Symmetry-driven Reciprocal Frame Algorithm

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Abstract

Recent advances in parametric design have enabled algorithmic generation of Reciprocal Frame Structures (RFs) – spatial structures composed by mutually supported elements –, yet challenges remain due to the complex interplay of variables such as form, curvature, grid pattern, and material [1, 2, 3]. Many current strategies rely on overlapping the linear elements resembling beams (commonly known as nexors), emphasizing eccentricity to maintain structural stability [4, 5, 6, 7, 8]. While effective for simple and temporary structures, these methods often struggle with curvature or irregular geometries and limit flexibility in element design. Some efforts, such as [9]'s pavilion, show alternative non-overlapping approaches where eccentricity is secondary. Following a similar logic, this paper proposes a new algorithmic model that omits eccentricity as a key constraint. Instead, it leverages pinwheel patterns and eight of the seventeen wallpaper symmetry groups to define 3D RF geometries. Implemented in Grasshopper for Rhinoceros, this approach enables controlled variation in element dimensions, global form, and grid patterns, demonstrated through quad-fan and mirrored quad-fan configurations.

Symmetry, widely studied across disciplines such as mathematics, philosophy, and geometry, is understood as the one-to-one correspondence of parts within an object, often transformable onto itself through geometric operations [10, 11]. In architectural contexts, pinwheel patterns - spiral arrangements from cyclic rotations - are closely associated with RF configurations [12], with historical examples like Serlio's Floor. These pinwheels, when reflected or translated, can form the seventeen wallpaper groups, which classify all 2D repeating symmetrical patterns [13, 14] and were famously explored by M.C. Escher [15].

By applying all seventeen groups with consistent symmetry units, and imposing conditions, this study identified eight groups capable of generating RFs, yielding nine standard and two special symmetric patterns. These include quad-fans, mirrored quad-fans, tri-fans, and their reflected versions. While these patterns define all possible planar RF symmetries, adapting them to 3D surfaces, especially freeform ones, requires more complex operations. Inspired by Escher's non-Euclidean distortions [16], this can be achieved by preserving topological symmetry logic. For practical use, elements must remain linear or planar, fan thickness must not exceed cross-section height, and fans must result from proper discretization of curved surfaces, commonly through meshing - transforming the RF generation challenge into one of data structuring and manipulation.

An algorithm for generating Reciprocal Frames (RFs) was implemented in Grasshopper, based on the discretization of a NURBS surface (a mathematical representation defined by control points, vectors and B-splines) into a mesh and six input parameters: number of isocurves in U and V directions (n_U and n_V), rotation angle (α), extension length (e), height (h), and thickness (t). To handle the complexity of the UV mesh, lines are indexed into a single list (□ UV□ _T) and categorized into inner, boundary, and corner lines. Inner lines are further sorted into U and V directions (U_TU and V_TV) using arithmetic progressions (APs) to maintain consistent indexing across mesh refinements. These Aps are defined by three input parameters: a starting number (S), a step size (N), and the count of total values (C). To optimize certain relations, these inputs may not be single values, but instead, lists of numbers.

U_TU and V_TV lines are rotated around their normal vector based on α, generating the characteristic pinwheel pattern of quad-fan RFs. Further geometric operations involve calculating intersections between U and V line sets and their corresponding planes, producing precise fan connection points and lines. After a succession of geometric operations using APs, the final trimmed lines (□ Uf□ _TU and □ Vf□ _TV) are produced, each with an accurate length and direction based on the full parametric configuration.

Final geometrical operations transform 2D lines into 3D nexors. To do this, each trimmed line is shortened at both ends by half the thickness (t) to account for material width. Then, it is translated in both directions along its normal vector to establish its full height (h), creating two parallel lines. Each pair of lines defines the central plane of a nexor which is extruded in both directions by half of the thickness (t),

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resulting in a fully defined 3D element. This process preserves all index values and element positions, ensuring structural coherence across parametric variations.

The algorithmic workflow operates through a top-down approach, which begins with an existing surface onto which a reciprocal pattern is systematically applied. This method contrasts with traditional bottom-up techniques, which often rely on calculating the eccentricity between individual nexors. By adjusting the defined parameters, the user can control key aspects of the system, such as the density and orientation of the pattern, the degree of structural overlap, the spatial depth of the elements, and the overall geometric expression of the reciprocal frame. As a demonstrative case, the algorithm was applied to a complex NURBS surface using the following parameter values: n_U = 12, n_V = 10; angular spread α ranging from 9° to 29°; extension length e, defined as the negative of half of the original curve value; elements height h varying from 15 cm to 60 cm; and a constant thickness t = 15 cm (Figure 1).

The algorithm described outlines how RFs can be generated as a wallpaper pattern of pinwheels applied to a NURBS surface. The automation enabled by the algorithm allows for quick comparisons and the exploration of various geometric designs, enabling different RF configurations for the same or different shapes by adjusting the controlling parameters. This approach can also be extended to other pinwheel patterns, such as triangular or nonagonal ones. Future research will focus on structural performance, fabrication and assembly processes, and connections between nexors

Keywords
Reciprocal frame, Algorithmic model, Pinwheel pattern, Wallpaper symmetry groups, Computational design

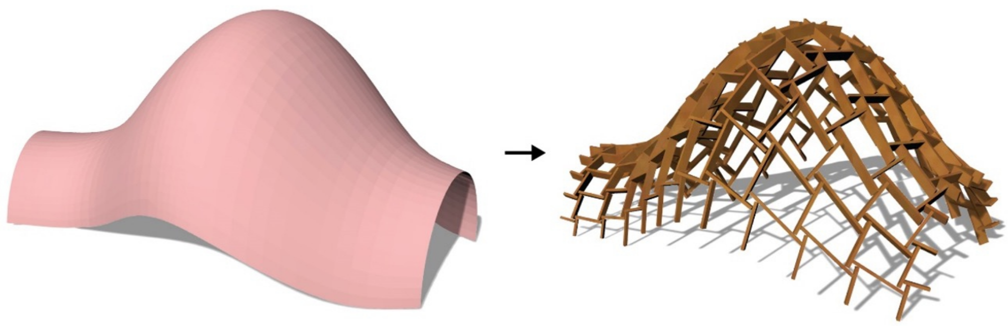

Figure 1: Transformation process of a (a) NURBS generic surface into a (b) RF structure, defined by n_U = 12, n_V = 10, α = 9° to 29°, e = negative of half of the curve length h = 15 to 60 cm and, t = 15 cm. Source: CASTRIOTTO, CELANI and TAVARES, 2022, p. 184.

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Declaration on Generative AI

The authors did not use any type of generative AI in this paper.

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