no clear criteria to determine if one norm overrides the other. This situation constitutes a genuine deontic conflict; specifically, the deontic conclusion that "C is obligatory" is ambiguous due to two conflicting reasoning chains, one supporting this conclusion and another opposing it.

While such scenarios are familiar in deontic reasoning, there is less exploration of how this type of conflict can impact subsequent deontic reasoning chains. Now, consider a third norm n_3, independent of n_1 and n_2, which requires D if C is permitted. The sanction for violating the third norm is S_3.

Now, consider the situation where A and B are true, but C and D are not. What sanctions, if any, apply in this case? Additionally, how does the outcome change if n_3 requires C to be not forbidden or explicitly authorised, rather than merely permitted? Is handling this scenario independent of the argumentation semantics adopted for modelling deontic reasoning?

Deontic logic offers a powerful framework for analysing and resolving such ambiguities. However, most Deontic Logic systems are not well-suited to deal with conflicting norms. Formal argumentation systems provide the tools to address these challenges. They offer a structured way to represent deontic statements, reason about their implications, and evaluate the validity of arguments. By explicitly stating premises, rules of inference, and conclusions, formal methods promote transparency, clarity, and consistency in deontic reasoning. This rigorous approach helps mitigate the risks of implicit bias, ad hoc judgments, and inconsistencies that plague informal reasoning. The next sections will explore how formal argumentation frameworks can enhance our ability to navigate the complexities of deontic logic in diverse contexts, from legal judgments to ethical dilemmas.

The layout of the paper is as follows. Section 2 addresses the challenges of deontic argumentation, particularly focusing on weak permission and its theoretical nuances. In Section 3, we introduce the notion of Deontic Argumentation Theory and recall the results of [2] for weak permission under the grounded semantics. Section 4 presents our new argumentation semantics tailored for weak permission. Lastly, Section 5 reviews related work in the domain, and Section 6 summarises our findings and contributions.

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2. Deontic Argumentation: Challenges

Deontic argumentation presents unique challenges. For example, dealing with weak permission within this context highlights both theoretical nuances and practical implications. As is well known, deontic logic distinguishes between weak and strong permissions (for an outline [3, 4, 5]). Weak permission is inferred when an action or a state of affairs is not explicitly prohibited, introducing a potential gray area in argumentation, especially when deontic conflicts arise. This section spots these challenges, motivating the need for advanced deontic argumentation frameworks as developed in our research.

2.1. Understanding Weak Permission

The concept of weak permission is rooted in an action being permitted merely due to a lack of explicit prohibition. When neither prohibition nor obligation is decisively concluded (as, for example, when the relative strength of conflicting rules is undetermined), both conflicting actions might be weakly permitted.

The implications of weak permission are significant in deontic reasoning systems like Defeasible Deontic Logic [4, 6, 7]. A research challenge is to adequately resolve scenarios where weak permissions are invoked amidst conflicts. This issue persists in both practical legal interpretations and theoretical frameworks. Consequently, deontic systems must carefully navigate these areas to maintain logical coherence and practical applicability.

In many real-world cases, like judicial decision-making, the role of weak permission becomes critical. Judges and legal practitioners frequently encounter scenarios where multiple, potentially conflicting obligations and permissions must be weighed and interpreted. The arguments must consider all potential influences and outcomes, requiring a robust framework for integrating diverse types of permissions and obligations.

Our research identifies and addresses these challenges through the design of a sophisticated argumentation framework that integrates deontic reasoning with formal argumentation techniques. This approach looks to provide clarity in situations where weak permissions exist alongside conflicting obligations.

2.2. Some Examples

Assume we work with a Defeasible Deontic Logic in which deontic rules have the form

$$r\colon a \Rightarrow \mathsf{obl}(b) \qquad\qquad s\colon \mathsf{obl}(c), d \Rightarrow \mathsf{perm}(e).$$

If r is applicable, we derive $\mathsf{obl}(b)$; if we know that $\mathsf{obl}(c)$ and d are the case, then we conclude that e is permitted, i.e., that $\mathsf{perm}(e)$. Suppose that we also have the following rule:

$$t\colon a, f \Rightarrow \mathsf{obl}(\neg b).$$

If both r and t are applicable and we do not have additional information, we cannot conclude either $\mathsf{obl}(b)$ or $\mathsf{obl}(\neg b)$ but that both b and $\neg b$ are weakly permitted, i.e., $\mathsf{perm}_w(b)$ and $\mathsf{perm}_w(\neg b)$.

Now, consider judicial adversarial scenarios where the lack of the prohibition is derived during a legal proceeding. Multiple scenarios can be identified.

Article 520 of the Italian Code of Criminal Procedure states that a judge must issue a sentence of acquittal if (i) the fact does not constitute a crime or (ii) is not provided by law as a crime. In this case, the absence of a prohibition is derived from the absence of a crime. In the first case, there is an obligation or prohibition, but the facts upon which the proceeding is based are not covered by the obligation or prohibition. Thus, we see that $\mathsf{obl}(b)$ is in force, but f (the fact of the case) holds but $b \not\equiv f$, something which is established via argumentation.

For instance, in the Italian legal system, a public officer who intentionally signs a forged document is punished with imprisonment. This means that the person is obligated not to sign a document knowing it is forged. However, if the person accidentally signs the document or signs it without knowing it is forged, the fact is not covered by the prohibition. In this case, we can say that the action of signing is weakly permitted because it does not result in punishment.

A more complicated example is the following.

