Phase categories as a tool for modeling uncertainty in higher education programs and program learning outcomes*

Hryhorii Hnatiienko[1,†], Oleg Ilarionov[1,†], Hanna Krasovska[1,†], Larysa Myrutenko[1,*,†], and Anzhelika Tkachuk[1,†]

[1] *Taras Shevchenko National University of Kyiv, 64/13 Volodymyrska str., 01601 Kyiv, Ukraine*

Abstract

A new tool for modeling uncertainty in educational programs in the form of phase categories is proposed. Uncertainty naturally exists in the reflections between the topics, outcomes, and competencies of educational programs. A review of the literature exploring the formalization and evaluation of educational programs is conducted. A diagram of the relationships between the elements of the model within an educational program is provided. Approaches to applying category theory to describe educational programs in the form of a rigorous mathematical structure are described, where objects are interpreted as topics or outcomes, and morphisms are interpreted as logical dependencies between them. A number of approaches to the reasonable determination of the values of weight coefficients of mutual influences between functor mappings are proposed. Several lemmas are formulated and proved, which mathematically justify the authors' proposed toolkit for modeling uncertainty. Examples of the application and interpretation of the described phase models are given. The prospects for further research and the possible development of uncertainty modeling in educational programs using phase-category tools are considered.

Keywords

program learning outcomes, uncertainty, modeling and structuring of educational content, weight coefficients, personal educational trajectories

1. Introduction

The higher education system in Ukraine is undergoing profound reforms related to the transition to a competency-based approach and the updating of state standards. The new standards no longer define program learning outcomes (PLOs), but only outline integral competencies and a list of general and professional competencies. The formulation of SLOs is entrusted directly to the developers of educational programs, which, on the one hand, increases the autonomy of educational institutions, but on the other hand, creates the challenge of ensuring transparent and reasonable correspondence between SLOs and the competencies defined by the standards.

In such conditions, the content of educational disciplines becomes crucial. Disciplines are carriers of knowledge and skills that, through the formation of SLOs, ensure the achievement of graduate competencies. Gaps or duplications in course topics or imbalances in their structure directly affect the quality of training, even if formally the program covers all the necessary PRN and competencies. In this regard, it is important to find formalization tools that can assess the quality of educational content not only at the level of an individual course, but also in the context of the entire educational program.

Traditional tools, such as knowledge graphs and ontological models, effectively model semantic relationships, but are limited in expressing compositional and complex transformations between

* *CPITS-II 2025: Workshop on Cybersecurity Providing in Information and Telecommunication Systems, October 26, 2025, Kyiv, Ukraine*

* Corresponding author.
† These authors contributed equally.
hnatiienko@knu.ua (H. Hnatiienko); oleg.ilarionov@knu.ua (O. Ilarionov); hanna.krasovska@knu.ua (H. Krasovska); myrutenko.lara@gmail.com (L. Myrutenko); angelikatkachuk@knu.ua (A. Tkachuk)
0000-0002-0465-5018 (H. Hnatienko); 0000-0002-7435-3533 (O. Ilarionov); 0000-0003-1986-6130 (H. Krasovska); 0000-0003-1686-261X (L. Myrutenko); 0000-0002-3380-2427 (A. Tkachuk)

elements of the educational process. Category theory, which emerged as a mathematical apparatus for describing abstract structures and relationships, is increasingly being used in computer science, logic, and system modeling. Its potential lies in its ability to formalize compositions, mappings between structure levels, and consistent transformations, making this approach promising for the educational sphere. The use of the categorical apparatus opens up opportunities for accurate modeling of dependencies between the topics of the educational component, program learning outcomes, and competencies, as well as for identifying critical nodes and gaps in the educational content.

2. The current state of research on the problem

The issues of formalization and evaluation of educational programs are actively researched in education, computer science, and applied mathematics. Modern approaches use network models, ontologies, and knowledge graphs, which allow structuring educational content and reflecting the connections between its components [1, 2]. Such tools provide visualization of educational programs and partial quantitative analysis, but they are limited in expressing compositional and multilevel representations between learning outcomes, program outcomes, and competencies.

The use of mathematical methods for modeling and structuring educational content is one of the leading areas of contemporary research. Examples include the analysis of educational programs through network models [3], the development of curricular analytics for quantitative program evaluation [1], and the construction of models for formalizing the interaction between educational content and student learning trajectories [2]. Although most of the work focuses on mathematics education, similar approaches are beginning to be actively implemented in the field of information technology [4, 5], in particular for evaluating the effectiveness of courses using Bayesian networks [6] and optimizing the integration of IT into digital education [7].

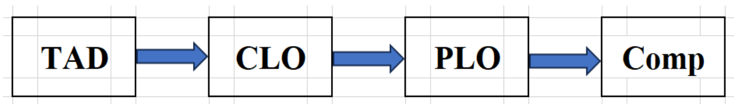

Figure 1: 1907 Franklin Model D roadster. Photograph by Harris & Ewing, Inc. [Public domain],

The following abbreviations are used in Figure 1: TAD is an academic discipline topic; CLO is a learning outcome; PLO is program learning outcome; Comp is competency.

In this context, ontological models and knowledge graphs, which have proven to be effective tools for organizing educational content [8, 9], play an important role. They allow formalizing the conceptual apparatus of a discipline, reflecting hierarchical and semantic relationships between concepts, and creating visual knowledge maps for navigation and search. In modern research, these approaches are used to build domain ontologies of courses [10], model curricula and syllabi [11], support educational processes [12], and integrate educational materials into knowledge graphs [13].

Despite this, in the context of a competency-based approach, ontologies and knowledge graphs reveal a number of limitations. They are mostly declarative and do not describe the procedural aspect of the formation of results and competencies, nor do they take into account the sequence or alternatives of educational trajectories. Graph structures do not provide a strict compositional logic, which makes it impossible to formally describe educational paths. The relationships between the different levels of the educational structure, shown in Figure 1, are established by experts and cannot be verified for consistency at. In addition, ontologies do not have built-in mechanisms for quantitative assessment of program coverage or balance.

These limitations necessitate the use of more formalized models capable of describing compositional relationships and mappings between different levels of the educational process. Such possibilities are provided by category theory, which provides a rigorous mathematical apparatus for modeling structures and relations. Its fundamental principles are set out in the classic works of

Mac Lane [14] and Awodey [15]. In computer science, the categorical apparatus is actively used to formalize programming languages and data types [16]. Further research [17, 18] emphasizes that categories provide a natural way to describe compositional structures, which creates the basis for their application in educational processes.

