A Bayesian Justification for the Scenario Approach to Legal Proof

Mario Günther[1,2,†], Conrad Friedrich[1,*,†]

[1] Ludwig-Maximilians-Universität München, Munich Center for Mathematical Philosophy, Geschwister-Scholl-Platz 1, 80539 München, Germany

[2] Carnegie Mellon University, Department of Philosophy, Baker Hall 161, 5000 Forbes Avenue Pittsburgh, PA 15213, USA

Abstract

We probabilify the scenario approach to legal proof. The scenario approach searches for the scenario that strikes the best balance in explaining the available evidence, in fitting to general background beliefs, and in its degree of internal coherence. Our account provides a unified measure of the three dimensions in terms of probabilities, and so is proof that the scenario approach can be probabilified. Indeed, our account can be summarized by a version of Bayes Theorem: the most likely scenario in light of the evidence is the best. We thereby provide a Bayesian justification for the scenario approach.

Keywords

Legal Epistemology, Legal Proof, Scenario Approach, Bayesianism

1. Introduction

The scenario approach is a promising normative account of legal proof [1, 2]. It is commonly presented as an incompatible alternative to Bayesian accounts of legal proof [3]. The scenario approach says that you should find a defendant liable if and only if (iff) the *best scenario* implies that the defendant is liable. A scenario is best among the available scenarios if it strikes the best balance between

(i) explaining the available evidence,

(ii) fitting to the general background beliefs, and

(iii) exhibiting internal coherence.

Here we probabilify the scenario approach. The upshot is that the most probable scenario is the best. Unlike the original scenario approach, our probabilistic account can explain what scenario strikes the best balance. Our account shows that the normative scenario approach is compatible with Bayesian accounts of legal proof.

Our goal is to provide a normative foundation for the scenario approach. Our project is *not* to describe legal practice, as for example Pardo and Allen [4] and Allen and Pardo [5] do in an informative way. In this vein, Cheng [6] aims to probabilify the merely descriptive story model of juror decision making offered by Pennington and Hastie [7]. In contrast, van Koppen and Mackor [1] build on the story model to develop the scenario approach as a normative account of legal proof. Unlike Cheng, we aim to probabilify the normative scenario approach.

As a consequence of our goal, we focus on the context of justification rather than the context of discovery throughout. The scenario approach has proven its worth for constructing scenarios to be compared in a court of law. It has been applied to criminal cases in the Netherlands [1]. However, we

AI4EVIR: Workshop on AI for evidential reasoning, December 9, 2025, Turin, Italy.

*Corresponding author.

† These authors contributed equally.

mario.guenther@lmu.de (M. Günther); conrad.friedrich@lmu.de (C. Friedrich)

http://www.mario-guenther.com/ (M. Günther); https://conradfriedrich.github.io/ (C. Friedrich)

0000-0001-6208-448X (M. Günther); 0009-0009-1515-513X (C. Friedrich)

do not think that the mere generation of scenarios is normatively relevant. Hence, we abstract away from the context of discovery.

We will present the scenario approach for legal proof in Section 2 and our probabilistic version thereof in what follows. Section 3 outlines our notion of best explanation and Section 4 our notion of fit with the general background beliefs. Section 5 shows that our approach turns out to be equivalent to Bayes Theorem. Section 6 explains our notion of internal coherence and Section 7 how internal coherence fits into our probabilistic account of legal proof. Section 8 draws normative implications of our account in comparison to coherence accounts and descriptive accounts of legal proof.

2. The Scenario Approach

On the scenario approach, fact-finders should compare and assess scenarios to find out which one is best in the legal case at hand. A scenario is a story of what might have happened. Each story either implies that the defendant is liable, or else implies that the defendant is not liable. The scenarios are assessed on three dimensions.

(i) How well does a scenario explain the available evidence?

(ii) How well does a scenario fit to the general background beliefs?

(iii) How well does a scenario internally cohere?

A scenario is, other things being equal, better than another if it explains the available evidence better, if it fits better to the general background beliefs, and if it internally coheres better. A scenario is best if it strikes the best balance on the three dimensions. In this sense, the scenario approach is holistic.

Take, for example, a simplified version of the Simonshaven case, a Dutch criminal case provided by Mackor (2021). The defendant Ed and his wife Jenny arrived at the Simonshaven forest by car and went for a walk, as eyewitnesses testified. There is evidence that Jenny was hit by a blunt object and died. There are two scenarios. According to the prosecution scenario, Ed killed his wife. On the defense scenario, a madman jumped out of the bushes who beat up both Ed and Jenny. As a result, Ed lost his consciousness for some time and Jenny died.