Example 1. *In 2019, the Sea Watch 3, an NGO vessel, was on a mission in the Mediterranean Sea to rescue migrants. The Italian Government issued a decree that banned the Sea Watch 3 from rescue operations (for migrants) in the Italian Contiguous Zone waters.*

$$r_1\colon\ \mathit{distress}, \mathit{proximity} \Rightarrow \mathsf{obl}(\mathit{assistance})$$
$$r_2\colon\ \mathit{SeaWatch}, \mathit{migrants}, \mathit{ItalianContiguousZone} \Rightarrow \mathsf{obl}(\neg \mathit{assistance})$$

Maritime Law stipulates that a vessel in proximity to a distress vessel must assist the vessel in distress (rule r_1). On the other hand, r_2 encodes the prohibition on rescuing migrants in Italian Contiguous Zone waters. Moreover, Maritime Law requires vessels permitted to refrain from assisting vessels in distress to alert the closest relevant authorities and keep clear of the rescue area.

$$r_3\colon\ \mathsf{perm}(\neg \mathit{assistance}) \Rightarrow \mathsf{obl}(\mathit{alertAuthorities}) \& \mathsf{obl}(\mathit{keepClear})$$

Suppose a migrant vessel is in distress in the Italian Contiguous Zone, and Sea Watch 3 is nearby. Does Sea Watch 3 have the obligation to alert the authorities and keep clear of the rescue area? Here, we have a conflict over two opposite norms, and it is unclear which one takes precedence over the other.[1] *Also, suppose that Sea Watch 3 alerted the competent authorities but remained in the operation area (and eventually assisted with the rescue operation). Do the Sea Watch 3 actions contravene the norms? As we said, there are two options. The permission does not follow from the conflict. In this case, the Sea Watch is not required to leave the area, and it does not contravene the norms. In the second approach, the permission from the conflict follows (as a weak permission). Leaving the area is a legal requirement, and Sea Watch 3 does not comply with the norms.*

What about if weak permission does not follow from a conflict? We can consider an alternative formulation of the last provision. Assume that it stipulates that vessels for which the obligation to assist does not hold must keep clear of the rescue area.

$$r'_3\colon\ \mathsf{not}\ \ \mathsf{obl}(\mathit{assistance}) \Rightarrow \mathsf{obl}(\mathit{alertAuthorities}) \& \mathsf{obl}(\mathit{keepClear})$$

Given the conflict, we can argue that the obligation to offer assistance does not hold, and the Sea Watch 3 has to follow the obligations given by r'_3.

2.3. Motivating Advanced Deontic Argumentation Techniques

The primary motivation for our research is the limitations in existing argumentation semantics as applied to deontic logic, such as grounded and stable semantics, which inadequately handle weak permissions under conflict. These methods often prevent justifications for weak permission due to their inherent limitations in conflict resolution, notably when multiple deontic rules apply simultaneously.

To address these limitations, we propose an argumentation semantics that effectively incorporates weak permission by allowing a more nuanced evaluation of arguments. Our approach ensures that both obligations and permissions are considered robustly within argumentative frameworks, promoting a balance between legal precision and interpretative flexibility.

3. Grounded Semantics and Weak Permission

We recall the results of [2] that show that grounded semantics does not support weak permission when there is a conflict between two obligations. Moreover, we set the definition for the notion of a deontic

[1] In response to the Sea Watch 3 incident, an Italian Tribunal established that International Maritime Law prevailed over the Italian Government Decree for the specific case. However, International Maritime Law scholars debated over the proper course of action.

argumentation theory. The definitions of argument and attacks are common to the grounded semantics and the semantics we are going to develop in the next section.

The language of a deontic argument is built from a set of literals (Lit), where a literal is either an atom (l) or its negation ($\neg l$). In addition, we extend the language with *deontic literals.* The set of deontic literals is defined as

$$\{\mathsf{dop}(l), \mathsf{not}\ \ \mathsf{dop}(l) | l \in \mathrm{Lit}\}$$

where dop is a deontic operator, more precisely $\mathsf{dop} \in \{\mathsf{obl}, \mathsf{perm}, \mathsf{perm}_w\}$. Given a literal l, we use $\sim l$ to denote the complement of l; more precisely, if l is an atomic proposition, then $\sim l = \neg l$. If l is a negated atomic proposition $l = \neg m$, then $\sim l = m$.

Arguments are built from rules, where a rule has the following format:

$$a_1, \dots a_n \Rightarrow c$$

where $\{a_1, \dots, a_n\}$ is a (possibly empty) set of literals and deontic literals, and c is either a literal or deontic literal but not a weak permission (i.e., $c \neq \mathsf{perm}_w(l)$, otherwise it would be an explicit permission).

Definition 1. *A* Deontic Argumentation Theory *is a structure*

$$(F, R)$$

where F is a (finite and possibly empty) set of literals (the fact or assumption of the theory), and R is a (finite) set of rules.

The key concept of an argumentation theory is the notion of an argument. Definition 2 below defines what an argument is. Each argument A has associated with it, its conclusion $C(A)$ and its set of sub-arguments $\mathrm{Sub}(A)$.

Definition 2. *Given a Deontic Argumentation Theory (F, R), A is an* argument *if A has one of the following forms:*

1. $A = \mathsf{perm}_w(l)$ *for any literal $l \in \mathcal{L}$, the conclusion of the argument* $C(A) = \mathsf{perm}_w(l)$*, and* $\mathrm{Sub}(A) = \{A\}$.
2. $A = a$ *for* $a \in F$*;* $C(A) = a$ *and* $\mathrm{Sub}(A) = \{A\}$.
3. $A = A_1, \dots, A_n \Rightarrow c$*, if there is a rule* $a_1, \dots, a_n \Rightarrow c$ *in R such that for all* $a_i \in \{a_1, \dots, a_n\}$ *there is an argument A_i such that* $C(A_i) = a_i$*;* $C(A) = c$ *and* $\mathrm{Sub}(A) = \{A\} \cup \mathrm{Sub}(A_1) \cup \dots \cup \mathrm{Sub}(A_n)$.
4. $A = B \Rightarrow \mathsf{perm}(l)$*, if B is an argument such that* $C(B) = \mathsf{obl}(l)$*;* $C(A) = \mathsf{perm}(l)$*, and* $\mathrm{Sub}(A) = \{A\} \cup \mathrm{Sub}(B)$.