The key constructs of category theory—categories, functors, and natural transformations—allow for the systematic modeling of multilevel educational structures. Subjects can be interpreted as objects, logical dependencies between them as morphisms, and reflections in learning outcomes and competencies as functors. Natural transformations, in turn, open up opportunities for comparing different teaching strategies or learning scenarios.

Of particular importance is the concept of Olog [19, 20], which demonstrates how knowledge can be represented as a category with morphisms corresponding to semantic relations and form mappings between different ontological structures. This brings the categorical approach closer to a tool for modeling educational content.

Thanks to this, category theory expands the possibilities of analysis: it allows not only to formally describe the structure of educational content, but also to quantitatively assess the coverage of competencies by topics, identify duplications or gaps, and explore the variability of educational trajectories. Thus, the categorical approach emerges as a promising direction for systematic assessment of the quality of training at the level of disciplines and educational programs.

3. Application of category theory to describe educational programs

We will apply category theory to describe educational programs in the form of a strict mathematical structure, where objects are interpreted as topics or results, and morphisms as logical dependencies between them. This approach allowed the authors to perform a formal check of the consistency and coverage of the programs. Despite its novelty, this model operates exclusively with binary relations: the presence or absence of a connection. It does not take into account the variability of influences inherent in the real educational process.

To account for the fuzziness in modeling educational systems, methods based on fuzzy set theory are gaining increasing attention [21]. In pedagogical analytics and educational research, they are used to describe uncertainty in student knowledge assessments, in the formulation of learning outcomes, and in the construction of individual educational trajectories [22, 23]. However, these approaches usually remain at the level of individual fuzzy logic methods and are not integrated into the overall structural model of the educational program.

A promising direction for combining structural and fuzzy approaches is the use of fuzzy categories. The idea was first proposed in works that generalize classical category theory by introducing degrees of membership for morphisms [24, 25]. In computer science, phase categories are used to model fuzzy databases, intelligent systems, and formalize uncertainty in algorithms [26, 27]. Such models allow you to simultaneously preserve composability and introduce a quantitative measure of the strength of the connection.

Despite the existence of developments in the field of mathematics and computer science, there have been no attempts to apply phase categories directly to the modeling of educational programs [28]. This creates a scientific gap: there is no formalized toolkit that would simultaneously take into account both the multilevel structure of the educational process and the fuzziness of the connections between its elements.

Thus, analysis of the literature shows that:

1. Classical categorical models provide a rigorous apparatus for describing educational structures, but do not take into account uncertainties.
2. Fuzzy logic methods allow working with fuzzy estimates, but do not integrate into the structural description of programs.

3. Phase categories have the potential to combine both approaches, creating a generalized model for analyzing educational programs.

3.1. Basic definitions

Category. A category is a fundamental structure of category theory that generalizes the concepts of sets and functions, focusing on the relationships between elements [29, 30]. It allows modeling systems where not only individual components are important, but also the ways they interact. In an educational context, a category can represent the structure of a discipline: topics as objects, and logical or educational dependencies between them as morphisms. This makes it possible to formalize the sequence of studying the material, identify cycles or alternative paths, which is critical for a competency-based approach where the emphasis is on the integration of knowledge.

Definition 1. Category C consists of:

1. A set of objects $Obj(C)$
2. A set of morphisms $Hom_C(A, B)$ for each pair of objects $A, B \in Obj(C)$
3. The composition operation of morphisms: $\circ: Hom_C(B,C) \times Hom_C(A,B) \longrightarrow Hom_C(A,C)$
4. Identity morphisms $id_A \in Hom_C(A,A)$ для кожного $A \in Obj$©.

These elements satisfy the axioms:

1. Associativity: for all $f \in Hom(A,B)$, $g \in Hom(B,C)$, $h \in Hom(C,D)$: $h \circ (g \circ f) = (h \circ g) \circ f$.
2. Unity: for every $f \in Hom(A,B)$: $id_B \circ f = f = f \circ id_A$.

In a pedagogical context, objects of a category are interpreted as topics or modules of a discipline, and morphisms as logical or educational dependencies between them. The composition of morphisms reflects the possibility of constructing educational trajectories through the sequential mastery of topics.

For example, let's imagine a simple category for the discipline "Mathematics": objects—"Arithmetic," "Geometry," "Algebra"; morphisms—"dependency: Arithmetic → Algebra" (because basic operations are necessary for equations). Composition allows us to create the trajectory "Arithmetic → Geometry → Algebra," which models logical progress in learning. This approach helps to identify "critical nodes"—topics on which many others depend—and optimize the program to avoid gaps.

4. Theoretical foundations of phase categories

Classical category theory operates with objects and morphisms that either exist or do not exist. This ensures rigor, but in the case of educational programs, it proves insufficient: the influence of a subject on learning outcomes cannot always be described as binary. For example, one topic may almost completely shape a certain outcome, another may only partially shape it, and a third may shape it indirectly.

4.1. Phase sets

To describe such situations, the apparatus of fuzzy sets [21] is used, in which each element is assigned a degree of membership $\mu \in [0,1]$. In a pedagogical context, μ can reflect the strength of influence or the level of reliability of an expert assessment.

4.2. Fuzzy category

The phase category C_f generalizes the classical category by introducing a degree of membership for each morphism. Thus, instead of the statement "there is a morphism from A to B," we have "there is a morphism from A to B with degree μ.

Example 1. Interpretation in education.

1. Objects: subject topics, learning outcomes (CLO), program outcomes (PLO), competencies (Comp).
2. Morphisms: relationships "provides," "contributes," "corresponds," with an attached degree μ showing the intensity of influence.

4.3. Composition of morphisms

Composition in the phase-category preserves the logic of the classical category, but takes into account weaker links. A typical method is the minimum rule: $\mu(f \circ g)=min(\mu(f), \mu(g))$.

This means that the contribution of a topic to competence through intermediate results cannot exceed the weakest link in the corresponding chain.

Example 2. If the topic "Agile methodologies" with an intensity of $\mu=0.9$ forms the *CLO* "uses Scrum/Kanban," and this *CLO* with an intensity of $\mu=0.7$ corresponds to PLO_{11}, then the total contribution of the topic to PLO_{11} is 0.7.

