Composite geometric modeling for directional brachytherapy: a novel approach for capsule placement optimization*

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Abstract

This study presents a novel geometric modeling approach for directional brachytherapy, focusing on optimizing capsule placement within anatomically complex target regions. The proposed method introduces composite capsule representations, combining cylindrical bodies with spherical radiation zones, and employs normalized phi-functions to enforce spatial and angular constraints. A hybrid optimization strategy is developed, integrating online and offline packing techniques to construct and refine feasible configurations incrementally. The mathematical formulation treats the placement problem as an identical item packing problem (IIPP), incorporating nonlinear programming to manage capsule interactions and orientation control. Computational experiments were conducted across four examples with varying capsule counts, geometric parameters, and constraint settings. Results demonstrate the method's adaptability to different anatomical conditions and effectiveness in non-parallel alignment. The approach supports anatomically conformal treatment planning.

Keywords

directional brachytherapy, geometric design, cylindrical capsule, non-linear optimization, phi-function technique

1. Introduction

Brachytherapy is a form of internal radiotherapy where sealed radioactive sources are placed directly within or near the tumor [1]. This technique allows for high-dose radiation delivery to malignant tissues while minimizing exposure to surrounding healthy structures. It is widely used in the treatment of prostate, cervical, breast, and head-and-neck cancers due to its precision and localized effect. A critical aspect of brachytherapy planning is the spatial arrangement of radioactive capsules within the target volume. These capsules are typically cylindrical, and their placement must be optimized to achieve a uniform and therapeutically effective dose distribution [2].

One of the foundational approaches in brachytherapy planning involves template-guided or grid-based placement of radioactive sources. These methods rely on predefined geometric patterns to ensure consistent spacing and orientation of capsules. While effective in standardized anatomical contexts, they often lack adaptability to patient-specific geometries. For example, in prostate

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brachytherapy, fixed templates may not account for organ deformation or irregular tumor shapes, leading to suboptimal dose coverage [3].

Recent developments have focused on inverse planning and optimization algorithms that allow for more flexible source placement. These methods use dose-volume constraints and iterative solvers to determine optimal capsule positions within the target volume. Modern approaches, such as those described by Harris et al. [4], integrate imaging modalities and individualized planning strategies to enhance dose conformity and minimize toxicity in prostate HDR brachytherapy. Another promising direction is the use of geometric modeling and spatial optimization. These approaches treat the capsule placement problem as a packing or tiling challenge, where cylinders must be arranged within a bounded volume without overlap and with controlled orientation. Tanderup et al. [5] demonstrated how such modeling improves dose conformity in cervical cancer treatment. Furthermore, patient-specific anatomy modeling has become increasingly important. Advanced imaging and segmentation techniques allow for the creation of 3D anatomical models, which serve as the basis for personalized capsule placement.

A comprehensive review by Morén, Larsson, and Carlsson Tedgren [6] analyzes the mathematical models used in high-dose-rate brachytherapy treatment planning and highlights the evolution from purely dosimetric optimization to geometry-aware approaches. The authors categorize existing methods into several classes, including linear and mixed-integer programming, quadratic programming, stochastic metaheuristics, multi-criteria optimization, and robust optimization. Linear and mixed-integer programming models are widely used to optimize dwell times while satisfying dose-volume constraints. These models are effective for enforcing strict clinical requirements but typically assume fixed source positions. In contrast, quadratic programming focuses on minimizing dose deviations and is computationally efficient, though less suited for handling hard geometric constraints. Stochastic and metaheuristic methods—such as simulated annealing, genetic algorithms, and particle swarm optimization—offer greater flexibility in exploring complex solution spaces. These approaches are particularly valuable when capsule placement must be optimized in irregular anatomical regions or when incorporating geometric constraints such as minimum inter-capsule distances and angular dispersion. Multi-criteria optimization frameworks allow planners to balance competing objectives, such as maximizing tumor coverage while minimizing exposure to organs at risk.