We call the weak permission arguments (arguments defined by clause (1) above) imaginary arguments. *Arguments defined by the other clauses are called* natural arguments. *The set of all arguments for a Deontic Argumentation Theory is $Args$.*

Condition (1) encodes the idea that weak permission is the failure to derive an obligation to the contrary (more on this when we discuss the notion of attack between arguments). Thus, by default, every literal is potentially weakly permitted, and we form an argument for this type of conclusion. Condition (2) gives the simplest form of an argument. We have an argument for a if a is one of the assumptions/facts of a case/theory. Condition (3) allows us to form arguments by forward chaining rules. Thus, we can form an argument from a rule, if we have arguments for all the elements of the body of the rule. The way the condition is written allows us to create arguments from rules with an empty body. Finally, condition (4) corresponds to the D axiom of Standard Deontic Logic.

Definition 3. *Let A and B be arguments. A* attacks *B ($A > B$) iff*

1. $B = \mathsf{perm}_w(l)$ *and* $C(A) = \mathsf{obl}(\sim l)$*;*

2. $\exists B' \in \mathrm{Sub}(B), C(B') = l$ *and* $C(A) = {\sim}l$;
3. $\exists B' \in \mathrm{Sub}(B), C(B') \in \{\mathsf{obl}(l), \mathsf{perm}(l)\}$ *and* $C(A) = \mathsf{obl}({\sim}l)$;
4. $\exists B' \in \mathrm{Sub}(B), C(B') = \mathsf{obl}(l)$, *and* $C(A) = \mathsf{perm}({\sim}l)$.

As we alluded to in Section 1, the idea of weak permission is the negation as failure of the obligation to the contrary. Thus, in Definition 2, we create an argument for the weak permission for any literal l; however, this argument is attacked by any argument for $\mathsf{obl}({\sim}l)$ (condition (1) above and notice this is the only case where the attack is not symmetrical). The rest of the conditions define an attack when the two arguments have opposite conclusions: condition (2) covers the case of plain literals, while conditions (3) and (4) are reserved for deontic literals. Specifically, we have opposite deontic conclusions when one of the two is an obligation for a literal, and the other is either an obligation or a permission for the opposite literal. Furthermore, an argument attacks another argument when the conflict is on the conclusion of the second argument (this corresponds to the notion of rebuttal) or when there is a conflict with one of the sub-arguments of the attacked argument (known as undercutting attack).

Example 2. *Let us consider the Deontic Argumentation Theory* (F, R) *where* $F = \{a\}$ *and* R *contains the following rules:*

$$r_1\colon \Rightarrow \mathsf{obl}(p) \qquad r_3\colon \mathsf{obl}(p) \Rightarrow \mathsf{obl}(q) \qquad r_5\colon \mathsf{perm}_w(p) \Rightarrow s$$
$$r_2\colon \Rightarrow \mathsf{obl}(\neg p) \qquad r_4\colon \Rightarrow \mathsf{obl}(\neg q) \qquad r_6\colon \mathsf{perm}(q) \Rightarrow t$$

The set of natural arguments contains $A_0\colon a$ *(given that* a *is a fact) and the following arguments built from the above rules:*

$$A_1\colon \Rightarrow \mathsf{obl}(p) \qquad A_2\colon \Rightarrow \mathsf{obl}(\neg p) \qquad A_3\colon A_1 \Rightarrow \mathsf{obl}(q)$$
$$A_4\colon \Rightarrow \mathsf{obl}(\neg q) \qquad A_5\colon A_1, \Rightarrow \mathsf{perm}(p) \qquad A_6\colon A_2 \Rightarrow \mathsf{perm}(\neg p)$$
$$A_7\colon A_3 \Rightarrow \mathsf{perm}(q) \qquad A_8\colon A_4 \Rightarrow \mathsf{perm}(\neg q) \qquad A_9\colon A_0, I_1 \Rightarrow s$$
$$A_{10}\colon A_7 \Rightarrow t$$

Moreover, for each propositional letter a, p, q, s, t *we have two imaginary arguments, one for the letter and one for its negation. Specifically, we consider the following imaginary arguments:*

$$I_1\colon \mathsf{perm}_w(p) \qquad I_2\colon \mathsf{perm}_w(\neg p) \qquad I_3\colon \mathsf{perm}_w(q) \qquad I_4\colon \mathsf{perm}_w(\neg q)$$

Arguments A_0, A_1*, and* A_4 *do not have proper subarguments, and have only themselves as their subarguments.*

$$\mathrm{Sub}(A_3) = \{A_1, A_3\} \qquad \mathrm{Sub}(A_5) = \{A_1, A_5\} \qquad \mathrm{Sub}(A_6) = \{A_2, A_6\}$$
$$\mathrm{Sub}(A_7) = \{A_1, A_3, A_7\} \qquad \mathrm{Sub}(A_8) = \{A_4, A_8\}$$
$$\mathrm{Sub}(A_9) = \{A_0, I_1, A_9\} \qquad \mathrm{Sub}(A_{10}) = \{A_1, A_3, A_7, A_{10}\}$$

The attack relation is as follows (for each argument A*, we list the arguments* A *attacks):*

$$A_1\colon A_2, A_6, I_2 \qquad A_2\colon A_1, A_3, A_5, I_1, A_{10}$$
$$A_3\colon A_4, A_8, I_4 \qquad A_4\colon A_3, A_7, I_3, A_{10}$$
$$A_5\colon A_2, A_6 \qquad A_6\colon A_1, A_3, A_5, A_7, A_{10}$$
$$A_7\colon A_4, A_8 \qquad A_8\colon A_3, A_7, A_{10}.$$

We are ready to recall the standard definitions of Dung argumentation semantics [8].