4.4. Consistency with the classical model and composition axioms in phase categories

Phase category C_f generalizes the classical category by allowing morphisms to have degrees of membership $\mu \in [0,1]$. If all $\mu \in \{0,1\}$, the phase category reduces to the classical category, where morphisms are binary (either present or absent). However, to guarantee the rigor of the phase-categorical approach, it is necessary to prove that the composition of morphisms in phase categories satisfies the axioms of a category: associativity and unity. In this subsection, we formally prove these axioms using the composition rule $\mu(f \circ g)=min(\mu(f), \mu(g))$, and illustrate their application in an educational context.

Assessing the quality of educational programs in modern higher education is a complex, multi-level process that involves verifying the compliance of programs with state standards, professional requirements, and stakeholder expectations. To formalize the relationships within education, we will apply category theory according to the scheme shown in Figure 1.

This model allowed us to present educational content in the form of a rigorous mathematical structure and enabled a quantitative analysis of its consistency.

However, this approach has limitations related to the binary nature of representations: each relationship between elements of the educational program was interpreted as either present or absent. In real life, relationships are rarely so clear-cut. The impact of a single subject on learning outcomes can be significant or only partial; the formulation of outcomes often contains elements of uncertainty; expert assessments of the alignment of program outcomes with competencies are subjective and vague. As a result, the classical categorical model does not fully reflect the complexity and variability of the educational process.

To overcome this limitation, the article proposes using fuzzy categories, which generalize classical categories by introducing degrees of membership of relations $\mu \in [0,1]$.
In such a model, each morphism is assigned not only the fact of existence, but also a numerical indicator of the strength or reliability of the connection. This allows formalizing the fuzziness inherent in both the description of learning outcomes and expert assessments of their achievement.

The purpose of this work is to develop a phase-categorical model for evaluating educational programs that:

1. Preserves the compositional and rigorous nature of the categorical approach.
2. Takes into account the vagueness and unevenness of the connections between educational elements.
3. Creates a basis for the development of software with support for analytics and visualization.

4.4.1. Definition of a phase-category

Let C_f be a phase-category consisting of:

1. A set of objects $Obj(C_f)$, which in an educational context are interpreted as topics (C_i), learning outcomes ($CLOj$), program outcomes (PLO_m), or competencies $(Comp_m)$.
2. The sets of phase morphisms $Hom_{Cf}(A,B)\subseteq [0,1]$ for each pair of objects $A,B\in Obj(C_f)$, where $\mu(f)\in [0,1]$ is the degree of membership of the morphism $f: A\rightarrow B$.
3. Morphism composition operations: *for* $f: A\rightarrow B$, $g: B\rightarrow C$, the composition $g\circ f: A\rightarrow C$ is defined as $\mu(g\circ f)=min\ (\mu(f),\mu(g))$.
4. Identity morphisms: for each $A\in Obj(C_f)$, there exists $id_A:A\rightarrow A$ with $\mu(id_A) = 1$.

These elements must satisfy the axioms of the category: associativity of composition and unity of identity morphisms.

Lemma 1: Associativity of composition. For any morphisms $f: A\rightarrow B$, $g: B\rightarrow C$, $h: C\rightarrow D$ in the phase category C_f, the following holds

$$h\circ (g\circ f)=(h\circ g)\circ f,$$

i.e.

$$\mu(h\circ (g\circ f))=\mu((h\circ g)\circ f).$$

Proof: Consider the composition of morphisms in a phase category. According to the composition rule:

$$\mu(g\circ f)=min\ (\mu(f),\mu(g)).$$

Then for the left side of the axiom:

$$h\circ (g\circ f):A\rightarrow D,\ \mu(h\circ (g\circ f))=min\ (\mu(h),\mu(g\circ f))=min\ (\mu(h),\ min\ (\mu(f),\mu(g))).$$

Since the min operation is associative on [0,1], we have:

$$min\ (\mu(h),\ min(\mu(f),\ \mu(g)))=min\ (min\ (\mu(h),\ \mu(g)),\ \mu(f)).$$

For the right side:

$$(h\circ g)\circ f:A\rightarrow D,\ \mu((h\circ g)\circ f)=min\ (\mu(h\circ g),\mu(f))=min\ (min\ (\mu(h),\ \mu(g)),\ \mu(f)).$$

Since both sides are equal in terms of associativity *min*, then:

$$\mu(h\circ (g\circ f))=\mu((h\circ g)\circ f).$$

Therefore, the associativity axiom is satisfied.

Example 3 in a pedagogical context: Let the objects in category C_{Topics} be topics of the discipline "Software Product Development Technology" (C_1: Software Life Cycle, C_2: Development Methodologies, C_3: Architectural Solutions), and the morphisms be:

1. $f: C_1\rightarrow C_2$, $\mu(f)=0.9$ (the life cycle strongly influences methodologies).
2. $g: C_2\rightarrow C_3$, $\mu(g)=0.7$ (methodologies partially determine architecture).
3. $h: C_3\rightarrow C_4$, $\mu(h)=0.8$ (architecture affects documentation).
4. Composition $g\circ f:C_1\rightarrow C_3$, $\mu(g\circ f)=min\ (0.9,0.7)=0.7$.
5. $h\circ (g\circ f):C_1\rightarrow C_4$, $\mu(h\circ (g\circ f))=min\ (0.8,\ 0.7)=0.7$.
6. $(h\circ g)\circ f:C_1\rightarrow C_4$, $\mu(h\circ g)=min\ (0.8,\ 0.7)=0.7$, $\mu((h\circ g)\circ f)=min\ (0.7,\ 0.9)=0.7$.

The equality $\mu(h\circ (g\circ f))=\mu((h\circ g)\circ f)=0.7$ confirms associativity, and the value $\mu=0.7$ shows that the strength of the relationship is limited by the weakest link (methodology → architecture).

Lemma 2: Uniqueness of identical morphisms. For any morphism $f: A\rightarrow B$ in phase category C_f, the following holds

$$id_B\circ f=f,\ \ f\circ id_A=f,$$

that is

$$\mu(id_B\circ f)=\mu(f),\ \mu(f\circ id_A)=\mu(f).$$

Proof: By definition, $\mu(id_A) = 1$ for any $A\in Obj(C_f)$. Consider the composition:

$$\mu(id_B\circ f)=min\ (\mu(id_B),\ \mu(f))=min\ (1,\mu(f))=\mu(f).$$

Similarly:

$$\mu(f\circ id_A)=min\ (\mu(f),\ \mu(id_A))=min\ (\mu(f),\ 1)=\mu(f).$$

Since $\mu(id_B \circ f)=\mu(f)$ and $\mu(f \circ id_A)=\mu(f)$, the unity axiom is satisfied.