Both the prosecution and the defense scenarios can explain the evidence about Jenny and Ed's whereabouts and Jenny's injuries leading to her death. Both scenarios are internally coherent at least in so far their elements are logically consistent with one another. However, the madman scenario fits less well with our background beliefs—"our general assumptions about the world"—than a scenario where a husband kills his wife (Mackor, 2021, p. 2414). It is much more likely that women are killed by their (ex)partners than by madmen jumping out of bushes. Hence, the prosecution scenario is better than the madman scenario. Or so goes Mackor's plausible assessment.

In criminal cases, the standard of proof is beyond a reasonable doubt—a more demanding standard than in civil cases. The scenario approach specifies the criminal standard of proof as follows: a fact-finder should find a defendant guilty iff the best scenario implies the defendant's guilt and is much better than any scenario which implies his innocence.

Is the prosecution scenario the best and much better than the madman scenario? The scenario approach has difficulties to come to a clear verdict because it does not say what it means that a scenario is 'much better' than another. Furthermore, the scenario approach does not explain what it means that a scenario strikes the best balance on the three dimensions. For example, suppose we have two scenarios that are equally well explained by the evidence. Scenario 1 fits better with general background beliefs, but scenario 2 is more coherent. Which is "the best scenario" in this case? In what follows, we provide the resources to measure how good a scenario is as compared to others.

In particular, we make precise both what it is for a scenario to be the *best* scenario and also what it is for that scenario to be *much better* than other scenarios. It will turn out that a scenario is best if it is more likely than any other scenario after conditionalizing on all available evidence. We discuss how this coincides with the central dimensions of the scenario approach in the following sections 3 through 7. The latter is given by a standard decision theoretic argument, presented in section 7.

3. Degree of Explanation

How can we measure the degree to which a scenario explains the available evidence? A standard explication of explanation is probability raising [8]. Applied to the scenario approach, a scenario S explains the available evidence E iff

$$P(E \mid S) > P(E),$$

where $P(E \mid S)$ is the conditional probability of E given that S and $P(E)$ is the probability of E. We define the degree to which a scenario S explains evidence E as follows:

$$Exp(S, E) := \frac{P(E \mid S)}{P(E)}, \text{ if } P(E) > 0.$$

If the scenario explains the evidence, $Exp(S, E)$ will be greater than 1. If the scenario does not explain the evidence, $Exp(S, E)$ will be less than or equal to 1. Furthermore, a scenario S' explains the available evidence E better than a scenario S iff

$$Exp(S', E) > Exp(S, E).$$

We can infer a best-explaining scenario S^* by searching for an argument S that maximizes $Exp(S, E)$—in symbols $S^* = argmax_S(Exp(S, E))$ for all relevant scenarios S.

Let E be the available evidence in the Simonshaven case—the eyewitness testimony and the autopsy of Jenny's injuries. Furthermore, let S_E be the scenario where Ed kills Jenny, and S_M the scenario where a madman kills Jenny. The evidence is roughly as likely given the Ed scenario as it is given the madman scenario: $P(E \mid S_E) \approx P(E \mid S_M)$. Hence, both S_E and S_M explain the available evidence E roughly to the same degree on our probabilistic account. This result is in line with Mackor (2021, p. 2415): "the defense scenario can also explain the evidence and it can do so roughly equally well as the prosecution scenario."

We rank scenarios by their degree of explanation. Any top-ranked scenario explains the evidence best. Inferring top-ranked scenarios comes apart from notions of inference to the best explanation that are taken to be truth-conducive [9, 10, 11, 12]. A reason for a hypothesis being true is on these notions that it best explains the evidence. The degree to which a hypothesis explains some evidence is, however, logically independent from its truth. A scenario, in which aliens come from outer space, kill Jenny, and prepare all the evidence just as we found it, explains the evidence well. But it does not follow from its high explanatory degree that the scenario is true. Indeed, we are not warranted to infer that the scenario is likely true because of its low initial probability of truth.[1]

Our account developed below chooses a best scenario that strikes the best balance between its degree of explanation and its initial probability. The best scenario is the scenario most likely to be true, given the evidence. Our truth-conducive notion of inference to the best explanation will simply turn out to be Bayesian inference: choose the scenario with the highest posterior probability.

