On the Logic of Theory Base Change: Reformulation of Belief Bases (Extended Abstract)*

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Abstract

In the logic of theory change, the AGM model has acquired the status of a standard model. However, the AGM model does not seem adequate for some contexts and application domains. This inspired many researchers to propose extensions and generalizations to AGM. Among these extensions, one of the most important are belief bases. Belief bases have more expressivity than belief sets, as explicit and implicit beliefs have different statuses. In this paper, we present *reformulation*, a belief change operation that allows us to reformulate a belief base making some particular sentences explicit without modifying the consequences of the belief base. We provide a constructive method and its axiomatic characterization.

Keywords

Knowledge Representation and Reasoning, Reasoning with Beliefs, Change Theory

1. Introduction

The primary objective of the area of belief change is to identify appropriate methods for modeling belief states of rational agents and the changes that occur in these states when the agent receives new information as a result of its interaction with the world. One of the most significant extensions to the seminal work of AGM [2] is the use of belief bases [3] instead of belief sets. Belief bases consist of sets of sentences not necessarily closed under logical consequence. This allows for more expressive power, as they allow to distinguish between basic beliefs and beliefs that are inferred from basic beliefs:

Example 1. *Consider the following two belief bases:* $B1 = \{p, p \leftrightarrow q\}$ *and* $B2 = \{p, q\}$*. They have the same logical consequences, and, therefore, generate the same belief set. However, the difference between* $B1$ *and* $B2$ *is not just notational,but rather expresses in a different manner the relationship between* p *and* q*.*

It can be considered thus, that while $B1$ and $B2$ are *statically equivalent*, they fail to be *dynamically equivalent* [4] since their revision may lead to different outcomes. Up to now, there are no operations to transform $B1$ into the logically equivalent $B2$ (or the other way round). The aim of the paper is to define a constructive reformulation method and to study its axiomatic characterization. Fundamentally, the aim is to define an operator R that satisfies the following postulate, stating that no new beliefs are added or removed from the belief set:

(R1) $Cn(R(B, p)) = Cn(B)$. (Logical Equivalence)

Before outlining the technical details of our proposal, consider the next example in the legal domain. Assume the proposition p states that “Users under 18 cannot enter into binding contracts” and the

NMR 2025: 23rd International Workshop on Nonmonotonic Reasoning, November 11–13, 2025, Melbourne, Australia
*Published in AAAI 2025: the 39th Annual AAAI Conference on Artificial Intelligence [1].
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sentences $p \leftrightarrow q$ states that "Users that are not able to enter binding contracts means do not have full legal capacity". Now, assume we have the following belief base $B = \{p, p \leftrightarrow q\}$. From B we can infer that q: "Under-18s do not have full legal capacity", however, if the rules change and certain exceptions for 16-17 years-olds are allowed (for instance by obtaining parent's consent), the revision of B by this new information may create undesired effects. Since q is implicit, the revision mechanism might attempt to accommodate the new rule by altering or even discarding $p \leftrightarrow q$ or by weakening p. This creates ambiguity: do 16–17 year-olds with parental consent have full legal capacity, or is legal capacity still tied to minority status? On the other hand, an operator that makes q explicit first can act as a "clarifier", preserving the rules' consequences while improving clarity, usability, and allowing for more transparent change operations.

2. Construction of Reformulation Operations

First, we recall the two well-known constructive models of contraction functions on belief bases as we our proposal aims to exploit these well-known operators as the basis for the contruction of the new one.

Definition 1 (Partial meet base contraction [2, 5])**.** Let B be a belief base, $B \perp p$ be the set of all maximal subsets of B that do not imply p, and let $\gamma : (2^{\mathcal{L}} \times \mathcal{L}) \rightarrow 2^{2^{\mathcal{L}}}$ be a selection function that satisfies: $\gamma(B \perp p)$ is a non-empty subset of $B \perp p$ (unless $B \perp p$ is empty, in which case $\gamma(B \perp p) = \{B\}$). The *partial meet contraction* on B that is generated by γ is the operation $-_\gamma$ such that for all sentences p:

$$B -_\gamma p = \bigcap \gamma(B \perp p).$$

An operation $-$ on B is a partial meet contraction if and only if there is a selection function γ for B such that for all sentences $p : B - p = B -_\gamma p$.

Definition 2 (Kernel base contraction [6])**.** Let B be a belief base, $B \perp\!\!\!\perp p$ be the set of all minimal subsets of B that imply p σ an incision function for B, such that (i) $\sigma(B \perp\!\!\!\perp p) \subseteq \bigcup (B \perp\!\!\!\perp p)$ and (ii) if $\emptyset \neq B' \in B \perp\!\!\!\perp p$, then $B' \cap \sigma(B \perp\!\!\!\perp p) \neq \emptyset$. A *kernel base contraction* $-$ is defined as follows:

$$B - p = B \setminus \sigma(B \perp\!\!\!\perp p).$$

We are now ready to provide the construction of our reformulation operator.