Definition 4. *Let* (F, R) *be a Deontic Argumentation Theory, and* S *be a set of arguments. Then:*

- S *is conflict free iff* $\forall X, Y \in S : X \not> Y$.
- $X \in Args$ *is acceptable with respect to* S *iff* $\forall Y \in Args$ *such that* $Y > X : \exists Z \in S$ *such that* $Z > Y$.

- *S is an admissible set iff S is conflict free and $X \in S$ implies X is acceptable w.r.t. S.*
- *S is a complete extension iff S is admissible and if $X \in Args$ is acceptable w.r.t. S then $X \in S$.*
- *S is the grounded extension iff S is the set inclusion minimal complete extension.*
- *S is a stable extension iff S is conflict free and $\forall Y \notin S, \exists X \in S$ such that $X > Y$.*

Definition 5. *Let $D = (F, R)$ be a Deontic Argumentation Theory, an argument A is* sceptically justified *under a semantic T iff $A \in S$ for all sets of arguments S that are an extension under T.*

Definition 6. *A literal or a deontic literal $l \in \mathcal{L}$ is a* Justified conclusion *under a semantics T iff for every extension S under T, there is an argument A such that $A \in S$ and $C(A) = l$.*

Example 3. *Let us consider a Deontic Argumentation Theory where $F = \emptyset$ and R contains the two rules*

$$r_1 : \Rightarrow \mathsf{obl}(a) \qquad\qquad r_2 : \Rightarrow \mathsf{obl}(\neg a)$$

This theory has the following arguments:

$$\begin{array}{lll} A_1 : \mathsf{perm}_w(a) & A_3 : \Rightarrow \mathsf{obl}(a) & A_5 : A_3 \Rightarrow \mathsf{perm}(a) \\ A_2 : \mathsf{perm}_w(\neg a) & A_4 : \Rightarrow \mathsf{obl}(\neg a) & A_6 : A_4 \Rightarrow \mathsf{perm}(\neg a) \end{array}$$

For the attack relation, we have the following instances

$$\begin{array}{lllll} A_3 > A_2 & A_3 > A_4 & A_3 > A_6 & A_5 > A_4 & A_5 > A_6 \\ A_4 > A_1 & A_4 > A_3 & A_4 > A_5 & A_6 > A_3 & A_6 > A_5 \end{array}$$

It is easy to verify that $\{\}$ is a complete extension (and trivially, it is the minimal complete extension w.r.t. set inclusion). Thus, it is the grounded extension of the theory. Accordingly, there is no argument in the grounded extension such that its conclusion is either $\mathsf{perm}_w(a)$ or $\mathsf{perm}_w(\neg a)$. Hence, $\mathsf{perm}_w(a)$ and $\mathsf{perm}_w(\neg a)$ are not justified conclusions under the grounded semantics.

When we consider the stable semantics, we have the following two extensions:

$$\{A_1, A_3, A_5\} \qquad\qquad \{A_2, A_4, A_6\}$$

Clearly, $\mathsf{perm}_w(a)$ is a conclusion of the first extension but not of the second one; conversely, $\mathsf{perm}_w(\neg a)$ is a conclusion of the second extension but not of the first one. Consequently, $\mathsf{perm}_w(a)$ and $\mathsf{perm}_w(\neg a)$ are not justified conclusions under the stable semantics.

The above example should suffice to show that weak permission is not supported by grounded and stable semantics when a deontic conflict exists: we have a scenario where we fail to conclude that an obligation, but at the same time, we cannot conclude the weak permission of the opposite. However, we can generalise the result; the result holds for any theory with a conflict between two applicable obligation rules. This is formalised by the following definition.

Definition 7. *A Deontic Argumentation Theory is* conflictual *when it contains a pair of rules $b_1, \ldots, b_n \Rightarrow \mathsf{obl}(c)$ and $d_1, \ldots d_m \Rightarrow \mathsf{obl}(\neg c)$, such that there are arguments B_i with conclusion b_i, $1 \leq i \leq n$, and D_j with conclusion d_j, $1 \leq j \leq m$. We will call c the* conflicted *literal.*

Theorem 1. *Let $D = (F, R)$ be a conflictual theory. For any conflicted literal l, $\mathsf{perm}_w(l)$, $\mathsf{perm}_w(\neg l)$ are not justified conclusions under grounded and stable semantics.*

Proof. Let l be a conflicted literal. By the definition of argument, the theory D and the fact that the theory is conflicted, we have the following arguments:

$$\begin{array}{ll} A_1 : \mathsf{perm}_w(l) & A_3 : B_1, \ldots, B_n \Rightarrow \mathsf{obl}(l) \\ A_2 : \mathsf{perm}_w(\neg l) & A_4 : D_1, \ldots, D_m \Rightarrow \mathsf{obl}(\neg l) \end{array}$$

such that $A_3 > A_2$ and $A_4 > A_1$. Given that there are arguments attacking A_1 and A_2, these two arguments are not in the minimal complete extension. Accordingly, $\mathsf{perm}_w(l)$ and $\mathsf{perm}(\neg l)$ are not justified conclusions under the grounded semantics.

Given the attack relationship among A_1, A_2, A_3 and A_4, we can conclude that there are at least two extensions E_1 and E_2 such that $A_1, A_3 \in E_1$ and $A_2, A_4 \notin E_1$, and $A_2, A_4 \in E_2$ and $A_1, A_3 \notin E_2$. Hence, there is an extension where no argument has $\mathsf{perm}_w(l)$ as its conclusion, and there is an extension where no argument has $\mathsf{perm}_w(\neg l)$ as its conclusion. Therefore, $\mathsf{perm}_w(l)$ and $\mathsf{perm}_w(\neg l)$ are not justified conclusions under the stable semantics. □

4. An Argumentation Semantics for Weak Permission

We take the notions defined in the previous section and extend it to a semantics that can deal with weak permission. The new semantics is based on and extends the argumentation semantics for Defeasible Logic [9] with conditions to deal with imaginary arguments.

