

Equivalence and Splitting Techniques for Ranking Functions in Knowledge Representation and Belief Change

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Abstract

My research revolves around conditionals and ordinal conditional functions (OCFs), which assign implausibility ranks to possible worlds. OCFs can be considered an implementation of total preorders (TPOs) representing epistemic states and are, therefore, relevant for knowledge representation, non-monotonic reasoning and belief revision. This paper briefly discusses the correspondence between OCFs and TPOs, as well as applications to belief change and nonmonotonic reasoning, connections to defeasible subsumptions in description logics, and neuro-symbolic applications.

Keywords

conditionals, ranking functions, epistemic states, knowledge representation, belief change, splitting techniques

1. Introduction

The purpose of this paper is to provide an overview over my research interests, the problems I plan to tackle as a PhD student, as well as some first results. This paper refers to both published and unpublished results.

My work mainly revolves around *conditionals*, denoted as $(B|A)$, which encode conditional or defeasible statements of the form “If A holds, then (usually/typically) B holds as well”. Semantics for knowledge bases consisting of conditionals are provided by epistemic states, often represented by total preorders (TPOs) on possible worlds. Ordinal conditional functions (OCFs), which assign ranks to possible worlds, are one way to implement such total preorders.

Conditional knowledge bases with TPO- and OCF-based semantics can be used for a variety of applications. In this paper, some applications of OCFs to belief revision and nonmonotonic reasoning are presented.

In Section 2, some basics regarding epistemic states, ranking functions, belief change, and inductive reasoning are recalled. Section 3 is about the relationship between TPOs and OCFs. In Section 4, syntax splitting and case splittings for conditional revision are explained. Section 5 contains applications of OCF-based semantics to other areas that I am exploring in my ongoing work. This paper concludes with a brief summary of the presented (preliminary) results of my research and future work in Section 6.

2. Belief Change and Inductive Reasoning based on Epistemic States

Belief change refers to updating an agent’s beliefs in the light of new information using a revision operator $*$, and inductive reasoning refers to the use of inference relations $\sim_{\Delta}$ based on some form of background knowledge Δ . These two fields are deeply connected through the utilization of conditional information.

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2.1. Iterated Belief Revision

The popular *AGM framework* [1] considers three types of one-step belief changes of belief sets: *expansion* (new information is fully accepted), *revision* (new information is accepted, but some current beliefs may need to be given up), and *contraction* (some current beliefs are eliminated).

When considering *iterated revision*, i.e. allowing multiple belief revision steps in a row, paying special attention to conditional beliefs is of crucial importance to guarantee reasonable results. Therefore, revision of *epistemic states*, which may contain more information than just the set of propositional formulas an agent believes at a given time, is necessary. This is the core idea of the so-called *DP framework* of iterated belief revision [2].

Epistemic states Ψ are often represented by *total preorders* (TPOs) $\preceq_\Psi$ which order a set of possible worlds Ω according to their plausibility. A possible world ω_1 is more plausible in Ψ than another world ω_2 iff $\omega_1 \preceq_\Psi \omega_2$. A conditional $(B|A)$ is accepted by such a TPO iff at least one possible world satisfying $A \wedge B$ is more plausible than all worlds satisfying $A \wedge \neg B$.

Ordinal conditional functions (OCFs), often called *ranking functions*, were introduced in [3]. An OCF is a function $\kappa : \Omega \rightarrow \mathbb{N}_0 \cup \{\infty\}$ such that $\kappa^{-1}(0) \neq \emptyset$, i.e., at least one world is assigned the rank 0. The rank of a formula A is determined by $\kappa(A) := \min\{\kappa(\omega) \mid \omega \models A\}$, and κ accepts a conditional $(B|A)$ (short: $\kappa \models (B|A)$) iff $\kappa(A \wedge B) < \kappa(A \wedge \neg B)$. Hence, OCFs can be considered an implementation of epistemic states.

A *c-representation* is a special kind of ranking function which is based on the underlying conditional structure of a given conditional knowledge base [4]. A simplified definition is given below.

Definition 1 (c-representation). *Let Δ be a set of conditionals with $\Delta = \{(B_1|A_1), \dots, (B_n|A_n)\}$. An OCF κ is a c-representation for Δ iff it satisfies $\kappa \models \delta$ for every $\delta \in \Delta$, and is of the form*

$$\kappa(\omega) = \kappa_0 + \sum_{1 \leq i \leq n} \eta_i, \quad (1)$$

for all ω with $\kappa(\omega) \neq \infty$, where $\kappa_0, \eta_i \in \mathbb{N}_0$ with $\eta_i > 0$ suitably chosen to ensure that $\kappa \models (B_i|A_i)$.

Closely related are *c-revisions* (or, more generally, *c-changes*), which yield high-quality revision results by adhering to a so-called *principle of conditional preservation*, which was axiomatized in [4] and the ideas for which go back to [2].

2.2. Intentional Forgetting

There are two main classic kinds of forgetting in the literature: AGM contraction of propositional beliefs [1] and variable elimination in logic programming [5].