Example 4 in a pedagogical context: Let in category C_{CLO} objects are learning outcomes (CLO_1: Applies Scrum/Kanban, CLO_2: Develops technical specifications), and the morphism $f: CLO_1 \longrightarrow CLO_2$, $\mu(f)=0.6$ (Scrum partially affects technical specifications). Then:

$id_{CLO2} \circ f: CLO_1 \longrightarrow CLO_2$, $\mu(idCLO2 \circ f)=min\ (1, 0.6)=0.6=\mu(f)$;

$f \circ id_{CLO1} : CLO_1 \longrightarrow CLO_2$, $\mu(f \circ id_{CLO1})=min\ (0.6, 1)=0.6=\mu(f)$.

This confirms that identical morphisms act neutrally, preserving the degree of connection.

4.4.2. Consistency with the classical model

If $\mu(f) \in \{0,1\}$ for all morphisms, then:

$\mu(f)=1$ corresponds to the presence of a connection (classical morphism)

$\mu(f)=0$ corresponds to its absence.

The composition $\mu(g \circ f)=min\ (\mu(f), \mu(g))$ reduces to the classical one: *if* $\mu(f)=1$, $\mu(g)=1$, then $\mu(g \circ f)=1$; if at least one $\mu=0$, then $\mu(g \circ f)=0$. Identical morphisms with $\mu(id_A)=1$ correspond to classical id_A. Thus, the phase category is a generalization of the classical one, preserving its axioms.

5. Phase-categorical model of the educational process

The proposed model aims to formalize the relationships between subject topics, learning outcomes, program outcomes, and competencies, taking into account the vagueness and unevenness of their influences. It generalizes the previous categorical model by using phase categories, which allows describing not only the structure but also the strength of the relationships.

5.1. Different levels of the model

The model is constructed as a sequence of four categories:

C_{Topics}—category of discipline topics.

C_{CLO}—ategory of *course learning outcomes.*

C_{PLO}—category of *program learning outcomes.*

C_{Comp}—category of competencies defined by higher education standards.

The objects in each category correspond to specific elements of the curriculum, and morphisms describe their internal dependencies (for example, logical transitions between topics or interrelationships between competencies).

5.2. Phase functors

To establish correspondences between the levels of the model (described in subsection 4.1: C_{Topics}, C_{CLO}, C_{PLO}, C_{Comp}), phase functors are introduced, which generalize classical functors by taking into account the degrees of membership $\mu \in [0,1]$ for each mapping. This allows modeling fuzzy relationships, such as the partial contribution of a topic to the learning outcome or the subjective reliability of the program outcome's correspondence to competence. Phase functors preserve composability and category axioms (associativity and unity), as proven in subsection 3.4, but add a quantitative measure of the strength of the relationship.

Definition of a phase functor. Let $C_f$and $D_f$be two phase categories. The phase functor $F_f: C_f \longrightarrow D_f$ is defined as:

1. Mapping of objects: for each object $A \in Obj(C_f)$, $F_f(A) \in Obj(D_f)$.
2. Morphism mapping: for each morphism $f: A \longrightarrow B$ with $\mu(f) \in [0,1]$, there exists a morphism $F_f(f): F_f(A) \longrightarrow F_f(B)$ with $\mu(F_f(f)) \in [0,1]$, where $\mu(F_f(f))=\mu(f)$ (or another function that preserves order, but in our model − direct transfer for simplicity).
3. Composition preservation: for $f: A \longrightarrow B$, $g: B \longrightarrow C$, $\mu(F_f(g \circ f))=min(\mu(F_f(f)), \mu(F_f(g)))$ $=min(\mu(f),\mu(g))$.

4. Preservation of identity morphisms: $\mu\left(F_f\left(id_A\right)\right) = \mu\left(id_{F_f(A)}\right) = 1$.

Three phase functors are introduced in our model:

1. F_0: $C_{Topics} \longrightarrow C_{CLO}$, which reflects the topics of disciplines in the learning outcomes of disciplines *(CLO).*
2. F_1: $C_{CLO} \longrightarrow C_{PLO}$, linking learning outcomes to program outcomes *(PLO).*
3. F_2 : $C_{PLO} \longrightarrow C_{Comp}$, which reflects program outcomes in competence *(Comp).*

Each mapping F_i is assigned a degree of membership $\mu(F_i(f)) \in [0,1]$, which characterizes the strength or reliability of the corresponding relationship. For example, if the topic "Agile methodologies" strongly influences the *CLO* "uses Scrum/Kanban" *(μ=0.9),* but the *CLO* only partially corresponds to the *PLO* "project management" (μ=0.7), then the composition is limited to $min\ (0.9, 0.7)=0.7$.

5.3. Proving the preservation of axioms for phase functors

Phase functors preserve category axioms, as shown in Section 3.4 for composition of morphisms. Here we prove that the phase functor F_f preserves associativity and unity under transfer.

Lemma 3: Preservation of associativity. Let $f: A \longrightarrow B$, $g: B \longrightarrow C$, $h: C \longrightarrow D$ *in* C_f.

Then $F_f(h \circ (g \circ f)) = F_f((h \circ g) \circ f)$.

Proof:

On the left side: $\mu(F_f(h \circ (g \circ f)))=\mu(h \circ (g \circ f))=min\ (\mu(h), min(\mu(g), \mu(f)))=min(\mu(h), \mu(g), \mu(f))$. From the right: similarly, $min(min(\mu(h), \mu(g)), \mu(f))=min(\mu(h), \mu(g), \mu(f))$. The equality holds by the associativity of min.

Lemma 4: Preservation of unity. Let $f: A \longrightarrow B$.

Then $F_f(id_B \circ f)=F_f(f)$ and $F_f(f \circ id_A)=F_f(f)$.

Proof:

$\mu(F_f(id_B \circ f)) = min\ (1,\mu(f)) = \mu(f) = \mu(F_f(f))$. Similarly, for $f \circ id_A$. ■

These proofs confirm that phase functors are a correct generalization of classical ones, preserving mathematical consistency.

Example 5 of application in an educational context. Consider the phase functor F_0: $C_{Topics} \longrightarrow C_{CLO}$ for the discipline "Software Product Development Technology." Objects: topics C_1 (Software Life Cycle), C_2 (Development Methodologies); *CLO:* CLO_1 (Understanding the Life Cycle), CLO_2(Applying Agile). Morphism f: $C_1 \longrightarrow C_2$, $\mu(f)=0.8$ (the life cycle partially determines the methodologies). Then $F_0(f)$: $CLO_1 \longrightarrow CLO_2$, $\mu(F_0(f))=0.8$, which reflects a fuzzy influence.

