Best-acceleration filter for moving object tracking

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Abstract

This paper presents the derivation and effectiveness of the best-acceleration (BA) filter, which minimizes the steady-state acceleration-error variance in an $\alpha-\beta-\gamma$ tracking filter based on a constant-acceleration motion model. The purpose is to enhance the tracking performance for a target object. Currently, the minimum-variance (MV) filter has already been proposed to minimize the steady-state position-error variance. The performance of the proposed BA filter was evaluated by comparing it with the MV filter. The results demonstrate the effectiveness of the BA filter, supporting the optimality of the BA filter in minimizing the steady-state acceleration-error variance.

Keywords

alpha-beta-gamma tracking filter, constant acceleration model, steady-state error, optimal acceleration prediction

1. Introduction

Monitoring systems for robots and intelligent vehicles that use remote sensors such as cameras, LiDAR, depth sensors, and radar require accurate tracking of moving objects. For such applications, Kalman-filter-based trackers are commonly used to estimate position, velocity, and acceleration [1–5]. In the Internet of Things (IoT) era, all sensors embedded in measurement systems and their targets (e.g., cars, robots, and smart equipment) are connected, and their data can be fused. As a result, such systems can acquire various motion parameters not conventionally used and can now be exploited for tracking and navigation applications. Although tracking systems are generally applied to moving-object tracking, there are many other systems in IoT applications, including state-of-charge estimation of Li-ion batteries [6], state estimation and stability control in smart grids [7], microclimate forecasting that optimizes ventilation and irrigation in smart greenhouses [8], and vibration-based fault diagnosis for predictive maintenance of rotating industrial machinery [9]. Thus, the design of tracking systems is essential for the development of various IoT applications.

The signal processing methodology for a tracking system is known as a tracking filter. For moving object tracking, tracking filters estimate the future state of parameters such as position, velocity, acceleration, and other states of a target based on observed data from sensors such as LiDAR and radar. A tracking filter also aims to smooth the estimation results to achieve higher

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prediction accuracy. Representative tracking filters include the Kalman, α–β, and α–β–γ filters [1]. Figure 1 shows the structure of a tracking filter.

Figure 1: The structure of the tracking filter

This study focuses on the optimal design of the α–β–γ filter, which is one of the simplest tracking filters. The α–β–γ filter is a one-dimensional tracking filter that uses a constant-acceleration motion model and is known as the steady-state Kalman filter [2, 3]. Because users can freely configure the filter gains (α, β, γ), it requires significantly less computational effort than the more widely used Kalman filter and its variants. For this reason, even when other filters such as the Kalman filter are employed, it can be beneficial to use the α–β–γ filter to predict performance in advance.

For the optimal gain design of the α–β–γ filter, the MV filter criterion has been proposed [4]. The MV filter criterion minimizes the steady-state variance of the predicted position errors and is known to achieve smaller prediction errors than the well-used Kalman-filter criterion [5]. However, the MV filter criterion optimizes only the prediction of position; other parameters, including acceleration, are not optimized. In certain applications in intelligent vehicles and motion estimation using only position sensors, tracking filters aim to estimate and predict acceleration (e.g., estimating sports performance using acceleration information obtained via remote motion measurements [10]). However, the theoretical properties of a tracking filter that optimizes acceleration prediction are unknown.

In this study, we theoretically analyze the performance of the α–β–γ filter for the best acceleration prediction. The optimal gains that minimize the variance of the predicted acceleration are derived in closed form. We term the proposed acceleration-optimized α–β–γ filter the BA filter. The performance of the conventional MV filter and the proposed BA filter, both with optimal gains, is compared through theoretical analyses and numerical simulations.