The first step is to use the notion of attack between arguments to define the notions of *support* and *undercut*. Support means that all proper subarguments of an argument are in the set of arguments that supports it; undercut means that the set of arguments that undercuts an argument supports an argument that attacks a proper subargument of the argument. This means that the premises of the undercut argument do not hold.

Definition 8. *A set of arguments S* supports *an argument A if every proper subargument of A is in S.*

Definition 9. *A set of arguments S* undercuts *an argument B if S supports an argument A that attacks a proper natural subargument of B.*

Example 4. *Consider a Deontic Argumentation Theory where R contains the following rules:*

$$r_1\colon \Rightarrow a \qquad r_2\colon a \Rightarrow \mathsf{obl}(b) \qquad r_3\colon \mathsf{perm}_w(\neg b) \Rightarrow d \qquad r_4\colon \mathsf{perm}(\neg b) \Rightarrow e$$

At this stage, we can define the notion of *wp-acceptable* and *wp-rejected* arguments. The main difference is that we have to distinguish between natural and imaginary arguments. An imaginary argument is an argument for a weak permission, and weak permission means that it is impossible to have a valid argument for the obligation to the contrary. In other words, weak permission succeeds when we fail to derive the obligation to the contrary. Accordingly, we must be able to tell when an argument is acceptable or when an argument is rejected. Given that weak permission requires that we reject the obligation of the contrary, an imaginary argument is acceptable when all arguments for the prohibition are rejected. A natural argument is acceptable when we can show that the arguments that attack it cannot fire (their premises do not hold).

Definition 10. *An argument A is* wp-acceptable *by the sets of arguments R and S iff*

1. *A is an imaginary argument and $\forall B \in Args$, $B > A$, B is wp-rejected by R and S; or*
2. *A is a natural argument, S supports A, and $\forall C \in Args$, $C > A$, either S undercuts C or $C \in R$.*

The next definition specifies when an argument is rejected. Specifically, an imaginary argument is rejected when there is an argument for the obligation to the contrary that is acceptable (thus we did not fail to establish the obligation to the contrary). A natural argument is rejected when one of its subarguments is rejected or when there is an argument attacking it that is supported by the already accepted arguments. Thus, there is an applicable rule for the opposite of the conclusion of the (rejected) argument

Definition 11. *An argument A is* wp-rejected *by the sets of arguments R and S iff*

1. *A is an imaginary argument and $\exists B \in Args$, $B > A$, $B \in S$; or*
2. *A is a natural argument and*

a) $\exists B \in \mathrm{Sub}(A)$, $B \neq A$, *and* $B \in R$*; or*
b) $\exists B \in Args$, $B > A$, *and* S *supports* B.

The wp-semantics (or wp-extension) is then defined by the following fixed-point construction, where we iteratively build the sets of wp-acceptable and wp-rejected arguments. Notice that since we have to capture both what arguments are acceptable and what arguments are rejected, the extension is given by the pair of such sets of arguments.

Definition 12. *The* wp-extension *of a Deontic Argumentation Theory* T *is the pair*

$$(JArgs, RArgs)$$

such that

$$JArgs = \bigcup_{i=1}^{\infty} J_i^T \qquad RArgs = \bigcup_{i=1}^{\infty} R_i^T$$

where

- $J_0^T = \emptyset$; $R_0^T = \emptyset$;
- $J_{n+1}^T = \{A \in Args : A$ *is wp-acceptable by* R_n^T *and* $J_n^T\}$;
- $R_{n+1}^T = \{A \in Args : A$ *is wp-rejected by* R_n^T *and* $J_n^T\}$.

Whenever clear from the context, we will drop the references to the sets R and S when speaking of wp-accepted and wp-rejected arguments.

Example 5. *Let us consider again the Deontic Argumentation Theory of Example 2. The arguments and the instances of the attack relation are the same for the grounded semantics and the wp-semantics.*

Let us go step-by-step through the construction of the wp-extension. The first step is to determine the arguments that are wp-acceptable and wp-rejected by J_0^T *and* R_0^T*, namely the empty set. To this end, we start by listing what arguments are supported and undercut by* J_0^T *and* R_0^T*. It is easy to verify that the only arguments supported by* J_0^T *and* R_0^T *are*

$$A_0, A_1, A_2, A_4.$$

These are the arguments that do not have any proper subarguments (we ignored all imaginary arguments since they are not involved in any attack, and the condition to). The next step is to determine the arguments that are undercut, namely:

$$A_3, A_5, A_6, A_7, A_{10}.$$

A_6 *is undercut because it is attacked by* A_1*; the arguments* A_3, A_5, A_7 *are undercut because they are attacked by* A_2*, and, finally,* A_{10} *because it is attacked by* A_4*.*

Then, we can compute J_1^T *and* R_1^T*:*

$$J_1^T = \{A_0, A_4\} \qquad R_1^T = \{A_1, A_2, A_3, A_5, A_6, A_7, A_{10}\}$$

Argument A_0 *is trivially wp-acceptable because no argument attacks it, and it does not have any proper subarguments.* A_4 *is attacked by* A_3 *and* A_7*, but these two arguments are undercut by* J_0^T*. Arguments* A_1*,* A_5*,* A_7*, and* A_{10} *are wp-rejected because they are attacked by* A_2*, which is supported by* J_0^T*. Arguments* A_2 *and* A_6 *are wp-rejected because they are attacked by* A_1*, which is supported by* J_0^T*.* A_3 *is wp-rejected because it is attacked by* A_4*, which is supported by* J_0^T*.*

We can now move to the next step, where we compute J_2^T *and* R_2^T*. In addition to the arguments supported by* J_0^T*,* J_1^T *supports* A_8 *(*$A_4 \in J_1^T$*).*

$$J_2^T = J_1^T \cup \{I_1, I_2, I_4, A_8\} \qquad R_2^T = R_1^T \cup \{I_3\}$$

The imaginary arguments I_1, I_2 and I_4 are wp-acceptable because they are attacked respectively by A_2, A_1 and A_3, and these arguments are wp-rejected by R_1^T. Argument A_8 is wp-acceptable because it is now supported by J_1^T and it is attacked by A_3 and A_7. A_3 and A_7 are undercut by J_1^T. I_3 is wp-rejected because it is attacked by A_4, which is wp-acceptable by J_1^T.