4. Fit with General Background Beliefs

The second dimension on which a scenario can be better than others is its fit with a fact-finder's general background beliefs. The fit of a scenario S with the general background beliefs can be modelled by the probability $P_B(S) = P(S \mid B)$ of S given the background beliefs B. We may say that a scenario fits

---

[1]Cheng [6, pp. 1268&1277] elevates the comparative degree of explanation to a descriptive account of legal proof. He says a defendant is to be found guilty in the adversarial legal system of the US iff the degree to which the prosecution's story explains the available evidence is (much) higher than the degree to which the defense's story does: $\frac{Exp(S_E, E)}{Exp(S_M, E)} = \frac{P(E \mid S_E)}{P(E \mid S_M)} > k$ for some constant $k > 1$. As a consequence of Cheng's burden of proof, Ed is not to be found guilty if the prosecution presents either the madman or the aliens scenario. If this is indeed how the US legal system operates, we think it should be revised by taking the initial probability of the stories into account. Cheng [6, pp. 1272-5] is aware that his account effectively disregards the initial probabilities and so falls prey to the fallacious *base rate neglect* [13, 14]. His account should not be mistaken for a normative foundation of the law.

the background beliefs better than another iff it is more likely in light of the background beliefs than the other is. To be precise, a scenario S' fits the background beliefs B better than a scenario S iff

$$P(S' \mid B) > P(S \mid B).$$

The initial degrees of belief or credences of our rational fact-finder is represented by a probability distribution P. Her background beliefs B can be encoded by conditionalizing P on B. The resulting probability distribution P_B measures the likelihood of propositions in light of the background beliefs B before we receive evidence for the case at hand. As all probabilistic accounts we are aware of, we need to assume that our rational fact-finder has a distribution P_B reflecting "reasonable" background beliefs which comply with our intuitions—just like the scenario approach needs to assume "reasonable" background beliefs without any further specification. Reasonable background beliefs exclude, for example, "prejudices for or against the defendant" [3, p. 456].

As already mentioned, the belief that Jenny is killed by their (ex)partners is more plausible in light of our reasonable background beliefs than the belief that Jenny is killed by a madman jumping out of bushes (cf. Mackor 2021, p. 2416). On our model, this comparative plausibility assessment translates into a comparative assessment of probabilities:

$$P(S_E \mid B) > P(S_M \mid B).$$

The Ed scenario fits better to the background beliefs than the madman scenario.

Next, we explain how our rational fact-finder changes her credences upon receiving new evidence. We will, moreover, explain how the degree of explanation and fit with general background beliefs can be combined in an overall assessment of scenarios.

5. Bayes Theorem and Learning New Evidence

The best scenario strikes the best balance between explaining the available evidence and its fit with the background beliefs. We can think of the best scenario as the one which best explains the available evidence weighted by its fit with the background knowledge. So the goodness of a scenario S can be measured by the product of the degree to which it explains the available evidence E and the scenario's initial probability:

$$Exp_B(S, E) \cdot P_B(S), \text{ where } Exp_B(S, E) = \frac{P_B(E \mid S)}{P_B(E)}.$$

The formula measures how well a scenario explains the available evidence weighted by how likely the scenario is in the first place. It ranks the different scenarios according to their overall goodness. The formula is the right-hand side of Bayes Theorem—a theorem often used to calculate the probability of a proposition after learning a new piece of evidence:

$$P_B(S \mid E) = \frac{P_B(E \mid S)}{P_B(E)} \cdot P_B(S).$$

In general, a Bayesian agent learns new evidence by conditionalization. The probability distribution P'_B after learning a piece of evidence E is determined as follows:

$$P'_B(\cdot) = P_B(\cdot \mid E) = P(\cdot \mid E \cap B).$$

Bayes Theorem can be used to calculate the probability $P_B(S \mid E)$ of each scenario S after learning the available evidence E. A Bayesian agent can thereby compare the probability $P_B(S_E \mid E)$ of the Ed scenario with the probability $P_B(S_M \mid E)$ of the madman scenario. In what follows, we show that the most likely scenario is the best scenario according to our probabilistic account.

6. Internal Coherence

So far, we have understood any scenario S as a proposition—a set of possible worlds at which the scenario S is true. However, scenarios have an internal structure. They consist of chronologically ordered and causally connected elements. Hence, scenarios can be modeled as sets of propositions in time, some of which are causally related. Evaluating the internal coherence of a scenario is to evaluate how well the scenario's elements cohere. If they cohere well, they fit well to each other.

How can we measure the internal coherence of scenarios? Our basic idea is this: if the elements of a scenario fit well to one another, they are taken together as a whole more likely than they would be taken together as individuals. Let's denote the elements of a scenario S by $S_1, S_2, \ldots, S_n$. The probability of two elements S_1 and S_2 of a scenario taken as individuals is simply the product of their individual probabilities: $P_B(S_1) \cdot P_B(S_2)$. Taking elements of a scenario as individuals means treating them *as if* they were probabilistically independent. S_1 is independent of S_2 relative to P_B just in case $P_B(S_1 \cap S_2) = P(S_1) \cdot P_B(S_2)$. If S_1 and S_2 are independent,

$$\frac{P_B(S_1 \cap S_2)}{P_B(S_1) \cdot P_B(S_2)} = 1,$$

provided $P_B(S_1)$ and $P_B(S_2)$ are not zero.[2] We can use the ratio as a measure of pairwise coherence. Two propositions are coherent iff the ratio is strictly higher than 1 iff they are positively relevant to each other: $P_B(S_1 \cap S_2) > P(S_1) \cdot P_B(S_2)$. The ratio measures the degree by which the two propositions as a whole are more likely than they would be if they were independent.