2. $\alpha - \beta - \gamma$ tracking filter

2.1. Definition and performance indices

The α–β–γ filter is defined based on a constant-acceleration motion model, with a sampling interval denoted as T. The filter gains are denoted by α, β, and γ, while the measured position at time step k is denoted by $x_{o,k}$. The smoothed position, velocity, and acceleration are defined as [1, 2].

$$x_{s,k} = x_{p,k} + \alpha(x_{o,k} - x_{p,k}) \tag{1}$$

$$v_{s,k} = v_{p,k} + \frac{\beta}{T}(x_{o,k} - x_{p,k}) \tag{2}$$

$$a_{s,k} = a_{p,k} + \frac{\gamma}{T^2}(x_{o,k} - x_{p,k}) \tag{3}$$

The predicted position, velocity, and acceleration are defined as [1, 2]

$$x_{p,k} = x_{s,k-1} + T v_{s,k-1} + \frac{T^2}{2} a_{s,k-1} \tag{4}$$

$$v_{p,k} = v_{s,k-1} + T a_{s,k-1} \tag{5}$$

$$a_{p,k} = a_{s,k-1} \tag{6}$$

By iterating these smoothing and prediction processes, the filter sequentially estimates and predicts the target state.

One of the fundamental performance indices of the α–β–γ filter is trackability of the target moving according to the filter's motion model considering the measurement noises. Since this performance is evaluated based on errors that are caused by random measurement noises, it can be evaluated in terms of the variance of the prediction errors. The true values of the target object's position, velocity, and acceleration are denoted by x_k, v_k, and a_k. We define the steady-state errors as follows. Because the steady-state error does not depend on time because of the steady-state assumption, the time index is omitted. The variance of the steady-state position error is expressed as

$$\sigma_p^2 = E[(x_p - x)^2] \tag{7}$$

Similarly, the variance of the steady-state acceleration error is expressed by

$$\sigma_a^2 = E[(a_p - a)^2] \tag{8}$$

By eliminating x_p, v_p, and a_p using equations (1)–(6), the variances σ_p^2 and σ_a^2 can be expressed in terms of the filter gains α, β, γ, and the measurement error variance A as

$$\sigma_p^2 = \frac{8\beta^2}{(2-\alpha)g(\alpha,\beta,\gamma)} A + \frac{\alpha}{(2-\alpha)} A \tag{9}$$

$$\sigma_a^2 = \frac{4\beta\gamma^2}{g(\alpha,\beta,\gamma)} \frac{A}{T^4} \tag{10}$$

where,

$$\begin{aligned} g(\alpha,\beta,\gamma) &= 2\alpha\beta(4 - 2\alpha - \beta) - \gamma(8 - 8\alpha - 2\beta + \alpha\beta + 2\alpha^2) \\ &= (2\alpha\beta - \gamma(2-\alpha))(4 - 2\alpha - \beta) \end{aligned} \tag{11}$$

2.2. Conventional minimum-variance filter

The conventional MV filter assumes γ is constant, and determines the gains α and β that minimize σ_p^2. By taking the partial derivatives of equation (9) with respect to α and β and setting them to zero, we obtain the two following equations [4]

$$4\alpha\beta = \gamma(8 - 4\alpha - \beta) \tag{12}$$

$$\beta^3 + 2(3\alpha - 8)\beta^2 + 4(3\alpha^2 - 12\alpha + 16)\beta - 8\alpha^2(2 - \alpha) = 0 \tag{13}$$

From equations (12) and (13), the gains α, β and γ are determined.

3. Derivation of proposed best acceleration filter

Whereas the conventional $\mathbf{MV}$ filter minimizes the position prediction error, this study derives an α–β–γ filter that minimizes the steady-state acceleration prediction error. In this paper, the proposed filter is referred to as the $\mathbf{BA}$ filter.

Similar to the $\mathbf{MV}$ filter, we find the gains α and β that minimize σ_a^2 subject to γ being constant. We take partial derivatives of equation (10) with respect to α and β, and find their minimum points.