Definition 3 (Reformulation)**.** Let B be a belief base, p a formula, and $-$ a base contraction function. The *reformulation* of p in B is:

$$R(B, p) = B - p \cup \{p \rightarrow q \ : \nvdash q \leftrightarrow p \text{ and } q \in B \setminus B - p\} \cup \{p\}$$

when $B \vdash p$, and $R(B, p) = B$ when $B \nvdash p$.

Note that in the definition above, $-$ can be any belief base contraction function. If $-$ is a partial meet base contraction then $R(B, p)$ is a *partial meet-based reformulation* operation, and if $-$ is a kernel base contraction then $R(B, p)$ is a *kernel-based reformulation* operation.

Basically, the operation eliminates all the implicit ways to obtain p from the belief base via a contraction function and adds p explicitly. All the explicit sentences that were erased in the contraction can be recovered, implicitly, thanks to the addition of $\{p \rightarrow q \ : \nvdash q \leftrightarrow p \text{ and } q \in B \setminus B - p\}$.

2.1. Postulates for Reformulation

In this section we introduce postulates that constitute desirable properties of reformulation operations. In addition to **(R1)**, the following are a set of minimum requirements.

(R2) If $B \vdash p$, then $p \in R(B, p)$. (Explicit Success)

(R2b) If $\nvdash p$ then $R(B,p) \setminus \{p\} \nvdash p$. (Isolation Success)

(R3) If $q \in B$ and $q \notin R(B,p)$, then there exists H such that $H \subseteq B$ and $(B \cap R(B,p)) \setminus \{p\} \subseteq H$ and $H \nvdash p$ but $H \cup \{q\} \vdash p$. (Relevance)

(R5) If $q \in B$ and $q \notin R(B,p)$, then there exists $H \subseteq B$ such that $H \nvdash p$ but $H \cup \{q\} \vdash p$. (Core-retainment)

(R4) If $p \in Cn(B')$ if and only if $q \in Cn(B')$ for all subsets B' of B, then $B \cap (R(B,p) \setminus \{p,q\}) = B \cap (R(B,q) \setminus \{p,q\})$. (Uniformity)

(R6) If $q \in B \setminus R(B,p)$, then $p \to q \in R(B,p)$. (Recovery)

(R6b) If $q \in R(B,p) \setminus (B \cup \{p\})$, then there exists r such that $r \in B \setminus R(B,p)$ and $\vdash q \leftrightarrow (p \to r)$. (Dual Recovery)

The postulates above resemble those that characterize contraction operations. The main difference is that the reformulation operator aims to reorganize the belief base by making a sentence explicit but the inferences of the original base must remain. Explicit and isolation success ensure that p is made exclusively explicit, while $R3$ ($R5$) and $R6$ ($R6b$) ensure that only knowledge that implicitly infers p is removed. Finally, $R4$ ensures that the result of reformulating B by p depends only on which subsets of B imply some element of B; that is, if two sentences have the same epistemic attitude regarding all the subsets of B, then their reformulations coincide (with the exception of the proper sentences).

Representation Theorems: In the full paper [1] we present two representation theorems for the reformulation operation defined in Definition 3, each corresponding to the two possible constructions of base contraction operations, namely partial meet and kernel, respectively. Basically, given a contraction function $-$ a reformulation operation R based on $-$ satisfies (R2), (R2b), (R3) (R5, respectively), (R4), (R6), and (R6b) if and only if $-$ is a partial meet (kernel, resp.) base contraction function.

3. Other Versions of Reformulation

In [1], we introduce several variants of the reformulation operation defined in Definition 3. We highlight here one based on the fact that if $B{-}p \vdash p \to q$ then, again if we aim for non-redundancy, it is not necessary to reincorporate $p \to q$. We can thus refine the operation as follows:

Definition 4 (Reformulation Operation 2)**.** Let B be a belief base, p a formula, and $-$ a base contraction function. The *reformulation Operation 2* of p in B is:

$$R_{M_2}(B,p) = \ B{-}p \cup \{p\} \cup \{p \to q \ : \ q \in B \setminus B{-}p \text{ and} B{-}p \nvdash p \to q\}$$

when $B \vdash p$, and $R_{M_2}(B,p) = B$ when $B \nvdash p$.

4. Conclusion and Related Work

The aim of this paper was to introduce an operation that reformulates a belief base by making explicit a particular sentence that was originally implicit. Such an operation had not been investigated before in the theory of belief base change operations. We have defined this operation of *reformulation* in terms of a base contraction operator. We have studied its axiomatic characterization in two versions that are respectively based on partial meet contraction and kernel base contraction. We have also proposed three alternatives that further refine reformulation and that enhance non-redundancy. The precise axiomatic characterization for these alternatives is left to future work.

Acknowledgments

MVM was partially supported by the Spanish project PID 2022-139835NB-C21 funded by MCIN/AEI/10.13039/501100011033, PIE 2023-5AT010 and iTrust (PCI 2022-135010-2) CHIST-ERA under grant 2022/04/Y/ST6/00001. EF was partially supported by FCT-Fundação para a Ciência e a Tecnologia, Portugal, through project PTDC/CCI-COM/4464/2020 and by NOVA LINCS ref. UIDB/04516/2020 and ref. UIDP/04516/2020 with the financial support of FCT.IP. https://doi.org/10.54499/UIDP/04516/2020.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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