Take, for example, two elements of the Ed scenario that cohere well: Jenny was attacked and died in the forest (S_{E2}), and Ed killed Jenny in the forest (S_{E3}). The two propositions are positively relevant to each other—and to a high degree. The conditional probability that Jenny was attacked and died in the forest given that Ed killed Jenny in the forest is very high if not maximal. The unconditional probability that Jenny was attacked and died in the forest is much lower (based only on our background beliefs B). Hence,

$$P_B(S_{E2} \mid S_{E3}) = \frac{P_B(S_{E2} \cap S_{E3})}{P_B(S_{E3})} \gg P_B(S_{E2}).$$

Scenarios may have more than two elements. We generalize our measure of pairwise coherence to Shogenji's (1999) coherence measure:

$$Coh_B(S_1, S_2, \ldots, S_n) := \frac{P_B(S_1 \cap S_2 \cap \ldots \cap S_n)}{P_B(S_1) \cdot P_B(S_2) \cdot \ldots \cdot P_B(S_n)}.$$

A scenario is internally coherent to the degree that its probability as a whole is larger than the product of the probabilities of its individual elements. The product of the individual probabilities figures as a neutral base-line: it measures how likely the scenario is upon the hypothetical assumption that the elements are independent of each other. If all of the elements are indeed independent, the probability of their conjunction—formalized by set intersection—equals the product of the individual probabilities; and so the coherence measure assigns the scenario the neutral value 1. If the elements of the scenario fit well together, their conjunction is more likely than the base-line; and so the coherence of the scenario is greater than 1. Finally, if the elements do not fit well together, their conjunction is less likely than the base-line; and so the coherence of the scenario is less than 1.

For illustration, consider the following propositions adapted from Shogenji [15, p. 341]:

D_1: Dunnit killed the victim at time t in city A.
D_2: Dunnit was reportedly seen at time t in city B at the other end of the country.
D_3: Dunnit has an identical twin sibling living in B.

A scenario consisting of just D_1 and D_2 is not coherent: how could Dunnit have been in city A and been sighted in city B at the same time? The probability of their conjunction is low, and in particular lower than the product of the individual probabilities. The coherence measure is then less than 1.

[2] We will not mention the non-zero proviso in what follows.

What does the coherence measure say about the scenario consisting of D_1, D_2, and D_3? Adding D_3 makes the scenario more coherent because D_3 explains why Dunnit was reportedly seen and yet is in a different city: the witness mistook Dunnit for their sibling. The explanation is reflected in the fact that the conditional probability $P_B(D_1 \cap D_2 \mid D_3)$ is much higher than $P_B(D_1 \cap D_2)$. Hence, the coherence of the $D_1 \cap D_2$-scenario is much lower than the coherence of the $D_1 \cap D_2 \cap D_3$-scenario:

$$\frac{P_B(D_1 \cap D_2)}{P_B(D_1) \cdot P_B(D_2)} \ll \frac{P_B(D_1 \cap D_2 \mid D_3) \cdot P_B(D_3)}{P_B(D_1) \cdot P_B(D_2) \cdot P_B(D_3)}.$$

The twin scenario is internally more coherent.

Let's apply the coherence measure to the Simonshaven case. Both, the Ed and the madman scenario, seem to internally cohere quite well. There is no internal tension between the elements, respectively. One element of the Ed scenario S_E is that Ed and Jenny go for a walk in the forest (S_{E1}), another that Jenny was attacked and died (S_{E2}), and a third that Ed killed Jenny in the forest (S_{E3}). The coherence of the Ed scenario is measured by

$$\frac{P_B(S_{E1} \cap S_{E2} \cap S_{E3})}{P_B(S_{E1}) \cdot P_B(S_{E2}) \cdot P_B(S_{E3})}.$$

The scenario coheres because the elements of the scenario are more likely to be true together than individually.

The madman scenario S_M includes that Ed and Jenny go for a walk in the forest ($S_{M1} = S_{E1}$) and that Jenny was attacked and died ($S_{M2} = S_{E2}$). The difference to the Ed scenario is that a madman jumped out of the bushes and attacked both Ed and Jenny so that Ed lost his consciousness and Jenny died (S_{M3}). The coherence of the madman scenario is measured by

$$\frac{P_B(S_{M1} \cap S_{M2} \cap S_{M3})}{P_B(S_{M1}) \cdot P_B(S_{M2}) \cdot P_B(S_{M3})}.$$

The scenario is again coherent.