By taking the partial derivative with respect to α, we have

$$\frac{\partial \sigma_a^2}{\partial \alpha} = -\frac{4\beta\gamma^2}{[g(\alpha,\beta,\gamma)]^2}\frac{\partial g(\alpha,\beta,\gamma)}{\partial \alpha} \times \frac{A}{T^4} \tag{14}$$

Since $\frac{\partial \sigma_a^2}{\partial \alpha} = 0$ is assumed as a condition,

$$\frac{\partial g(\alpha,\beta,\gamma)}{\partial \alpha} = 0 \tag{15}$$

is satisfied. By taking the partial derivative of equation (11) with respect to α, we get the following equation:

$$\frac{\partial g(\alpha,\beta,\gamma)}{\partial \alpha} = 2\beta(4 - 4\alpha - \beta) - \gamma(-8 + \beta + 4\alpha) \tag{16}$$

From equation (15) and (16), we get the following equation:

$$2\beta(4 - 4\alpha - \beta) - \gamma(-8 + \beta + 4\alpha) = 0 \tag{17}$$

Next, we also calculate the partial derivative with respect to β as

$$\frac{\partial \sigma_a^2}{\partial \beta} = \frac{4\gamma^2}{[g(\alpha,\beta,\gamma)]^2}\left\{g(\alpha,\beta,\gamma) - \beta\frac{\partial g(\alpha,\beta,\gamma)}{\partial \beta}\right\} \times \frac{A}{T^4} \tag{18}$$

Since $\frac{\partial \sigma_a^2}{\partial \beta} = 0$ is assumed as a condition,

$$g(\alpha,\beta,\gamma) - \beta\frac{\partial g(\alpha,\beta,\gamma)}{\partial \beta} = 0 \tag{19}$$

is satisfied. By taking the partial derivative of equation (11) with respect to β,we can get the following equation as

$$\frac{\partial g(\alpha,\beta,\gamma)}{\partial\beta} = 8\alpha - 4\alpha^2 - 4\alpha\beta - \gamma(\alpha-2) \tag{20}$$

By substituting equation (20) into equation (19), the following equation is obtained:

$$g(\alpha,\beta,\gamma) - \beta\frac{\partial g(\alpha,\beta,\gamma)}{\partial\beta} = 2\left\{\alpha\beta^2 - \gamma(2-\alpha)^2\right\} = 0 \tag{21}$$

By simplifying Equation (21), we obtain

$$\alpha\beta^2 - \gamma(2-\alpha)^2 = 0 \tag{22}$$

From the above, we obtained equations (17) and (22). Using these equations, we express σ_a^2 in terms of γ. From equation (22), we obtain

$$\gamma = \frac{\alpha\beta^2}{(2-\alpha)^2} \tag{23}$$

By substituting equation(23) into equation (17), we obtain

$$0 = 2\beta(4-4\alpha-\beta) - \frac{\alpha\beta^2}{(2-\alpha)^2}(-8+\beta+4\alpha) \tag{24}$$

Simplifying equation (24),we obtain

$$\begin{aligned} 0 &= 2\beta(4-4\alpha-\beta)(2-\alpha)^2 - \alpha\beta(-8+\beta+4\alpha) \\ &= -\alpha\beta^2 - 2(3\alpha-2)(\alpha-2)\beta + 8(\alpha-2)^2(\alpha-1) \end{aligned} \tag{25}$$

By using the solution formula, β is expressed in terms of α, and we obtain

$$\begin{aligned} \beta &= \frac{-(3\alpha-2)(\alpha-2) \pm \sqrt{(3\alpha-2)^2(\alpha-2)^2 - 8\alpha(\alpha-2)^2(\alpha-1)}}{\alpha} \\ &= \frac{-(3\alpha-2)(\alpha-2) \pm (\alpha-2)^2}{\alpha} \end{aligned} \tag{26}$$