Recent advances have introduced geometric and computational methods for capsule placement in brachytherapy, including packing algorithms that treat the problem as one of fitting multiple cylinders within a bounded volume. These approaches leverage spatial modeling, dose optimization, and constraint-aware placement strategies. For example, Yousif et al. reviewed model-based dose calculation algorithms that incorporate anatomical geometry and material properties to improve dose accuracy [7]. Study [8] presents a GPU-accelerated Monte Carlo tool for HDR brachytherapy that significantly reduces computation time while maintaining high accuracy. Investigation [9] presents a composite geometric modeling framework for brachytherapy planning, where capsule placement is treated as a constrained packing problem with arbitrary orientations.

In most conventional models, the radiation source is assumed to be distributed along the axis of the capsule. However, in certain cases, the source may be localized at one end of the capsule typically at the center of one of its circular bases [7]. This configuration introduces asymmetry in the radiation field and necessitates more refined modeling techniques to accurately predict dose distribution. Such configurations are characteristic of directional brachytherapy, where the source is designed to emit radiation preferentially in one direction. Modeling these sources requires accounting for anisotropic dose distributions, which differ significantly from the symmetric kernels used in conventional TG-43-based planning. Chapter [10] discusses the use of focal and directional brachytherapy in prostate cancer, emphasizing the importance of accurate geometric modeling, imaging guidance, and orientation control to optimize dose delivery and minimize toxicity.

A critical technical consideration in directional brachytherapy is the impracticality of placing cylindrical capsules strictly parallel to the surface of living tissue, particularly along the tumor boundary. Such parallel arrangements are not only anatomically challenging due to tissue curvature and access limitations, but they can also lead to localized overdose, especially when directional sources are used. Studies such as [6] emphasize the importance of incorporating geometric constraints to avoid overly regular or symmetric placement patterns.

We propose a novel geometric modeling framework based on the packing of composite objects: a cylinder representing the physical capsule, and a sphere centered at one of its bases representing the localized radiation source. This modeling approach introduces several key advantages. The model captures directional emission patterns, enabling more accurate dose calculations. By adjusting the spatial relationship between capsules and their source zones, planners can fine-tune dose gradients and enforce angular dispersion constraints. The spherical component defines a localized region of radiation influence, allowing for precise control over capsule proximity and minimizing dose overlap. Our composite geometric modeling framework allows flexible capsule orientations and avoiding strict parallelism. This enables more adaptive and anatomically conformal placement strategies, reducing the risk of dose hotspots and improving overall treatment efficiency.

2. Problem statement

To ensure safe and effective placement of composite brachytherapy capsules, we model each capsule as a union of two geometric components $O_i = C_i \cup S_i$: a cylinder of with radius r and half-height h representing the physical implant, and a sphere with radius $\rho \geq r$, centered at the radiation source, for $i \in I_N = \{1,2,\dots,N\}$ where N is a sufficiently large number.

The center of each cylinder is located at the intersection of its axis and base. The center of each sphere is located in the intersection point of its axis and the cylinder base. The position and orientation of each object O_i is defined by the tuple $u_i = (v_i, \Theta_i)$, where $v_i = (x_i, y_i, z_i) \in R^3$ are the spatial coordinates of the cylinder center, $\Theta_i = (\varphi_i, \omega_i)$ are the orientation angles. Let $u = (u_1, u_2, \dots, u_N)$ be the configuration vector of all objects. The object O_i located in u_i is denoted as $O_i(u_i)$, $i \in I_N$.

The direction of the axis of each cylinder is represented by a unit vector $n_i \in R^3$, depending from its orientation angles as follows: $n_i = (\sin\omega_i \cos\varphi_i, \sin\omega_i \sin\varphi_i, \cos\omega_i)$.

The spheres represent the influence zones of emitted radiation. This composite structure allows us to simultaneously control the physical feasibility of placement and the dosimetric impact on surrounding tissues. The cylinder defines the spatial footprint of the implant within the target region, while the sphere serves to regulate the proximity of the radiation source to other sources and to healthy anatomical structures.

The target region T (tumor volume) is modeled as a convex polyhedron given by the intersection of half-spaces $A_j x + B_j y + C_j z + D_j \leq 0$, $j \in J_m = \{1,2,\dots,m\}$, where m is the number of half-spaces:

The optimization objective is to maximize the number of capsules $n \leq N$ that can be placed inside T without overlap, while maintaining a minimum distance d between pairs of cylinders and the frontier of T.