5.4. Diagram of functor mappings

To visualize phase-functor mappings, a diagram is provided (Figure 2), where edges with weights μ illustrate the strength of connections between model levels. The diagram shows the complete chain from topic C_2 to competency SC_{10} via *CLO* and *PLO*, with composition $\mu(F)=min(\mu(F_0), \mu(F_1), \mu(F_2))$.

Figure 2 illustrates phase-functor mappings between model levels. Each functor F_i is labeled with a weight μ indicating the strength of the link (e.g., $\mu(F_0) = 0.9$ for a strong influence of the topic on CLO). The composition F(dotted arrow) shows the overall path from topic C_2 to competency SC_{10}, where $\mu(F)=0.7$ is limited by the weakest link (F_1. This allows for visual analysis of imbalances; for example, if $\mu(F_1)<0.5$, then the entire chain is considered weak, signaling a gap in the program.

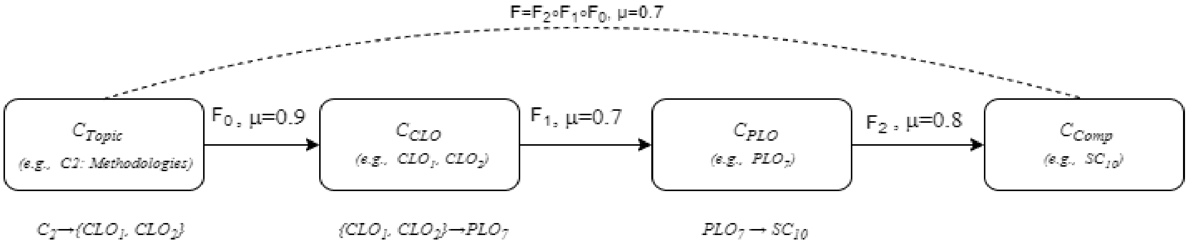

Figure 2: Phase-functor mappings between model levels

Phase functors provide flexibility to the model, allowing for the integration of expert assessments or automated calculations of μ (e.g., based on NLP analysis of syllabi).

5.5. Weight coefficients of interactions between functor mappings

Determining the weight of edges is a separate important and complex task. Its solution requires the use of different approaches or the development and justification of new methods. Various approaches can be proposed to determine the weight of edges:

1. Expert
2. A posteriori
3. Test
4. Axiomatic
5. Combined—as a justified combination of the four approaches mentioned above.

We will briefly describe the content and directions of research related to the above approaches to determining the numerical values of weight coefficients of mutual influences between functor mappings, schematically shown in Figure 2.

The expert approach is the most acceptable today, since this area of research is insufficiently developed and the best tool in such cases is expert judgment and adequately developed expert technologies [31].

The *a posteriori* approach consists in a posteriori determination of the level of influence on Comp competencies through CLO and PLO based on the known results of applying the phase-categorical model of the educational process proposed by the authors to specific educational programs implemented in specific higher education institutions [32].

The test approach can be applied to already implemented and operating educational programs by testing students who have mastered the relevant programs and formally received documentary confirmation of this [33, 34].

Note: Testing of students can be carried out both during their studies and for graduates of higher education institutions [35]. Testing can be applied to students of different courses and different levels of academic achievement. In this case, it is necessary to apply clustering methods [36] to preliminarily reduce the dispersion when analyzing the results obtained.

The axiomatic approach is based on the a priori use of expert judgments and consists in the preliminary determination of the level of influence depending on the “weight” of the components, i.e., the number of results, disciplines related to the educational program in force at the higher education institution [37].

The combined approach is a reasonable combination of the four approaches mentioned above, or the use of only some of these four.

5.6. Interpretation of phase reflections

1. If μ is close to 1, the relationship almost completely determines the correspondence between the elements (for example, the topic directly shapes a specific learning outcome).
2. If μ has an intermediate value, the relationship is partial or depends on additional conditions (e.g., the topic contributes to the outcome only in combination with others).
3. If μ is low (e.g., $\mu < 0.3$), the influence is weak or indirect.

5.7. Composition in the phase model

Thanks to the compositional nature of the categories, it is possible to trace the contribution of topics to the final competencies through all intermediate levels. The corresponding degree of belonging is calculated as a minimum across all links:

$$\mu_{topic} \rightarrow Comp = min(\mu_{topic \rightarrow CLO}, \mu_{CLO \rightarrow PLO}, \mu_{PLO \rightarrow Comp}).$$

This reflects the principle that "the strength of a chain is determined by its weakest link." Thus, even if the connection between the topic and the learning outcome is strong, but the subsequent reflection in competence is weak, the overall contribution of the topic to competence will be limited.

5.8. Weight maps

The result of building the model is a set of "weight maps" between all levels of the educational process. They can be represented as adjacency matrices with weights μ or as graphical diagrams where the thickness of the edges is proportional to the strength of the connection. Such maps provide a transparent visualization of the coverage of competencies and allow you to identify:

1. the topics with the greatest contribution to competencies.
2. critical "weak spots" in the program.
3. duplication or imbalance in the structure of learning outcomes.

6. Example of the application of the phase-categorical model

To demonstrate how the phase-categorical model works, let us consider the program learning outcome PLO_{11} of the educational program "Computer Science." In the classical category model, this outcome is linked to a number of subject areas through learning outcomes (CLO). However, the links are interpreted as binary—either present or absent. The use of phase categories allows each link to be given a numerical assessment of its strength of influence.

6.1. Initial data

The model uses three subject areas that are part of different educational components (EC):

1. EC_{21} "Agile methodologies."
2. EC_{28} "Software Documentation."
3. EC_{31} "Requirements Engineering."

Each topic is reflected in the corresponding learning outcomes (*CLO*), which in turn are linked to PLO_{11}. For each reflection, experts determined the degree of relevance μ, which reflects the strength of influence.

6.2. Assignment of weighting coefficients

1. Topic "Agile methodologies" → *CLO* "applies Scrum/Kanban" (μ=0.9).

2. Topic “Software Documentation” ⟶ *CLO* “develops technical specifications” (μ=0.6).
3. Topic “Requirements engineering” ⟶ *CLO* “specifies requirements” (μ=0.8).
4. *CLO* “uses Scrum/Kanban” ⟶ PLO_{11} (μ=0.7).
5. *CLO* “develops technical specifications” ⟶ PLO_{11} (μ=0.5).
6. *CLO* “specifies requirements” ⟶ *PLO* (μ=0.9).