Both scenarios are internally coherent to a similar degree (relative to P_B). To see this, observe that

$$P_B(S_{E1} \mid S_{E3}) \approx P_B(S_{E1} \mid S_{M3}).$$

Ed killing Jenny in the forest requires them to be in the forest. Going for a walk in the forest is a good reason for being in the forest. Hence, the probability that Ed and Jenny go for a walk in the forest given that Ed kills Jenny there is very high. Similarly, a madman killing Jenny and knocking Ed unconscious in the forest requires both of them to be there. A walk in the forest is as good a reason for being there as before. Hence, the probability that they are in the forest given that a madman killed Jenny in the forest and knocked Ed unconscious is very high.

The proposition that Jenny was attacked and died in the forest is entailed by the proposition that Ed killed Jenny there. It is also entailed by the proposition that the madman killed her there. Hence, we obtain

$$P_B(S_{E1} \cap S_{E2} \mid S_{E3}) \approx P_B(S_{E1} \cap S_{E2} \mid S_{M3}).$$

And further

$$\frac{P_B(S_{E1} \cap S_{E2} \mid S_{E3}) \cdot P_B(S_{E3})}{P_B(S_{E1}) \cdot P_B(S_{E2}) \cdot P_B(S_{E3})} \approx \frac{P_B(S_{E1} \cap S_{E2} \mid S_{M3}) \cdot P_B(S_{M3})}{P_B(S_{E1}) \cdot P_B(S_{E2}) \cdot P_B(S_{M3})}.$$

As S_{E1} and S_{E2} are identical to S_{M1} and S_{M2}, respectively, we obtain that both scenarios are internally coherent to a similar degree.

7. Bayes Theorem, Internal Coherence, and Legal Proof

How does Shogenji's measure of internal coherence fit into our account of legal proof summarized by Bayes Theorem? The probability of the conjunction of a scenario's elements equals their individual probabilities weighted by their coherence:

$$P_B(S_1 \cap \ldots \cap S_n) = Coh_B(S_1, \ldots, S_n) \cdot P_B(S_1) \cdot \ldots \cdot P_B(S_n).$$

The degree $P_B(S) = P_B(S_1 \cap \ldots \cap S_n)$ to which the whole scenario fits to the background beliefs can be broken down into the product of its elements's individual probabilities and how well they cohere. Indeed, the probability $P_B(S)$ of a scenario is more likely the more likely its elements are individually and the better they cohere.

We can make the elements of a scenario explicit by rewriting Bayes Theorem from above:

$$P_B(S_1 \cap \ldots \cap S_n \mid E) = \frac{P_B(E \mid S_1 \cap \ldots \cap S_n)}{P_B(E)} \cdot P_B(S_1 \cap \ldots \cap S_n).$$

Finally, we substitute the degree of explanation and the coherence measure into Bayes Theorem:

$$P_B(S \mid E) = \underbrace{Exp_B(S,E)}_{\text{Degree of explanation}} \cdot \overbrace{\underbrace{Coh_B(S_1, \ldots, S_n)}_{\text{Internal coherence}} \cdot P_B(S_1) \cdot \ldots \cdot P_B(S_n)}^{\text{Fit with background beliefs}}.$$

The labels at the braces indicate which parts of the formula represent which features of assessing scenarios. The best scenario given the available evidence is the scenario which strikes the best balance between explaining the available evidence and fit to the general background beliefs. The internal coherence can be seen as a part of the fit with the background beliefs. The best scenario *is* the most likely, given the available evidence.

We show now that, given the available evidence, any best liability scenario S_L^* coincides with the probability that the defendant is liable: $P_B(S_L^* \mid E) = P_B(L \mid E)$. For this to be seen, recall that any liability scenario S_L implies the defendant's liability L: $S_L \subseteq L$. Hence, $P_B(S_L \mid E) \leq P_B(L \mid E)$. In light of the evidence, the probability of liability is an upper bound on the best liability scenario. We can, furthermore, construct a best liability scenario. A best liability scenario S_L^* implies the defendant's liability and is implied by the strengthening of the evidence which implies his liability: $S_L^* \subseteq L$ and $S_L^* \supseteq L \cap E$. Such a best liability scenario is guaranteed to exist non-trivially just in case $L \cap E \neq \emptyset$. The same argument can be run for any best non-liability scenario $S_{\neg L}^*$, where $\neg L$ is the complement of L. In a civil trial, we can always consider whether or not the defendant is liable instead of the best liability and non-liability scenarios. This means it is a normatively valid strategy for the defense to poke holes in the prosecution's story without putting forth a particular story $S_D \subset \neg L$.[3]

Our probabilification of the scenario approach straightforwardly leads to a Bayesian account of legal proof if the goal is to minimize expected costs in legal fact-finding. As Cheng [6] points out, a true finding of liability (TL) and a true finding of non-liability (TN) do not incur any cost. Erroneous findings, however, incur costs. In "a civil trial, the legal system expresses no preference" between a false finding of liability (FL) and a false finding of non-liability (FN). (p. 1261) Hence, the fact-finder faces the decision summarized by the decision matrix in Table 1, where C is the cost function and c is a positive value.