Based on the phi-function method, the mathematical model of the problem is formulated as an identical item packing problem (IIPP) [11]:

$$n^* = max \sum_{i \in I_N} \Psi_i(u_i) \text{ s.t. } u \in G, \tag{1}$$

where the indicator function $\Psi_i(u_i)$ is defined as:

$$\Psi_i(u_i)=\begin{cases} 1 \text{ if } \Phi_i(u_i)\geq 0, \\ 0 \text{ otherwise,} \end{cases} \tag{2}$$

and the feasible region is

$$G=\{u\in R^{5n}: \Phi_{ij}(u_i,u_j)\geq 0, i<j\in I_N\} \tag{3}$$

$$\Psi_i(u_i)\alpha i_i \arccos\frac{n_i \cdot e_j}{|n_i|}_{iimaxmin} \quad i\in I_N j\in J_m \tag{4}$$

$$\Psi_i(u_i)\Psi_i(u_i)\beta i_{iii} \arccos\frac{n_i \cdot n_j}{|n_i|\cdot|n_j|}_{iiiimaxmin} \tag{5}$$

In (2), $\Phi_i(u_i)=min\{\Phi_{ic}(u_i)-d, \Phi_{is}(a_i)\}$ is a normalized phi-function for O_i and T^* where $T^*=R^3\setminus\int$ {T, $i\in I_N$ [12], $\Phi_{ic}(u_i)$ is a normalized phi-function for C_i and T^*, $\Phi_{is}(a_i)$ is a phi-function for S_i and T^*, $a_i=u_i+h_i n_i$. $\Phi_{ij}(u_i,u_j)=min¿\Phi_{ijcc}(u_i,u_j)-d, \Phi_{ijcs}(u_i,a_j), \Phi_{ijsc}(a_i,u_j), \Phi_{ijss}(a_i,a_j)\}¿$ is a normalized phi-function for O_i and O_j, $i<j\in I_N$. Here, $\Phi_{ijcc}(u_i,u_j)$ is a phi-function for C_i and C_j, $\Phi_{ijcs}(u_i,a_j)$ is a phi-function for C_i and S_j, $\Phi_{ijsc}(a_i,u_j)$ is a phi-function for S_i and C_j, $\Phi_{ijss}(a_i,a_j)$ is a normalized phi-function for S_i and S_j.

Each phi-function serves a specific purpose:

$\Phi_{ic}(u_i)$ ensures that the cylindrical component of capsule O_i is fully contained within T, maintaining anatomical validity,

$\Phi_{is}(a_i)$ ensures that the spherical radiation zone does not extend beyond the target region, protecting healthy tissue from unintended exposure,

$\Phi_{ijcc}(u_i,u_j)$ enforces a minimum separation between the cylindrical bodies of C_i and C_j, preventing physical overlap and excessive local dosing.

$\Phi_{ijcs}(u_i,a_j)$ controls the distance between the cylinder C_i and the radiation source of another capsule, ensuring safe spatial separation,

$\Phi_{ijsc}(a_i,u_j)$ ensures that the radiation source presented by S_i is not too close to the cylinder S_i, preserving dose uniformity and avoiding interference,

$\Phi_{ijss}(a_i,a_j)$ maintains a safe distance between the radiation sources of different capsules, preventing overlapping influence zones and cumulative dose effects.

Together, these constraints define the feasible region $G=\{u\in R^{5n}: \Phi_{ij}(u_i,u_j)\geq 0, i<j\in I_N\}$ for capsule placement, ensuring geometric compatibility, clinical safety, and dosimetric effectiveness.

The precise number of composite capsules that can be accommodated within the target region T, subject to the imposed minimum separation constraints, is not known a priori. Nevertheless, a preliminary upper bound can be inferred by comparing the aggregate volume of the capsules to the volume of the target domain.

To approach this IIPP, we adopt a sequential placement strategy, whereby capsules are introduced one at a time into the domain, known as incremental block optimization [13] or online packing [15]. This method, often referred to as block optimization, allows for incremental construction of admissible configurations.

Central to this methodology is the formulation of normalized phi-functions, which serve as analytical tools for evaluating spatial admissibility. An additional complexity arises from the inclusion of orientation parameters, which define the angular disposition of each capsule and influence both geometric feasibility and dosimetric performance. In the present study, the cylindrical components of the capsules are approximated by convex polyhedral shapes for which phi-function are constructed and well explored [12]. When selecting a sufficiently high number of faces in the polyhedral approximation, the geometric distortion becomes negligible and does not compromise spatial separation or dosimetric accuracy.