For the next and final step, we compute J_3^T and R_3^T. This time, J_2^T adds support for argument A_9 (because $I_1 \in J_2^T$), and then A_9 is wp-acceptable, because it is attacked by A_2, but A_1 attacks A_2 and it is supported by J_2^T.

Theorem 2. *For any Deontic Argumentation Theory $T = (F, R)$ the wp-extension is unique.*

Proof. The proof that the wp-extension is unique is by induction on the steps of the construction of the wp-extension, that if an argument is wp-justified/ wp-rejected by R_n^T and J_n^T, then the argument is wp-justified/wp-rejected by R_{n+1}^T and J_{n+1}^T. The induction proves that the extension of the wp-extension is monotonically increasing, and then we can apply the Knaster-Tarski Theorem to conclude that the wp-extension is the unique least/greatest fixed-point.

Let us start by the case when an argument $A \in J_1^T$. We have two sub-cases.

1) A is an imaginary argument. Thus, it does not have any proper subargument (so no argument can be wp-rejected), and every argument attacking it is rejected by $\emptyset$ and $\emptyset$ (R_0^T and J_0^T). Let B be an argument attacking A. Since A is wp-acceptable, then B is wp-rejected; therefore there is an argument C, $C > B$ and C is supported by $\emptyset$. This means that C does not have any proper subarguments, which in turn means that any J_i^T supports C rejecting B in any iteration of the construction. Hence, $A \in J_1^T$.

2) A is a natural argument. By construction A is wp-acceptable by R_0^T and J_0^T; by definition, A is supported by J_0^T, namely $\emptyset$, thus, again, A does not have any proper subarguments. Accordingly, A is supported by any J_i^T. The arguments attacking it are undercut by J_0^T. This means that for every argument B attacking A, there is an argument C supported by J_0^T. But as we have just seen for A, the argument C is supported by any J_i^T. So, $A \in J_1^T$.

We can now move to the case of a wp-rejected argument. Thus, $A \in R_1^T$. Again, we have two sub-cases.

1) A is an imaginary argument. A is wp-rejected by R_0^T and J_0^T, if there is an argument attacking A that is supported by J_0^T. We have already seen that an argument supported by J_0^T is supported by any J_i^T. Accordingly, A is wp-rejected at any step of the construction of the wp-extension; hence $A \in R_1^T$.

2) A is a natural argument. Given that $R_0^T = \emptyset$, A cannot be in R_0^T because one of its proper subarguments is in R_0^T. Thus, there is an argument B attacking in that is supported by J_0^T. We can argue, as in the other case, that B is supported by any J_i^T, and thus, $A \in R_1^T$.

For the inductive step, as usual, we assume that the property holds for arguments in J_n^T and R_n^T. Hence, for the justified arguments we can conclude that if an argument is wp-accepted for some set of arguments J_k^T ($k < n$), then the argument is supported by J_n^T; at this stage we can replicate the reasoning of the inductive base to conclude that if an argument is wp-acceptable by R_n^T and J_n^T, then it is wp-acceptable by R_{n+1}^T and J_{n+1}^T.

For the case when an argument A is in R_{n+1}^T in addition to the cases discussed in the inductive base that carry over by the inductive hypothesis, we have to examine the situation that there a proper subargument A' of A is wp-rejected by R_n^T and J_n^T. This means that $A' \in R_n^T$, and that argument has been added at a step before n. Thus, we can appeal to the inductive hypothesis to conclude that A' would be in all R_m^T, $m \geq n$. Thus, $A \in R_{n+1}^T$.

We have just proved that the construction of the wp-extension is monotonically increasing. Then, by the Knaster-Tarski Theorem, the construction has a least fixed point, and the least fixed point is unique. □

The next step is to show that the wp-semantics is correct. For correctness we have to consider two properties. We have to show that the wp-extension is conflict-free, thus the set of wp-acceptable arguments does not contain arguments for and against a given conclusion and that no argument is both wp-justified and wp-rejected.

Theorem 3. *For any Deontic Argumentation Theory $T = (F, R)$:*

- *$JArgs \cap RArgs = \emptyset$ (no argument is both wp-justified and wp-rejected);*
- *the wp-extension is conflict-free.*

Proof. We start by proving that the intersection of the wp-acceptable argument and the wp-rejected sets of arguments is empty. The proof is by contradiction and induction. Assume that there is an argument A such that $A \in JArgs$ and $A \in RArgs$. This means that there is some n, $A \in J_n^T$ and $A \in R_n^T$. We prove by induction that this is impossible.

For the inductive base, suppose that $A \in J_0^T$ and $A \in R_0^T$. We have two cases: A is an imaginary argument or A is a natural argument.

A is an imaginary argument. $A \in R_1^T$ means that there is an argument B, such that $B > A$ and $B \in J_0^T$. But $J_0^T = \emptyset$, so there is no such argument B. Hence, $A \notin R_1^T$. Thus, A cannot be in the intersection of J_1^T and R_1^T.