According to the decision-theoretic principle of minimizing expected costs, you should find the defendant liable iff

$$P'(L) \cdot C(TL) + P'(\neg L) \cdot C(FL) < P'(L) \cdot C(FN) + P'(\neg L) \cdot C(TN).$$

Under the assumptions about the costs, the equation simplifies to $P'(\neg L) \cdot c < P'(L) \cdot c$, which in turn is equivalent to $P'(L) > 1/2$. $P'(\cdot)$ stands on our scenario account for $P_B(\cdot \mid E)$. Hence, you should

[3] Cheng [6, p. 1262] observes that our normatively valid strategy does not align with legal practice: "it will not do for the defendant's theory to be "not plaintiff's story." The defendant may offer multiple possible alternatives, but each of these alternatives will be judged separately, not simultaneously." From a normative point of view, however, this "focus on stories is almost certainly suboptimal, because it unnecessarily forces the factfinder to assess each story in isolation, rather than make her best global guess." (pp. 1272-3) Judging the stories separately neither maximizes accuracy nor minimizes the expected costs. This is a high price paid by the legal practice if Cheng's observation is right.

	L	$\neg L$
finding L	$C(TL) = 0$	$C(FL) = c$
finding $\neg L$	$C(FN) = c$	$C(TN) = 0$

Table 1
Decision matrix for finding liable (L) or not ($\neg L$).

find a defendant liable iff $P_B(L \mid E) > 1/2$. You should find a defendant liable iff you judge his liability more likely than his non-liability in light of the evidence and your background beliefs.

Our account of legal proof also covers the beyond a reasonable doubt standard. For criminal cases, we exchange the defendant's liability L with his guilt G in the argument of the preceding paragraph. A false finding of guilt is (much) more costly than a false finding of innocence in criminal cases: $C(FI) < C(FG)$ [6, p. 1275]. A wrongful finding of guilt is worse than a wrongful finding of innocence, perhaps about ten times as worse [16]. You should find the defendant guilty iff

$$P_B(G \mid E) > \frac{C(FG)}{C(FI) + C(FG)}.$$ [4]

For illustration, assume $C(FG) = 10$ and $C(FI) = 1$. You should then find the defendant guilty iff

$$P_B(G \mid E) > \frac{10}{1 + 10} \approx 0.91.$$

Reasonable costs for true and false findings of guilt and innocence, respectively, provide a threshold for the probability of a defendant's guilt such that finding guilty minimizes expected costs. For further details of this decision-theoretic argument, see Kaplan [17] and Günther [18]. We leave the question of what are reasonable costs for future research. Notably, our probabilified scenario account of legal proof can be amended by several accounts using Bayesian networks [19, 20, 21, 22, 23].

8. Normative Implications

We have shown that our scenario account coincides with a Bayesian account of legal proof. Legal verdicts are justified by the probability of liability and guilt, respectively. The probabilities after learning the available evidence are determined via Bayes Theorem. Thereby, our scenario account inherits the normative justification of Bayesianism in terms of Dutch book arguments [24] and epistemic utility arguments [25]. Bayesianism confers further good-making features on our account such as avoiding confirmation biases as well as the base rate and prosecutor's fallacies [26].

On our scenario account, the most likely scenario is the best: it strikes a best balance between explaining the evidence and fit to general background beliefs, including being internally coherent. In particular, our scenario account translates coherence into probability without any loss [27]. Unlike coherence accounts, however, our account merely integrates the coherence internal to scenarios into a wider account relying also on the degree of explanation and the scenarios's fit with background knowledge. All three dimensions of the scenario approach have their place in the overall probability of liability (guilt) after learning the evidence by Bayes Theorem. Ultimately, the probability of the defendant's liability (guilt) is the decisive normative ground for a finding of liability (guilt).