3. Solution approach

We propose a hybrid optimization strategy that combines elements of online packing and offline packing. This approach enables both incremental configuration construction and global adjustment of capsule positions and orientations. Strategy for solving the problem.

3.1. General strategy

In online packing phase, capsules are introduced sequentially into the target domain. Each new capsule is initially placed with reduced size and fixed orientation, ensuring non-overlap with previously placed capsules. This phase emphasizes feasibility under static conditions.

In offline packing phase capsule dimensions are gradually homotatically restored to their original size. Simultaneously, all capsules are allowed to move and rotate. This phase involves solving a nonlinear programming problem to optimize spatial arrangement while maintaining all geometric and dosimetric constraints. This dual-phase strategy allows for adaptive placement in complex anatomical regions, balancing computational efficiency with clinical accuracy.

3.2. Algorithmic steps

Step 1. Estimate capacity. Determine the number of capsules n that can potentially be placed within the target region T, ensuring that each capsule $P_i(u_i), i \in I_n$, fits geometrically.

Step 2. Incremental expansion. Set $n := n+1$ to test the feasibility of placing an additional capsule.

Step 3. Initialize Scaling. Set the scaling factor $g_n := 0.01$.

Step 4. Online phase. Randomly generate a vector u_n, ensuring $v_n \in T$, $0 \le \varphi_n \le 2\pi$, $0 \le \omega_n \le 2\pi$.

Step 5. Offline phase. Solve the following nonlinear optimization problem

$$\max_{\tau} g_i \text{ s.t. } \tau = (u_1, u_2, \dots, u_n, g_n) \in W \tag{6}$$

where

$$W = \left\{ \begin{array}{l} \tau \in \mathbf{R}^{5n+1} : \Phi_i(\mathbf{u}_i) \ge 0, i \in I_n / \{n\}, \Phi_n(\mathbf{u}_n, g_n) - d \ge 0, \\ \Phi_{ij}(\mathbf{u}_i, \mathbf{u}_j) - d \ge 0,\ i < j \in I_n / \{n\}, \\ \Phi_{in}(\mathbf{u}_i, \mathbf{u}_n, g_n) - d \ge 0,\ i \in I_n / \{n\}, \\ \alpha_{\min} \le \arccos \dfrac{\mathbf{n}_i \cdot \mathbf{e}_j}{|\mathbf{n}_i|} \le \alpha_{\max}\ , i \in I_n, j \in J_m, \\ \beta_{\min} \le \arccos \dfrac{\mathbf{n}_i \cdot \mathbf{n}_j}{|\mathbf{n}_i| \cdot |\mathbf{n}_j|} \le \beta_{\max}\ , i < j \in I_n, \\ g_n \le 1,\ i < j \in I_n \end{array} \right\}. \tag{7}$$

The scaling factor g_n means that the cylinder C_n is considered with variable radius $g_n r$ and half-height $g_n h$ and the sphere S_n is with radius $g_n \rho$ (see [16]).

Step 6. If $g_n^* = 1$ in a local minimum point of problem (6), (7), then return to Step 2. Otherwise, go to Step 7.

Step 7. An approximate solution to problem (1) is taken to be $n^* := n - 1$. The algorithm terminates.

The proposed hybrid optimization strategy combining online and offline packing offers a robust framework for directional brachytherapy planning. By involving normalized phi-functions and incremental block optimization, the method ensures that capsules are placed with respect to anatomical constraints and radiation safety. The online phase facilitates rapid feasibility checks by introducing capsules in reduced form, while the offline phase refines the configuration through nonlinear programming, allowing simultaneous adjustment of positions and orientations. The integration of angular constraints and composite object modeling (cylinder and sphere) allows for precise control over directional dose delivery, minimizing overlap and enhancing treatment quality. The algorithm's modular structure also supports scalability and adaptability to various anatomical sites and treatment protocols.

4. Numerical examples

To evaluate the performance and behavior of the proposed capsule placement algorithm, we conducted a series of computational experiments with varying numbers of capsules, minimum allowed distances and angular constraints. These experiments aim to illustrate how geometric parameters influence the final configuration and feasibility of directional brachytherapy plans.