From equation (26),

$$\beta = 2(2-\alpha) \tag{27}$$

or

$$\beta = \frac{4(\alpha-1)(2-\alpha)}{\alpha} \tag{28}$$

Assuming that equation (27) holds, γ is

$$\gamma = 4\alpha \tag{29}$$

By substituting equation (27) and (29) into equation (11),

$$g(\alpha, \beta, \gamma) = 0 \tag{30}$$

Equation (30) implies that the denominator of σ_a^2 becomes zero and is not appropriate as the solution. Assuming that equation (28) holds, γ is

$$\gamma = \frac{16(\alpha - 1)^2}{\alpha} \tag{31}$$

Equation (31) implies

$$0 = 16(\alpha^2 - 2\alpha + 1) - \alpha\gamma \tag{32}$$

By using the solution formula, α is expressed in terms of γ, and we obtain

$$\alpha = \frac{(\gamma + 32) \pm \sqrt{\gamma(\gamma + 64)}}{32} \tag{33}$$

From equation (33), we can express α as

$$\alpha = \frac{(\gamma + 32) - \sqrt{\gamma(\gamma + 64)}}{32} \tag{34}$$

or

$$\alpha = \frac{(\gamma + 32) + \sqrt{\gamma(\gamma + 64)}}{32} \tag{35}$$

By applying the arithmetic–geometric mean inequality,

$$\gamma + 32 \geq \sqrt{\gamma(\gamma + 64)} \tag{36}$$

Assuming that Equation (31) holds, and referring to Equation (36), we obtain

$$\alpha \leq 0 \tag{37}$$

Equation (37) indicates that Equation (31) is the inappropriate solution because the gain becomes negative. Therefore, equation (35) is appropriate to determine α. Thus, β is expressed in terms of γ as

$$\beta = \frac{4(\gamma + \sqrt{\gamma(\gamma + 64)})(64 - \gamma - \sqrt{\gamma(\gamma + 64)})}{(\gamma + 32 + \sqrt{\gamma(\gamma + 64)})^2} \tag{38}$$

By substituting equations (35) and (38) into Equation (10), we arrive at the minimum variance of the acceleration prediction error as

$$\sigma_a^2 = \frac{1024\{\gamma + \sqrt{\gamma(\gamma + 64)}\}\gamma^2}{\{(\gamma - 32) + \sqrt{\gamma(\gamma + 64)}\}^2\{-3\gamma + \sqrt{\gamma(\gamma + 64)}\}} \times \frac{A}{T^4} \tag{39}$$

The equations (35), (38), and (39) are the main results of this paper that clarify the gains of the α–β–γ filter with best acceleration prediction. Furthermore, the steady-state minimum variance of the prediction errors is expressed solely in terms of γ.

4. Evaluation

4.1. Numerical analysis results

We present numerical analyses to validate the derived results. For a model undergoing constant acceleration motion, we evaluate and compare the $\mathbf{MV}$ filter and the $\mathbf{BA}$ filter in terms of σ_a^2 and σ_p^2 using equations against the fixed γ. We calculate these error variances by substituting the optimal gains ($\mathbf{MV}$ filter: equations (12) and (13), $\mathbf{BA}$ filter: equations (35) and (38)). In the analyses, A and T are normalized to 1.

Figures 2 and 3 show the numerical analysis results of σ_a^2 and σ_p^2, respectively. Figure 2 shows the numerical analysis results of σ_a^2, and Figure 3 shows the numerical analysis results of σ_p^2. Figure 2 shows that the $\mathbf{BA}$ filter performs better than the $\mathbf{MV}$ filter in minimizing σ_a^2. Figure 3 shows that, within the γ range from 0 to 1, the $\mathbf{MV}$ filter performs better than the $\mathbf{BA}$ filter in minimizing σ_p^2. These results indicate the theoretical performance differences between the two types of filters.