For epistemic states, and ranking functions in particular, other types of forgetting can be considered as well [6]. For example, *conditionalization* (i.e. considering only specific models) and *marginalization* (i.e. restricting the signature) of ranking functions may be interpreted as forgetting operations.

In ongoing work¹ [7], we define several subclasses of *c-contractions* (i.e. forgetting operators in the form of c-changes) based on [8] and evaluate them according to postulates adapted from both the AGM and ASP literature on forgetting [9, 10]. The goal is to provide a unifying framework for forgetting in epistemic states.

2.3. Inductive Reasoning

In [11] inductive inference operators are defined using the axioms *Direct Inference* (DI) and *Trivial Vacuity* (TV).

Definition 2 (Inductive Inference Operators C). *An inductive inference operator $\mathbf{C}$ assigns to each conditional belief base Δ an inference relation $\vdash_\Delta$ such that the following properties are satisfied:*

¹First results were presented at BRAON 2023 on Madeira.

(DI) $(B|A) \in \Delta$ implies $A \sim_{\Delta} B$.

(TV) If $\Delta = \emptyset$, then $A \sim_{\Delta} B$ only if $A \models B$.

Often, inductive reasoning is performed by (explicitly or implicitly) checking the conditionals accepted by an epistemic state induced by Δ . Moreover, inductive reasoning can be seen as a special case of belief revision [12].

3. Relationship between Total Preorders and Ranking Functions

Clearly, there is a correspondence between ranking functions and total preorders. In [13], this relationship is made precise via transformations, which are defined as follows.

Definition 3 (transformation operators). *The transformation operator τ maps each OCF κ to a TPO $\preceq_{\kappa}$ such that for all possible worlds $\omega_1, \omega_2 \in \Omega$*

$$\omega_1 \preceq_{\kappa} \omega_2 \text{ iff } \kappa(\omega_1) \leq \kappa(\omega_2) \quad (2)$$

holds. The transformation operator ρ maps each TPO $\preceq_{\Psi}$ to a ranking function κ_{Ψ} by setting

$$\kappa_{\Psi}(\omega) = \min\{\kappa(\omega) \mid \kappa \in \tau^{-1}(\Psi)\}. \quad (3)$$

The operator ρ is a well-defined such that $\kappa_{\Psi}(\omega_1) \leq \kappa_{\Psi}(\omega_2)$ iff $\omega_1 \preceq_{\Psi} \omega_2$ holds, i.e., $\kappa_{\Psi} \in \tau^{-1}(\Psi)$. Moreover, it holds that $\tau \circ \rho = id$, but $\rho \circ \tau \neq id$ in general.

There are several open questions regarding the relationship between TPOs and OCFs: If one wanted to implement TPO revision using ranking functions, which OCF would be the most adequate to represent a TPO in a revision scenario? And in which cases is the chosen ranking representation irrelevant, and any ranking function representing the total preorder would yield the same revision result in the end? The difference between the possible ranking representations lies in the position of the empty layers in the OCF, i.e. the natural numbers to which no world is assigned. Do these empty layers have drawbacks or do they offer advantages over TPOs?

In [14], first steps were taken towards answering the questions stated above by formalizing a notion of *revision equivalence* of ranking functions, i.e. a property of equivalent ranking functions to remain equivalent after revising them by the same information, and by providing first results for how revision equivalence can be ensured. It turns out that the DP postulates alone are not strong enough to guarantee a preservation of equivalence.

Preservation of equivalence between ranking functions is, of course, not only relevant for revision, but also for other belief change scenarios. For example, in [7], equivalence postulates are considered for forgetting operators.

4. Splitting Techniques

Splitting techniques can be used to handle epistemic states, and OCFs in particular, in a modular way. For my research, I mainly focus on two kinds of splittings: syntax splittings and case splittings.

4.1. Syntax Splitting

The concept of *syntax splitting* for propositional revision goes back to [15]. The idea is to take relevance into account by partitioning the propositional signature and only changing those beliefs during revision for which the new information is syntactically relevant. In order to do so, the belief set (resp. the conditional knowledge base, or even the whole epistemic state) is split based on the partition of the syntax. Afterwards, belief change or inference can be performed within the relevant local context.

Syntax splitting is useful because it allows for revisions and reasoning to be executed more efficiently (at least when only a small part of the signature should be affected), since reducing the signature means exponentially reducing the amount of possible worlds that need to be considered.

c-Revisions fulfill (a strong version of) syntax splitting [16], and inductive inference relations based on c-representations (c-inference) satisfy adopted syntax splitting postulates for inductive reasoning [11]. In ongoing research [17] building on [16], we propose merging principles and strategies for marginalized epistemic states, with a focus on preserving syntax splittings.

4.2. Kinematics

The *kinematics principle* has its origins in probabilistic revision [18, 19] and was adapted as *ranking kinematics* for ranking functions in [20]. The idea behind ranking kinematics is that when revising by conditionals whose premises concern different exclusive cases (i.e. there exists a *case splitting*), one should be able to revise by each case independently from the other cases. Moreover, when revising by conditional information, the plausibility of a conditional’s premise should not affect the revision process. It was shown in [20] that c-revisions adhere to this principle.