Example 1: 30 Capsules Without Angular Constraints In the first scenario, we considered a configuration of 30 composite capsules, each consisting of: A cylindrical body with radius 1.0 and height 1.5 units. A spherical radiation zone with radius 1.0 unit, centered at a base of the cylinder. The placement domain was randomly generated to ensure that all capsules could be accommodated without violating spatial constraints. The bounding box of the domain was: Width: 8.03 units Length: 6.22 units Height: 12.49 units No angular constraints were imposed in this experiment, meaning that capsules were allowed to orient freely in space. The minimum allowable distance between capsules was set to zero, permitting direct contact between the capsules. The algorithm successfully placed all 30 capsules within the domain, and the total computation time was 45 seconds. This configuration is shown in Fig.1.

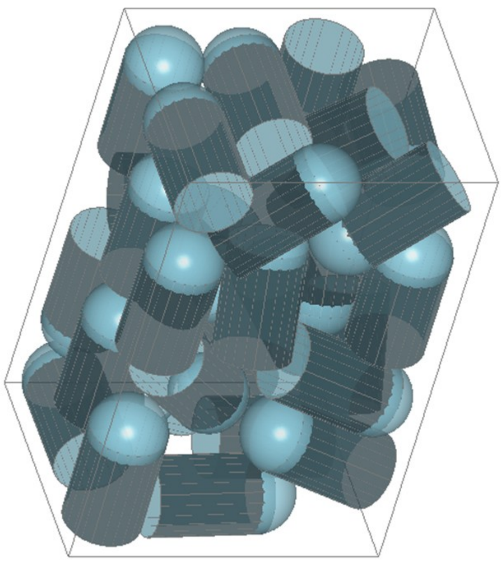

Figure 1: Packing of 30 composite objects according to Example 1.

Example 2. In the second example, we extended the experiment to 40 composite capsules, maintaining the same modeling principles but modifying the capsule geometry and domain size. Each capsule consists of a cylinder with radius 1.0 and height 3.0 units, reflecting a 1:3 ratio between radius and height. The spherical radiation zone retains a radius of 1.0 unit. The placement domain was expanded to accommodate the increased number and size of capsules. The bounding box of the domain was: width: 6.58 units Length: 14.42 units Height: 18.08 units. The runtime was about 4 minutes. This configuration is shown in Fig.2.

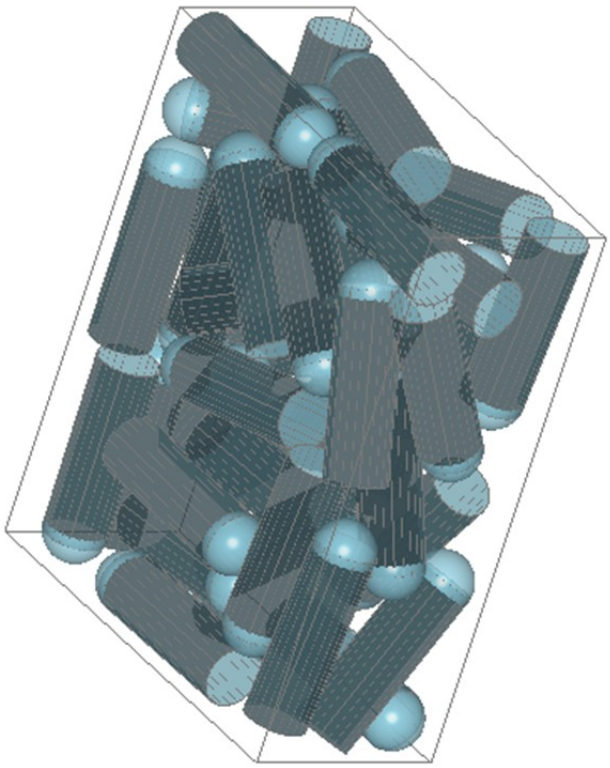

Figure 2: Packing of 40 composite objects according to Example 2.