The ultimate normative ground poses a question for the original scenario approach and coherence accounts more generally [28, 29]: why should any kind of coherence of a scenario, its capacity to

[4] Here is the proof:

$$P'(\neg G) \cdot C(FG) < P'(G) \cdot C(FI) \text{ iff } (1 - P'(G)) \cdot C(FG) < P'(G) \cdot C(FI) \text{ iff}$$
$$C(FG) - P'(G) \cdot C(FG) < P'(G) \cdot C(FI) \text{ iff } C(FG) < P'(G) \cdot C(FI) + P'(G) \cdot C(FG) \text{ iff}$$
$$C(FG) < P'(G) \cdot (C(FI) + C(FG)) \text{ iff } \frac{C(FG)}{C(FI) + C(FG)} < P'(G).$$

explain the available evidence, and its fit with the general background beliefs matter independently of its probability? Suppose a liability scenario exhibits more internal coherence than a non-liability scenario. However, the non-liability scenario is the best non-liability scenario and more likely than not. In such a legal proceeding, we have a hard time seeing on what normative grounds we should find the defendant liable. The same argument applies *mutatis mutandis* to the other two dimensions of the scenario approach taken in isolation.

We invite the proponents of the original scenario approach and other coherence accounts to explain how their theories deviate from ours and why this deviation is normatively justified. One could argue that we failed the mark of a normatively justified account of legal proof and so explain one's deviation. But this argument must be a normative argument. It is not sufficient to cite current legal practice.

Alternatively, one could say that one takes our normative justification on board (and, perhaps, deviates from it slightly for practical purposes). If so, a loose end remains. Our probabilified scenario account still succumbs to the problem of *statistical evidence* [30]—and so does any account that takes our normative justification on board. We could save our account by the broadly Bayesian solution offered by Günther [18] in terms of justified belief. It remains an open question for now whether the coherence accounts could be similarly amended. The problem of statistical evidence, the proposed solutions, and their drawbacks, deserve their own paper.

A probabilistic account of legal proof faces the challenge to explain the origin and justification of probability assignments. Where, for example, does the prior probability of the scenario come from? On the present account, the prior probability is analysed into the product of the measures of coherence and of the fit with general background beliefs, thus making some progress towards an explanation. Generally speaking, we develop a purely normative account, and are thus partial to see a probabilistic account of legal proof as more of a regulative ideal than a hands-on recipe for legal practice. Nevertheless, we are also open to accounts spelling out the details; minimally requiring probabilistic coherence, and additionally suggesting plausible objective Bayesian norms along probabilified legal norms like the presumption of innocence. Perhaps, the problem of the priors can be solved by the constraint that the initial probability of liablility and guilt, respectively, is exactly $1/2$. But these issues require a paper of their own.

Finally, a note on the *conjunction paradox* is in order. This alleged paradox arises when the allegation to be proven requires only that two or more claims must individually be proven for a finding of guilt or liability to be warranted [31]. For illustration, consider a civil case, where the law code requires that two claims, A and B, must each be proven to be more likely than not. Now, it is possible that the probability of A and B, respectively, is greater than $1/2$, even though the probability of the conjunction $A \cap B$ is less than $1/2$. So it is possible on our account of legal proof that the probability of each element of the charge is above the threshold while the probability of the charge as a whole is not. The question is whether or not the defendant should then be found guilty.

The conjunction paradox does not arise on our normative account of legal proof [32]. Our account implies that only the probability of the charge as a whole should matter. It does not only require that the elements or claims of an allegation must be proven individually. A defendant should be found liable or guilty just in case the entire allegation, the charge as a whole, meets the respective standard of proof.

The conjunction paradox is only a problem for descriptive legal probabilism. If existing law requires only that each element of a case meets the relevant standard of proof, there may be findings of liability (guilt) even though the probability that the defendant is liable (guilty) is below the threshold. If so, our account provides a normative reason to revise the existing law: the entire allegation must meet the relevant standard of proof. This would be a sensible and modest improvement to existing legal systems, where only each element must be proven in isolation.

9. Conclusion

We have probabilified the scenario approach to legal proof. The most likely scenario is best: it strikes the best balance between explaining the available evidence and fitting to the general background beliefs.

Internal coherence is a part of the fit with our background beliefs. Our probabilistic account makes precise how the three dimensions of the original scenario approach are to be weighted. It is, furthermore, equivalent to a Bayesian account of legal proof, inheriting a robust normative foundation. Our account is proof that reconciling the scenario approach and the Bayesian approach is possible. There is no need to put forth the scenario approach as a normative competitor to the Bayesian approach [28, 29, 1, 3]. We hope our results provide a further refinement of and a normative foundation for the already successfully applied scenario approach.

Declaration on Generative AI

No generative AI was used in the production of this work.

References

[1] P. J. van Koppen, A. R. Mackor, A scenario approach to the Simonshaven case, Topics in Cognitive Science 12 (2020) 1132–1151.

[2] A. R. Mackor, Different ways of being naked: a scenario approach to the naked statistical evidence problem, Journal of Applied Logics 8 (2021) 2407–2432.