A is a natural argument. $A \in J_1^T$ means that $J_0^T = \emptyset$ supports A, thus $\mathrm{Sub}(A) = \{A\}$ and A does not have any proper subarguments. Moreover, J_0^T undercuts all arguments attacking A. Suppose $B > A$, then J_0^T undercuts B, thus J_0^T supports an argument C that attacks a proper subargument of D of B, $D \in \mathrm{Sub}(B), D \neq B$. $A \in R_1^T$ means that there is an argument B, such that $B > A$ and $J_0^T = \emptyset$ supports B. Thus, $\mathrm{Sub}(B) = \{B\}$, and we get a contradiction since C cannot attack a proper subargument D of B.

For the inductive step, we assume that the property holds for n ($J_n^T \cap R_n^T = \emptyset$) and we show that it holds for $n + 1$. Assume that $A \in J_n^T$ and $A \in R_n^T$. Again, we have two cases: A is an imaginary argument or A is a natural argument.

A is an imaginary argument. $A \in J_{n+1}^T$ means that every argument attacking A is wp-rejected by R_n^T and J_n^T. Thus, any argument B, such that $B > A$, is in R_n^T. $A \in R_{n+1}^T$ means that there is an argument B, such that $B > A$ and $C \in J_n^T$. Hence, $B \in J_n^T \cap R_n^T$, against the inductive hypothesis.

A is a natural argument. $A \in J_{n+1}^T$ means that J_n^T supports A, and every argument B attacking A is undercut by J_n^T. Thus, for every argument $A' \in \mathrm{Sub}(A), A' \neq A$, A' is in J_n^T. Moreover, for an argument B such that $B > A$, J_n^T supports an argument C that attacks a proper subargument of B' of B ($B' \in \mathrm{Sub}(B), B' \neq B$). $A' \in \mathrm{Sub}(A), A' \neq A$ such that $A' \in R_n^T$ (and this is against the inductive hypothesis) or (2) there is an argument B such that $B > A$ and J_n^T supports B. Therefore, $\forall B' \in \mathrm{Sub}(A), B' \neq A, B' \in J_n^T$. But, B' is undercut by J_n^T, thus there is an argument C supported by J_n^T that attacks B'. Thus, $B' \in R_n^T$ against the inductive hypothesis.

For the wp-semantics, conflict-free means that we do not have pairs of arguments A and B such that $A > B$ and $A, B \in JArgs$.

Let us suppose that an extension is not conflict-free; this means that there is some n such that there are two arguments A and B such that $A, B \in J_{n+1}^T$, and $A > B$. By construction of the attack relation, we have that A is a natural argument. Thus, $A \in J_{n+1}^T$ means that J_n^T supports A, and every argument C attacking A is undercut by J_n^T.

B is either an imaginary argument or a natural argument. If B is an imaginary argument, $B \in J_{n+1}^T$ means that every argument attacking B is wp-rejected by R_n^T and J_n^T. But, A attacks B, thus A is wp-rejected by R_n^T and J_n^T, so $A \in R_{n+1}^T$; but we just proved that this is impossible, no argument is both wp-justified and wp-rejected.

If B is a natural argument, A attacks B, and J_n^T supports A, thus, $B \in R_{n+1}^T$. But, as we have just seen, this is impossible. □

We extend the notion of wp-justified conclusions to literals and deontic literals. A literal is wp-justified if it is the conclusion of a wp-justified argument.

Definition 13. *Given a Defeasible Argumentation Theory T, an argument A is* justified *under the wp-semantics (or wp-justified) iff $A \in JArgs$.*

A literal or a deontic literal is justified *under the wp-semantics (or wp-justified) iff it is the conclusion of a justified argument under the wp-semantics.*

Example 6. *When we consider the Deontic Argumentation Theory of Example 2, and the construction of the extension in Example 5, we can see that the arguments A_0, A_4, A_8, A_9, I_1, I_2 and I_4 are wp-justified. Thus, the wp-justified conclusions are*

$$a, \mathsf{obl}(\neg q), \mathsf{perm}(\neg q), s, \mathsf{perm}_w(p), \mathsf{perm}_w(\neg p), \mathsf{perm}_w(\neg q)$$

The final step is to show that weak permission is supported by the wp-semantics. This means that we can infer that a weak permission is in force even when a conflict between two obligations exists. Accordingly, we refine the definition of a conflictual theory to be more aligned with the wp-semantics.

Definition 14. *A Deontic Argumentation Theory is* wp-conflictual *it contains a pair of rules*

$$a_0, \dots, a_n \Rightarrow \mathsf{obl}(l) \qquad\qquad b_0, \dots, b_m \Rightarrow \mathsf{obl}(\neg l)$$

such that the arguments A_i ($0 \leq i \leq n$) and B_j ($0 \leq j \leq m$) have conclusions $C(A_i) = a_i$ and $C(B_j) = b_j$, and the A_i and B_j are wp-justified. We also say that l is a wp-conflictual literal.

Theorem 4. *Let T be a wp-conflictual theory and l a wp-conflictual literal such that the proper subarguments of the arguments for $\mathsf{obl}(l)$ and $\mathsf{obl}(\neg l)$ are wp-justified. Then $\mathsf{perm}_w(l)$ and $\mathsf{perm}_w(\neg l)$ are wp-justified.*

Proof. We can assume, without any loss of generality, that the arguments B_j and C_k have no attacks. Hence, the set of attacks for the theory is

$$A_1 > A_2 \qquad\qquad A_1 > \mathsf{perm}_w(\neg l)$$
$$A_2 > A_1 \qquad\qquad A_2 > \mathsf{perm}_w(l)$$

By definition all subarguments of A_1 and A_2 are wp-justified. A_1 is the only argument attacking $\mathsf{perm}_w(\neg l)$, A_2 attacks A_1 and A_2 is supported by the wp-extension. Hence, A_1 is wp-rejected. A_1 is the only argument attacking $\mathsf{perm}_w(\neg l)$. Hence, $\mathsf{perm}_w(\neg l)$ is a justified argument. We can repeat the same argument for $\mathsf{perm}_w(l)$. □

To conclude we show that the wp-semantics is an extension of the grounded semantics. To this end we need the following lemma.