6.3. Calculation of phase compositions

The total contribution of the topic to PLO_{11} is determined as the minimum of the values μ on the path “*topic* ⟶ *CLO* ⟶ PLO_{11}”. Results obtained:

1. Agile methodologies: *min (0.9, 0.7)=0.7.*
2. Software documentation: *min (0.6, 0.5)=0.5.*
3. Requirements engineering: *min (0.8, 0.9)=0.8.*

Thus, the topic “Requirements engineering” has the greatest contribution to the formation of PLO_{11}, while “Software documentation” plays a supporting role.

6.4. Reflection in competence

Next, we will consider the reflection of PLO_{11} in the competence of the educational program. For example, PLO_{11} can form:

1. SC_{10} “Ability to apply software engineering methods” (μ=0.8).
2. SC_{15} “Ability to engage in project activities” (μ=0.6).

Then the contribution of topics to competence is calculated using the same logic:

1. Agile methodologies ⟶ SC_{10}: *min (0.7, 0.8)=0.7.*
2. Agile methodologies ⟶ SC_{15}: *min (0.7, 0.6)=0.6.*
3. Software documentation ⟶ SC_{10}: *min (0.5, 0.8)=0.5.*
4. Software documentation ⟶ SC_{15}: *min (0.5, 0.6)=0.5.*
5. Requirements engineering ⟶ SC_{10}: *min (0.8, 0.8)=0.8.*
6. Requirements engineering ⟶ SC_{15}: *min (0.8, 0.6)=0.6.*

6.5. Visualization

To illustrate the results, a “phase map” was constructed in the form of a weighted graph, where the nodes correspond to topics (C_i), learning outcomes (*CLOj*), program outcomes (PLO_{11}), and competencies (SC_{10}, SC_{15}), and the thickness of the edges is proportional to μ (Figure 3). An alternative representation is an adjacency matrix, where the cells reflect the strength of the connection.

7. Analytical results

Testing the phase-categorical model using the example of PLO_{11} allowed us to draw a number of important analytical conclusions that cannot be identified in the classical binary model.

7.1. Identification of critical topics

The calculated membership values showed that the topic “Requirements Engineering” has the greatest contribution to the formation of PLO_{11} and related competencies (μ=0.8). This information is valuable for curriculum design, as it allows us to identify the disciplines and topics that play a key role in achieving the target results.

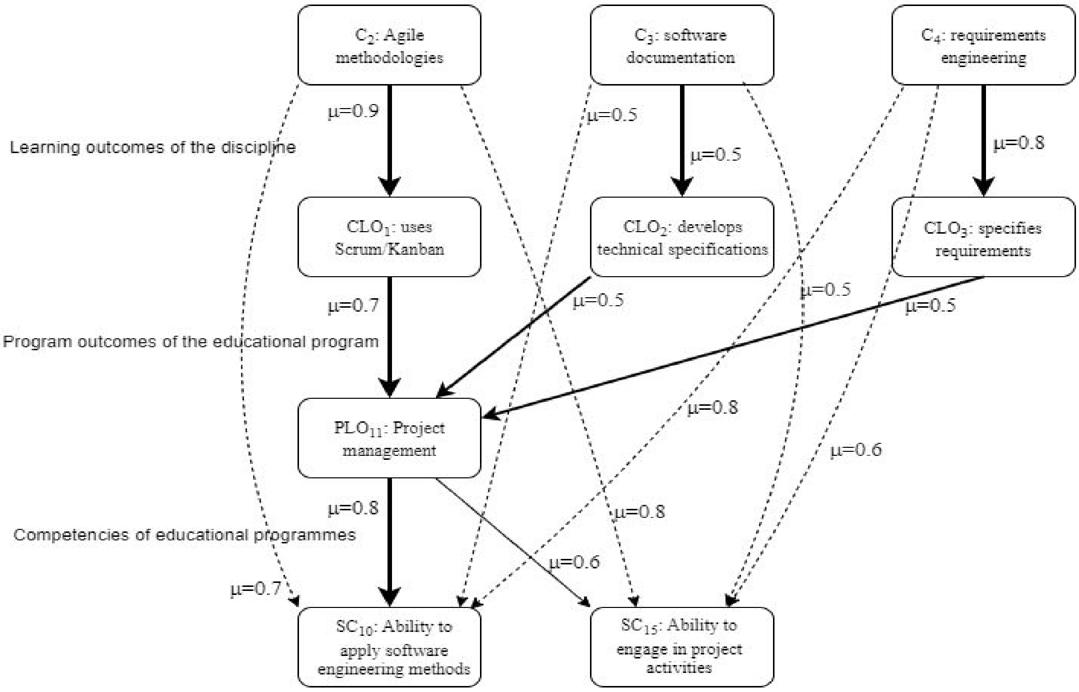

Figure 3: "Phase map" in the form of a weighted graph

7.2. Identification of weakly related elements

The topic "Software Documentation" showed a weak influence on PLO_{11} *(μ=0.5),* which may indicate the auxiliary nature of this content block. In the classical model, this connection would be marked as "present," but the phase approach allows us to record its low intensity and classify it as secondary.

7.3. Balance analysis

Comparing the weight contributions between topics allows us to assess the balance of the program. It was found that PLO_{11} is formed mainly due to the topic "Requirements Engineering," while other topics have a less pronounced influence. This indicates an imbalance, which can be both positive (focus on key competencies) and negative (risk of underestimating auxiliary skills).

Table 1 shows the adjacency matrix for the discipline "Software Product Development Technology," which quantitatively describes the relationships between topics, *CLOs, PLOs* and competencies. For example, the value $\mu=0.8$ for $C_4 \longrightarrow SC_{10}$ indicates a strong contribution of the topic "Requirements Engineering" to the competency "Software Engineering Methods," while $\mu=0.5$ for $C_3 \longrightarrow SC_{10}$ reflects a weaker connection for "Software Documentation." In both cases, the model allows for a clear identification of strong and weak contributions, which is the basis for analyzing balance and identifying gaps in the program (see subsection 6).

7.4. Gap analysis

The aggregated values of μ for competencies showed that some of them have low total coverage (<0.5). This indicates potential gaps in the program that need to be addressed by introducing additional topics or revising existing learning outcomes. For example, in the case of SC_{15} ("Ability to engage in project activities"), the value $\mu=0.5–0.6$ indicates insufficient development of this competency.