Based on the work mentioned above, it is part of my ongoing research to explore how the ideas behind the kinematics principle can be utilized for inductive reasoning. As a first result, we propose a kinematics postulate for inductive reasoning operators in [21].

Like syntax splittings, case splittings offer the possibility to perform revisions (and inference) more efficiently by reducing the set of possible worlds that need to be considered. The difference between the two splitting techniques is that syntax splitting can be used to focus on possible worlds over a reduced set of atoms, while case splittings can be used to focus on possible worlds satisfying a specific formula. Hence, a natural next step would be to combine these two kinds of splittings.

5. Other Applications of OCFs

Besides investigating fundamental properties of TPO- and OCF-based operators, it is part of my ongoing research to explore possible applications of the OCF framework.

5.1. First-Order Conditional Semantics for Defeasible DL Knowledge Bases

Similar to propositional conditionals, defeasible subsumptions $A \sqsubseteq B$ encoding statements of the form “Usually, As are Bs” are an extension for description logics (DLs), which are (usually decidable) fragments of first-order logic. Different semantics for defeasible DL knowledge bases have been proposed [22, 23, 24], but their connection to OCF-based semantics remains to be explored. One notable work in this regard is [25].

In [26], first steps are taken to explore the relationship between a ranking-based first-order conditional semantics, which was proposed in [27], and DL approaches to defeasible reasoning [23, 24]. Moreover, it is proven that the OCF-based approach fulfills several rationality postulates proposed for non-monotonic reasoning in DLs by [23, 28].

A key difference is that many DL semantics rely on an orderings over domain elements, representing typicality, while OCFs represent orderings over possible worlds. Defeasible subsumptions between concepts are often evaluated by considering the most typical elements for a specific concept. For first-order conditionals, however, representatives for conditionals (instead of individual predicates) are considered. The connection between typical elements (for predicates resp. concepts) and representatives (for conditionals resp. defeasible subsumptions) remains to be explored.

5.2. Neuro-Symbolic Applications

Neuro-symbolic approaches combine neural networks and formal logics, either for the purpose of improving a neural network’s inferential capabilities or as a means to explain the inner workings of

neural networks, which are often perceived as a “black box” by users.

In [29], we propose a methodology to extract propositional conditional knowledge bases from trained feed-forward neural networks. Similar to an approach by [30] for the extraction of defeasible DL knowledge bases from such networks, neurons are represented as atomic propositions, and connections between neurons are encoded as conditionals. However, our approach is purely qualitative. Our results show that the extracted knowledge bases do not invent inferences, i.e. everything inferred from those knowledge bases can be inferred from the neural network itself too. However, the converse does not necessarily hold. Thus, an interesting question is how far the accuracy of OCF-based qualitative explanations for neural networks can be pushed.

5.3. Epistemic Change Explanation

Even logic-based methods can appear as “black boxes” when the underlying mechanisms are unknown. When an epistemic state change is observed from the outside, i.e. only the prior and posterior state are known, it is not obvious whether the underlying belief change mechanism satisfies desired postulates.

In [31], we explore how TPOs and OCFs can be utilized in order to explain epistemic state changes, which correspond to changes in conditional beliefs. We investigate under which circumstances a transition from one epistemic state (represented as a TPO or OCF) to another can be modeled as a belief change process in (extensions of) the AGM/DP framework.

6. Summary and Future Work

The goal of my research is to contribute to the topics presented in this paper, with a focus on applying OCF-splitting techniques to belief revision and non-monotonic reasoning.

One part of my research is concerned with forgetting in epistemic states. In [7], a unifying framework for OCF-based forgetting operators will be presented, as well as an evaluation of postulates inspired by both the AGM and ASP literature.

Another interesting research question is how results for TPOs in the literature transfer to OCFs and vice versa, and whether qualitative versions of techniques developed for OCFs can be formulated in the style of [13]. Closely related is the investigation of limitations for the implementation of TPOs via OCFs. The paper [14] contains a first milestone in these directions by providing a formalization of revision equivalence and first results on how to ensure preservation of equivalence between ranking functions across revisions.

A proposal for a kinematics principle for inductive reasoning will be presented in [21]. While syntax splittings and case splittings already offer a lot of flexibility when processing OCFs, the question of whether and how these two approaches can be combined is still open. The goal here is to develop methods to split OCFs in both ways, process the parts individually and then merge them back together. Some results in this direction are expected to be published in the near future [17].

As for first-order conditionals, we present some first results on properties from the DL literature satisfied by first-order OCF-based semantics in [26]. My goal is to continue working towards achieving a deeper understanding for properties of semantics for first-order conditional knowledge bases, as well as the relationship between (OCF-based semantics for) first-order conditionals and defeasible subsumptions.

Since neural networks (or statistical learning methods in general) are powerful tools yet often opaque to the user, there is a lot of potential for both explanations and improvements of their behavior based on formal logics. My goal in that respect is to continue to explore how OCFs can be utilized to that end. A first approach for how a propositional conditional knowledge base with an OCF-based semantics can be extracted from a feed-forward neural network can be found in [29].

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Declaration on Generative AI

The author has not employed any Generative AI tools.

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