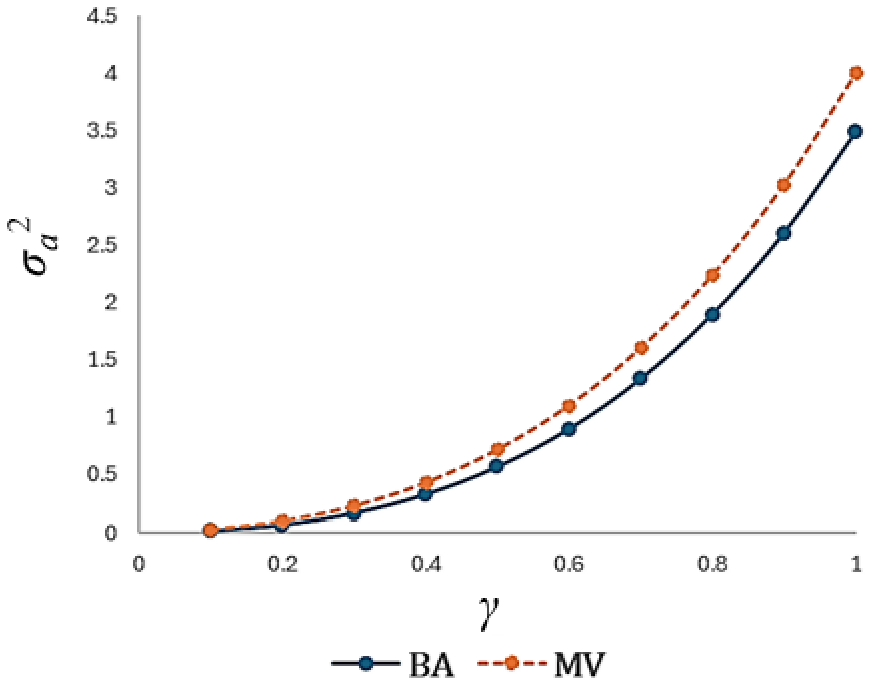

Figure 2: Analysis results of σ_a^2

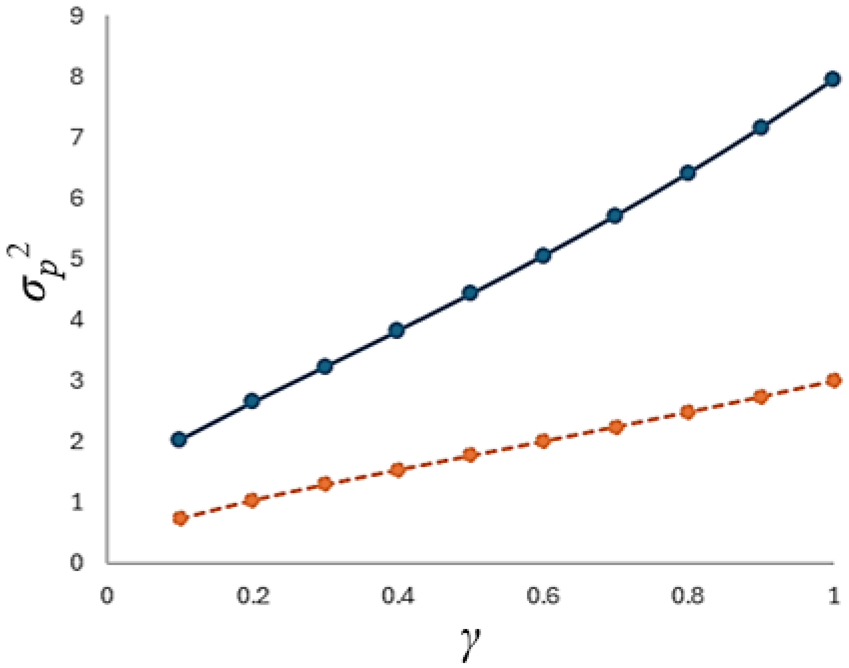

Figure 3: Analysis results of σ_p^2

4.2. Numerical simulation results

This subsection presents a numerical simulation to validate the derived results and to show the effectiveness under realistic sensing conditions with random noise and to corroborate the analysis. A target moved with a constant acceleration of 1 m/s^2 and a sampling interval T of 0.1 s and the observation period was 60 s. Observation noise was randomly applied within the range 0.01 to 0.3. Figures 4 and 5 show the simulation results for absolute prediction errors in acceleration and position, respectively. Figure 4 shows that the acceleration errors of the $\mathbf{BA}$ filter become smaller than those of the $\mathbf{MV}$ filter. As the steady-state error reflects the error in the limit as time approaches infinity, this indicates not only that the $\mathbf{BA}$ filter is more suitable for minimizing σ_a^2 but also that the results are consistent with Figure 2. Similarly, Figure 5 shows that the position errors of the $\mathbf{MV}$ filter consistently become smaller than those of the $\mathbf{BA}$ filter. This indicates not only that the $\mathbf{MV}$ filter is more suitable for minimizing σ_p^2, but also matches the results with Figure 3. Thus, the numerical simulation results prove the effectiveness of the proposed $\mathbf{BA}$ filter and its difference from the conventional $\mathbf{MV}$ filter.

5. Conclusion

In this paper, we derived the $\mathbf{BA}$ α–β–γ tracking filter that minimizes the steady-state variance of the predicted acceleration. The closed forms of the $\mathbf{BA}$ filter gains α, β and γ are given in equations (35) and (38). In addition, we conducted a performance comparison with the

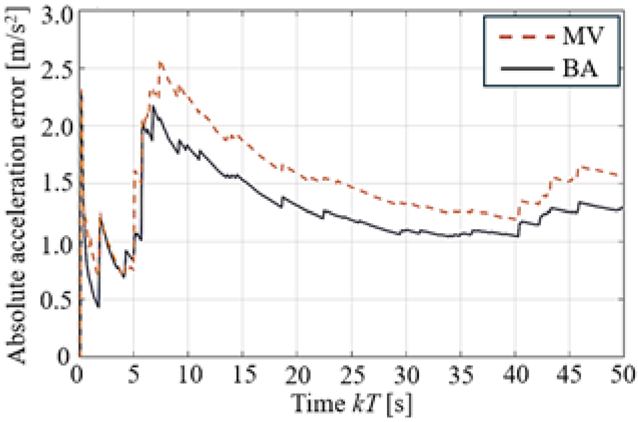

Figure 4: Simulation results for acceleration prediction error

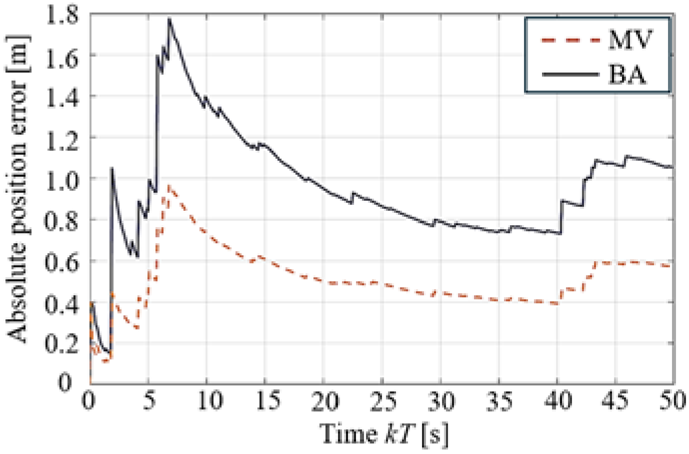

Figure 5: Simulation results for position prediction error

conventional $\mathbf{MV}$ filter that is designed to minimize position prediction errors. As a result, both numerical analysis and simulation demonstrate that the $\mathbf{BA}$ filter performs better than the $\mathbf{MV}$ filter for acceleration prediction.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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