Table 1
Adjacency matrix for the discipline "Software Product Development Technology," which quantitatively describes the relationships between topics, *CLOs, PLOs* and competencies

Non-English or Math	C_2	C_3	C_4	CLO_1	CLO_2	CLO_3	PLO_{11}	SC_{10}	SC_{15}
C_2: Agile methodologies	-	0	0	0.9	0	0	0	0.7	0.6
C_3: Software documentation	0	-	0	0	0.5	0	0	0.5	0.5
C_4: Requirements Engineering	0	0	-	0	0	0.8	0	0.8	0.6
CLO_1: Uses Scrum/Kanban	0	0	0	-	0	0	0.7	0	0
CLO_2: Develops technical specifications	0	0	0	0	-	0	0.5	0	0
CLO_3: Analyzes requirements	0	0	0	0	0	-	0.8	0	0
PLO_{11}: Project Management	0	0	0	0	0	0	-	0.8	0.6
SC_{10}: Software engineering methods	0	0	0	0	0	0	0	-	0
SC_{15}: Project activities	0	0	0	0	0	0	0	0	-

7.5. Comparison with the classical model

In the classical categorical-binary model, all three topics are equally considered to "ensure" PLO_{11}. In contrast, the phase model showed a qualitatively different picture:

1. it identified a theme with a dominant influence.
2. it showed weak and secondary connections.
3. it made it possible to quantitatively assess the balance and identify gaps.

This confirms that the phase-categorical model is a significant improvement on the classical approach and allows for more accurate and relevant analytics to ensure the quality of educational programs.

8. Discussion

The proposed phase-categorical model creates new opportunities for the analytics of educational programs, combining the rigor of the categorical approach with the flexibility of fuzzy set theory [38, 39]. The use of degrees of connection intensity eliminates the limitations of binary models, which is especially important for programs where individual topics have different weights in the formation of competencies. Compositional structure makes it possible to trace the influence from a topic to a competency through several levels of generalization without losing information about weak links. At the same time, the model remains compatible with classical categorical logic, which guarantees the preservation of formal rigor.

However, the use of phase categories is associated with a number of challenges. The key issue is the subjectivity of expert assessments when assigning weight coefficients, as different experts may interpret the significance of topics or results differently. Also, the lack of uniform methodological recommendations for scaling within the interval [0,1] can lead to ambiguous results. An additional

limitation is the complication of computational procedures as the number of elements increases, which requires the creation of effective algorithms and specialized visualization tools.
The software implementation of the model has particular promise. Its architecture may include modules for collecting data from syllabi and curricula, a database of weighting coefficients, an algorithmic block for calculating phase compositions, and means of visualizing the results in the form of graphs, heatmaps, or adjacency matrices. In addition to expert input of coefficients, it is possible to automatically determine their values using NLP analysis of syllabi or statistics on student academic performance. In this form, the model can be integrated into education quality management systems, accreditation platforms, and intelligent learning systems.

An important area of application is the support of adaptive learning. If weight coefficients are updated based on individual student performance, it becomes possible to form personal educational trajectories aimed at strengthening poorly developed competencies. This opens up the prospect of using phase categories not only in the context of auditing educational programs, but also in the design of adaptive learning technologies [40–43]. Thus, the proposed approach combines theoretical rigor, analytical flexibility, and practical significance, forming a promising direction in the development of educational analytics tools.

9. Conclusions

The article presents a generalization of the categorical model of educational programs through the application of phase categories. The proposed approach allows for the consideration of the vagueness and varying intensity of the connections between topics, learning outcomes, program outcomes, and competencies.

The study showed that:

1. Phase categories provide a formalized apparatus for describing uncertainty in representations of the educational process, while preserving the compositional and rigorous nature of classical categorical logic.
2. Using the example of the PLO_{11} program outcome, it was demonstrated that the model allows identifying critical topics with the greatest contribution to the formation of competencies, revealing weak and secondary connections, and performing *gap analysis.*
3. A comparison with the binary categorical model confirmed that the phase approach provides a much more accurate and flexible representation of the educational program.
4. The developed model creates a basis for practical implementation in the form of software that can support the audit of educational programs, visualization of competency coverage, and the construction of adaptive educational trajectories.

Further research directions include:

1. Developing algorithms for automated determination of weighting coefficients based on student performance statistics and NLP analysis of syllabi.
2. Integration of the model into learning management systems (LMS) and accreditation platforms.
3. Conducting comparative studies of different educational programs and international standards to assess the universality of the approach.

Declaration on Generative AI

While preparing this work, the authors used the AI programs Grammarly Pro to correct text grammar and Strike Plagiarism to search for possible plagiarism. After using this tool, the authors reviewed and edited the content as needed and took full responsibility for the publication’s content.

References

[1] G. L. Heileman, et al., Curricular Analytics: a Data-Driven Approach to Curriculum Evaluation and Redesign, in: ASEE Annu. Conf. & Expos., Salt Lake City, USA, 2018, 1–12.

[2] J.-B. LaGrange, Analyzing Curricula with Graph Theory, ZDM Math. Educ., 53 (2021) 1129–1142.

[3] J. M. Tuzón, et al., Complex Networks Approach to Curriculum Analysis and Physics Education, Phys. Rev. Phys. Educ. Res., 21(1) (2025) 010161.

[4] H. Hnatiienko, et al., An Intelligent Cloud-based System for Conduction of an Enrollment Campaign, in: Information Technologies and Security (ITS 2022), vol. 3503, 2022, 161–176.

[5] H. Hnatiienko, et al., Methods of Identifying the Correlation of Ukrainian Scientific Paradigms based on the Study of Defended Dissertations, in: Information Technology and Implementation (IT&I 2023), vol. 3646, 2023, 64–75.

[6] A. Kardan, R. Speily, M. Bahrani, Modeling the Effectiveness of Curriculum in Educational Systems using Bayesian Networks, arXiv preprint, 2015. doi:10.48550/arXiv.1506.02794

[7] O. Shkvyr, et al., Mathematical Modeling of Information Technology Integration in Digital Education: A Regional Perspective, Math. Model. Eng. Probl., 10(1) (2023) 24–33.

[8] H. Hnatiienko, et al., Some Aspects and Prospects of Artificial Intelligence Application in Educational Processes of the Agricultural Sector of the Economy, in: Applied Information Systems and Technologies in the Digital Society (AISTDS 2024), vol. 3942, 2024, 26–43.

[9] H. Hnatiienko, et al., Determining the Effectiveness of Scientific Research of Universities Staff, in: Information Technology and Interactions (IT&I 2020), vol. 2833, 2021, 164–176.