Example 3. In the third example, we investigated how angular and spatial constraints affect the structure of capsule placement plans. The setup was similar to Example 2, with 40 composite capsules, each consisting of a cylindrical body with a radius of 1.0 and a height of 3.0 units, maintaining the same 1:3 ratio. The spherical radiation zone attached to each capsule had a radius of 1.0 unit. Capsules were placed within a cubic domain measuring 15 × 15 × 15 units. Unlike previous examples, this configuration introduced a minimum allowable distance of 1.0 unit between the cylindrical bodies of any two capsules, ensuring physical separation and reducing the risk of localized overdose. Additionally, angular constraints were imposed between capsules: β_{min}, β_{max}. No angular constraints between capsules and domain boundaries. Runtime was about 5 minutes. The packing obtained is shown in Fig.3. This experiment demonstrates how the introduction of geometric constraints, both spatial and angular, can significantly influence the resulting configuration, leading to more clinically safer placement strategies.

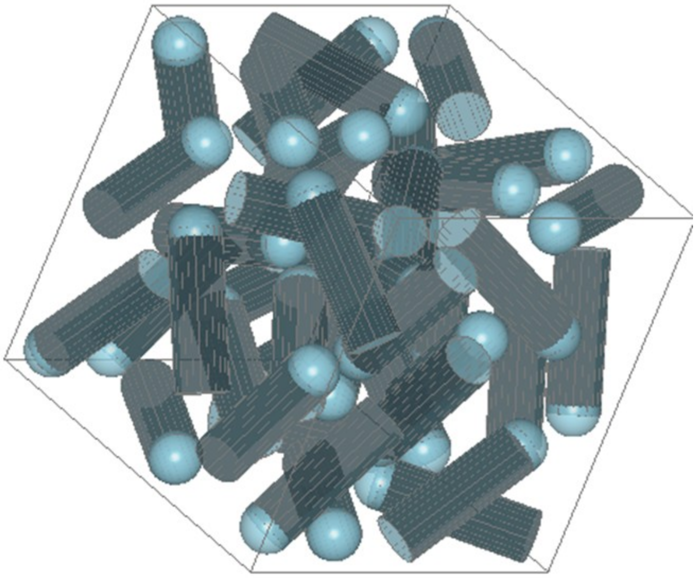

Figure 3: Packing of 40 composite objects according to Example 3.

Example 4. In the final example, we introduced both inter-capsule and boundary-related angular constraints to evaluate their combined effect on capsule placement. The setup remained consistent with Example 3 in terms of capsule geometry: 40 composite capsules were used, each consisting of a cylindrical body with a radius of 1.0 and a height of 3.0 units, maintaining the 1:3 ratio. The spherical radiation zone attached to each capsule had a radius of 1.0 unit. To accommodate the stricter constraints, the placement domain was enlarged beyond the dimensions used in previous examples. According to the extracted data, the bounding box of the domain measured approximately 16.07 × 15.15 × 20.18 units. The constraints applied in this scenario included a minimum distance of 1.0 unit between the cylindrical bodies of any two capsules. Angular constraints were also imposed between capsules, requiring the angle between their central axes and angular constraints between capsules and the domain boundaries $\alpha\, min_{min}$, $\alpha\, max_{max}$. The resulting configuration, as shown in Fig.4, exhibits no parallel alignment neither between capsules nor between capsules and the domain boundaries. The placement parameters are presented in Table 1. This outcome confirms that the algorithm effectively enforces angular dispersion, even under complex spatial conditions.

Figure 4: Packing of 40 composite objects according to Example 4.