Lemma 1. *All subarguments of a justified argument are justified.*

Proof. It is an immediate consequence of Definition 10. □

Theorem 5. *Given a deontic argumentation theory, every argument justified under the grounded semantics is also justified under the wp-semantics.*

Proof. It follows immediately from Lemma 1, since an argument that is justified by a set of arguments is also supported by the same set. □

5. Related Work

Formal argumentation has been investigated for a long time in the field of Artificial Intelligence and Law (for a comprehensive though a bit outdated survey, see [10]). However, the issue of incorporating deontic reasoning in argumentation frameworks has been neglected. Moreover, the issue of weak permission received almost no attention.

van der Torre and Villata [11] extends ASPIC$^+$ [12] with deontic operators and analyses the framework in terms of Input/Output logic. However, it does not consider weak permission.

The scope of [13] is formal argumentation for handling norms in multi-agent systems, presenting methods for representing and interpreting hierarchical normative systems, and addressing open texture in legal interpretation using fuzzy logic. No focus is devoted on deontic reasoning and weak permissions: deontic assertions are simply conclusion in ASPIC$^+$ setting.

The aim of [14] is to propose an argumentation framework to address different types of normative conflicts. Arguments are built by triples (b, i, d) where b is a formula for the brute facts of the arguments, i represents the institutional facts, and finally d is the deontic conclusion of the argument, which is either an obligation or a permission. While the work considers various notions of conflicts, it does not consider weak permission. Moreover, the argumentation framework adopts Dung's semantics, including grounded semantics. Thus, it cannot deal with weak permission when unresolved conflicts exist.

Riveret, Rotolo and Sartor [15] presented a modular rule-based argumentation system designed to represent and reason about conditional norms, including obligations, prohibitions, and weak and strong permissions. However, the work treated such notions as labels of literals without exploring their argumentative nature. Similar considerations apply to [16], in which deontic argumentation and probabilistic argumentation were merged. Riveret, Rotolo and Sartor [15] presented a modular rule-based argumentation system designed to represent and reason about conditional norms, including obligations, prohibitions, and weak and strong permissions. However, the work treated such notions as labels of literals without exploring their argumentative nature. Similar considerations apply to [16], in which deontic argumentation and probabilistic argumentation were merged.

The work by Straßer, Arieli and Berkel [17, 18] focuses on the relationships between formal argumentation and sequent style and Input/Output inference systems. However, in general, they do not consider permission (and weak permission) [18] or Standard Deontic Logic [17]. While [18] restricts its focus to grounded semantics, [19, 20] consider also other semantics (preferred and stable) but no permission, and given the results of [2] these approaches are not able to characterise weak permission. The work by Straßer, Arieli and Berkel [17, 18] focuses on the relationships between formal argumentation and sequent style and Input/Output inference systems. However, in general, they do not consider permission (and weak permission) [18] or Standard Deontic Logic [17]. While [18] restricts its focus to grounded semantics, [19, 20] consider also other semantics (preferred and stable) but no permission, and given the results of [2] these approaches are not able to characterise weak permission.

The proposal in [21] builds an argumentation framework in ASPIC$^+$ style for deontic reasoning based on Standard Deontic Logic extended with a preference relation. The approach considers permission as the dual of obligation, and then evaluates arguments based on the complete semantics. Notice that in case of a conflict, the complete semantics will provide two extensions, and the proof of Theorem 1 shows the two extensions contain opposite obligations, and weak permission is not sceptically justified.

The approach closer to our work is Defeasible Deontic Logic (DDL) [4]. DDL can with conflicts and weak permission; DDL offers a constructive proof-theoretic approach to deontic reasoning, though its proof theory has a strong argumentation flavour. However, no argumentation semantics has been advanced for DDL. The wp-semantics is based on the argumentation semantics for Defeasible Logic [9]. Given that wp-semantics is an extension of the semantics proposed by [9] for defeasible logic to account for deontic operators (including weak permission) we expect that the wp-semantics characterise DDL. A preliminary analysis in argumentation was developed in [22]. In this paper a first treatment of weak permission was developed by introducing the concept of argument agglomeration set, with respect to which an argument for a prohibition is rejected, thus proving the corresponding weak permission. However, no systematic study on argumentation semantics was offered, just confining the study to DDL.

Grounded semantics and the semantics proposed in [9] correspond (respectively) to ambiguity propagation and ambiguity blocking. [23, 24] point out that legal and deontic reasoning support ambiguity blocking and ambiguity propagation. In general, the two approaches are incompatible. [24] proposed an extension of DDL, DDL$^+$ that is a conservative extension of the variants. DDL$^+$ has been defined proof-theoretically. However, DDL$^+$ is a conservative extension of DDL. Accordingly, we plan to investigate an argumentation framework that allows up to integrate the two competing semantics.

While the wp-semantics covers the case of weak permission, it fails to explain the deontic operators (apart from weak permission being the failure to conclude the prohibition of the opposite). To fill this gap, we plan to extend the approach we proposed in [25] to account for the meaning of the deontic operators in a possible world (neighbourhood) semantics for Defeasible Deontic Logic [26].

6. Conclusions

In this paper, we have proposed a new semantics for deontic argumentation that can deal with weak permission. The semantics is based on the argumentation semantics for Defeasible Logic. We have shown that the semantics is unique, conflict-free, and that no argument is both justified and rejected. We have also shown that the semantics is able to characterise weak permission when there are unresolved conflicts addressing thus the issue with grounded semantics.

Declaration on Generative AI

Apple AI was used to proofread the document to spot typos and minor grammatical errors. The authors confirm that they have reviewed and edited the content as needed and take full responsibility for the content of the publication.

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