[3] A. R. Mackor, H. Jellema, P. J. van Koppen, Explanation-based approaches to reasoning about evidence and proof in criminal trials, in: B. Brożek, J. Hage, N. Vincent (Eds.), Law and Mind: a Survey of Law and the Cognitive Sciences, Cambridge University Press, 2021, pp. 431–470.

[4] M. S. Pardo, R. J. Allen, Juridical proof and the best explanation, Law and Philosophy 27 (2008) 223–268.

[5] R. J. Allen, M. S. Pardo, Relative plausibility and its critics, The International Journal of Evidence & Proof 23 (2019) 5–59.

[6] E. Cheng, Reconceptualizing the burden of proof, Yale Law Journal 122 (2013) 1254–1279.

[7] N. Pennington, R. Hastie, The story model for juror decision making, Cambridge University Press, 1993.

[8] W. C. Salmon, Statistical explanation, in: R. G. Colodny (Ed.), The Nature and Function of Scientific Theories: Essays in Contemporary Science and Philosophy, University of Pittsburgh Press, 1970, pp. 173–231.

[9] S. Psillos, Scientific realism: how science tracks truth, Routledge, New York, 1999.

[10] P. Lipton, Inference to the best explanation, in: W. Newton-Smith (Ed.), A Companion to the Philosophy of Science, Blackwell, 2000, pp. 184–193.

[11] T. Williamson, Abductive philosophy, Philosophical Forum 47 (2016) 263–280.

[12] I. Douven, Abduction, in: E. N. Zalta (Ed.), The Stanford Encyclopedia of Philosophy, summer 2021 ed., Metaphysics Research Lab, Stanford University, 2021.

[13] A. Tversky, D. Kahneman, Availability: a heuristic for judging frequency and probability, Cognitive Psychology 5 (1973) 207–232.

[14] M. Bar-Hillel, The base-rate fallacy in probability judgments, Acta Psychologica 44 (1980) 211–233.

[15] T. Shogenji, Is coherence truth conducive?, Analysis 59 (1999) 338–345.

[16] W. Blackstone, Commentaries on the laws of England: in four books, volume 2, J.B. Lippincott, 1753.

[17] J. Kaplan, Decision theory and the factfinding process, Stanford Law Review 20 (1968) 1065–1092.

[18] M. Günther, Legal proof should be justified belief of guilt, Legal Theory 30(3) (2024) 129–141.

[19] N. Fenton, M. Neil, D. A. Lagnado, A general structure for legal arguments about evidence using Bayesian networks, Cognitive science 37 (2013) 61–102.

[20] L. Van Leeuwen, R. Verbrugge, B. Verheij, S. Renooij, Building a stronger case: 3rd International Conference on Hybrid Human-Artificial Intelligence, HHAI 2024 (2024) 291–299.

[21] N. Fenton, M. Neil, B. Yet, D. Lagnado, Analyzing the Simonshaven case using Bayesian networks, Topics in Cognitive Science 12 (2020) 1092–1114.

[22] C. S. Vlek, H. Prakken, S. Renooij, B. Verheij, Building Bayesian networks for legal evidence with narratives: a case study evaluation, Artificial Intelligence and Law 22 (2014) 375–421.
[23] B. Verheij, F. Bex, S. T. Timmer, C. S. Vlek, J.-J. C. Meyer, S. Renooij, H. Prakken, Arguments, scenarios and probabilities: Connections between three normative frameworks for evidential reasoning, Law, Probability and Risk 15 (2016) 35–70.
[24] S. Vineberg, Dutch book arguments, in: E. N. Zalta, U. Nodelman (Eds.), The Stanford Encyclopedia of Philosophy, Fall 2022 ed., Metaphysics Research Lab, Stanford University, 2022.
[25] R. Pettigrew, Epistemic utility arguments for epistemic norms, in: E. N. Zalta, U. Nodelman (Eds.), The Stanford Encyclopedia of Philosophy, Summer 2024 ed., Metaphysics Research Lab, Stanford University, 2024.
[26] W. C. Thompson, E. L. Schumann, Interpretation of statistical evidence in criminal trials, Law and Human Behavior 11 (1987) 167–187.
[27] C. Dahlman, A. R. Mackor, Coherence and probability in legal evidence, Law, Probability and Risk 18 (2019) 275–294.
[28] P. Thagard, Coherence in thought and action, MIT press, 2002.
[29] A. Amaya, Coherence, evidence, and legal proof, Legal theory 19 (2013) 1–43.
[30] M. Redmayne, Exploring the proof paradoxes, Legal Theory 14 (2008) 281–309.
[31] L. J. Cohen, The Probable and the Provable, Clarendon Press, Oxford, 1977.
[32] B. Hedden, M. Colyvan, Legal probabilism: a qualified defence, Journal of Political Philosophy 27 (2019) 448–468.