[10] S. Boyce, C. Pahl, Developing Domain Ontologies for Course Content, in: Proc. 3rd IEEE Int. Conf. Adv. Learn. Technol., 2003, 402–406.

[11] M. Rani, K. V. Srivastava, O. P. Vyas, An Ontological Learning Management System, Comput. Appl. Eng. Educ., Wiley Online Library, 24(2) (2016). doi:10.1002/cae.21742

[12] V. Tkachenko, et al., Ontological Approach in Modern Educational Processes, in: Cybersecurity Providing in Information and Telecommunication Systems, vol. 3654, 2024, 73–82.

[13] C. Christou, et al., An Ontology for Representing Curriculum and Learning Material (Curriculum KG Ontology), arXiv preprint, 2025. doi:10.48550/arXiv.2506.05751

[14] S. Mac Lane, Categories for the Working Mathematician, 2nd ed., New York: Springer, 1998.

[15] S. Awodey, Category Theory, 2nd ed., Oxford: Oxford Univ. Press, 2010.

[16] B. C. Pierce, Basic Category Theory for Computer Scientists, Cambridge, MA: MIT Press, 1991.

[17] M. A. Arbib, E. G. Manes, Arrows, Structures, and Functors: The Categorical Imperative, New York: Academic Press, 1975.

[18] W. Lawvere, S. Schanuel, Conceptual Mathematics: A First Introduction to Categories, Cambridge: Cambridge Univ. Press, 2003.

[19] D. I. Spivak, R. E. Kent, Ologs: A Categorical Framework for Knowledge Representation, Purdue e-Pubs, 2012. https://arxiv.org/abs/1102.1889

[20] D. I. Spivak, Category Theory for the Sciences, Cambridge, MA: MIT Press, 2014.

[21] L. A. Zadeh, Fuzzy Sets, Inf. Control, 8(3) (1965) 338–353.

[22] S.-M. Chen, C.-H. Lee, J.-S. Pan, Fuzzy Logic Applications in Education and Learning, Int. J. Fuzzy Syst., 13(2) (2011) 75–85.

[23] C. Kahraman (Ed.), Fuzzy Logic in Its 50th Year: New Developments, Directions and Challenges, Cham: Springer, 2016, 404 p.

[24] A. P. Šostak, Fuzzy Categories, Czechoslovak Math. J., 28(3) (1978) 469–480.

[25] U. Höhle, Monoidal Closed Categories, Weak Topologies and Generalized Logics, Fuzzy Sets Syst., 104(1) (1999) 3–28.

[26] R. Bělohlávek, Fuzzy Relational Systems: Foundations and Principles, New York: Kluwer Academic / Plenum Publishers, 2002, 320 p.

[27] I. Stubbe, Categorical Structures Enriched in a Quantaloid: Categories, Distributors and Functors, Theory Appl. Categ., 14(1) (2005) 1–45.
[28] R. Zulunov, et al., Mathematical Models of the Modern Educational Space: Virtual Communication, Processing of Natural Language Information, Normalization of Speech Signal, in: Cybersecurity Providing in Information and Telecommunication Systems (CPITS 2025), vol. 3991, 2024, 223–338.
[29] B. Fong, D. I. Spivak, Seven Sketches in Compositionality: An Invitation to Applied Category Theory, Cambridge: Cambridge Univ. Press, 2019, 338 p.
[30] B. Milewski, Category Theory for Programmers, Version 98b71ac, 2023. https://bartoszmilewski.com/
[31] H. Hnatiienko, Methodological Framework for Addressing Manipulation in the Use of Expert Technologies for Decision-Making Challenges, in: Information Technologies and Security (ITS 2023), vol. 3887, 2023, 146–162.
[32] H. Hnatiienko, V. Snytyuk, A Posteriori Determination of Expert Competence under Uncertainty, in: Information Technologies and Security (ITS 2019), 2019, 82–99.
[33] V. Tsyganok, S. Kadenko, O. Andriichuk, Simulation of Expert Judgements for Testing the Methods of Information Processing in Decision-Making Support Systems, J. Autom. Inf. Sci., 43(12) (2011) 21–32.
[34] M. Povidaychyk, O. Povidaychyk, Modern Computer Technologies for Testing Students' Knowledge, Sci. Bull. Uzhhorod Natl. Univ. Ser.: Pedagogy. Social Work, 21 (2011) 160–163.
[35] H. Hnatiienko, et al., Application of Expert Decision-Making Technologies for Fair Evaluation in Testing Problems, in: Information Technologies and Security (ITS 2020), vol. 2859, 2021, 46–60.
[36] H. Hnatiienko, O. Suprun, Fuzzy Set Objects Clustering Method using Evolution Technologies, in: Information Technologies and Security, vol. 2318, 2018, 129–138.
[37] O. Voloshyn, S. Mashchenko, Decision-Making Models and Methods: Teaching Manual for Students of Higher Education Closing, 3rd ed., Kyiv: Liudmyla Publ., 2018, 292 p.
[38] C. Marsala, B. Bouchon-Meunier, Interpretable Monotonicities for Entropies of Intuitionistic Fuzzy Sets or Interval-Valued Fuzzy Sets, in: Joint Proc. 19th World Congr. IFSA, 12th Conf. EUSFLAT, 11th Int. Summer School AGOP, Atlantis Press, 2021, 48–54.
[39] A. Voloshin, G. Gnatienko, E. Drobot, A Method of Indirect Determination of Intervals of Weight Coefficients of Parameters for Metricized Relations between Objects, J. Autom. Inf. Sci., 35(1–4) (2003). doi:10.1615/JAutomatInfScien.v35.i3.30
[40] T. Babenko, H. Hnatiienko, V. Vialkova, Modeling of the Integrated Quality Assessment System of the Information Security Management System, in: Information Technology and Interactions (IT&I-2020), vol. 2845, 2021, 75–84.
[41] I. Hanhalo, et al., Adaptive Approach to Ensuring the Functional Stability of Corporate Educational Platforms under Dynamic Cyber Threats, in; Cybersecurity Providing in Information and Telecommunication Systems, vol. 3991 (2025) 481–491.
[42] V. Buriachok, V. Sokolov, Implementation of Active Learning in the Master's Program on Cybersecurity, Advances in Computer Science for Engineering and Education II, vol. 938 (2020) 610–624. doi:10.1007/978-3-030-16621-2_57
[43] V. Buriachok, et al., Implementation of Active Cybersecurity Education in Ukrainian Higher School, Information Technology for Education, Science, and Technics, vol. 178 (2023) 533–551. doi:10.1007/978-3-031-35467-0_32