Table 1
Placement parameters of cylindrical capsules

i	x_i	y_i	z_i	φ_i	ω_i
1	14.583	14.065	8.678	-0.636	-1.320
2	9.046	6.806	13.455	0.927	-0.345
3	17.138	15.987	21.653	-0.845	-0.922
4	10.083	12.167	25.694	0.927	-1.231
5	9.038	12.271	9.620	-0.927	-0.340
6	17.143	15.830	12.167	0.802	-0.622
7	13.424	6.111	20.102	-0.494	-0.431
8	14.875	5.326	9.719	0.201	0.900
9	10.492	17.314	10.218	-0.201	2.424
10	21.443	11.111	13.087	2.417	-0.525
11	10.083	7.847	8.678	-2.214	1.911
12	21.947	6.518	21.197	2.395	2.866
13	9.063	17.300	17.294	-2.935	0.215
14	17.784	6.747	21.599	2.214	-2.795
15	18.012	16.124	16.721	2.478	-2.096
16	21.947	16.028	20.200	2.435	2.875
17	13.410	16.837	18.216	-0.386	-0.591
18	17.061	6.340	15.279	2.502	-0.372
19	12.831	6.341	25.614	-2.554	1.291
20	14.033	9.485	10.969	3.616	2.288
21	9.038	16.373	23.253	-2.540	0.245
22	20.230	10.812	18.636	-0.305	2.147
23	19.830	6.373	25.696	-2.540	-1.326
24	21.947	15.771	13.684	2.309	0.302
25	17.216	10.774	15.678	-0.642	-0.328
26	10.033	13.290	15.342	-0.246	-2.541
27	11.780	9.663	16.585	0.201	-0.955
28	15.761	11.124	24.646	-2.361	0.529
29	21.273	5.876	14.950	-2.739	0.517
30	9.038	6.412	21.838	-0.601	-2.897
31	13.198	16.075	24.314	-2.363	-2.857
32	21.918	6.325	10.063	0.632	0.263
33	21.766	11.669	23.979	-0.570	0.317
34	19.066	9.357	8.695	-2.214	1.222
35	10.286	11.724	21.535	0.745	1.295
36	20.844	16.014	25.594	-2.318	1.215
37	12.195	5.326	15.791	0.201	-0.793
38	20.526	16.298	8.755	0.589	-1.294
39	14.550	11.846	19.646	0.927	2.088
40	12.822	15.882	13.379	-0.920	2.303

The numerical experiments demonstrate the algorithm's adaptability across synthetic cuboidal domains. Importantly, the developed approach supports arbitrary convex polyhedral representations and can be extended to handle non-convex anatomical regions.

5. Conclusion

This study presents an adaptable optimized geometric model for directional brachytherapy, enabling precise and constraint-aware placement of radioactive capsules within anatomically complex regions. By modeling each capsule as a composite object and applying normalized phi-functions, the method effectively enforces spatial separation, angular dispersion, and dose conformity. The hybrid optimization strategy, combining online and offline packing, demonstrates strong performance across varying geometric and clinical scenarios. It allows for incremental feasibility testing and global refinement, accommodating both fixed and dynamic constraints.

The computational experiments confirm that the algorithm reliably generates feasible, non-overlapping configurations while respecting angular and spatial constraints. The final example illustrates the method's ability to eliminate parallel alignments, both between capsules and with domain boundaries, which is critical for directional dose control and minimizing interference.

While the spherical radiation zone provides a mathematically tractable and physically intuitive model for directional emission, we acknowledge that real anisotropic dose distributions may require more refined representations. Future work may explore ellipsoidal or empirically derived dose kernels to better capture source-specific anisotropy.

The use of convex polyhedral approximations for cylindrical components enables the application of phi-functions but may introduce minor geometric distortions. Although these approximations are computationally efficient, future studies will investigate exact phi-function formulations for true cylinders to enhance placement accuracy.

This will also lay the groundwork for future advancements in real-time optimization, multi-objective treatment planning, and integration with robotic-assisted delivery systems.

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Declaration on Generative AI

The authors have not employed any Generative AI tools.

References

[1] B. R. Smith , S.A. Strand, D. Dunkerley, et al., Implementation of a real-time, ultrasound-guided prostate HDR brachytherapy program, J Appl Clin Med Phys. 22 (2021) 189–214. doi:10.1002/acm2.13363.

[2] B. Zhang, S. Zhang, L. Sun, Y. Wu, Y. Yang, Characteristics of preplan-based three-dimensional individual template-guided brachytherapy compared to freehand implantation, J Appl Clin Med Phys. 24(3) (2023) e13840. doi:10.1002/acm2.13840.

[3] S. Gerlach, F-A. Siebert, A. Schlaefer, Robust stochastic optimization of needle configurations for robotic HDR prostate brachytherapy, Med Phys. 51 (2024) 464–475. doi:10.1002/mp.16804.

[4] A. A. Harris, et al., High Dose Rate Prostate Brachytherapy, in: A. A .Solanki, R. C. Chen, (Eds.) Radiation Therapy for Genitourinary Malignancies. Practical Guides in Radiation Oncology, Springer, Cham, 2021, pp. 127–151. doi:10.1007/978-3-030-65137-4_6.

[5] K. Tanderup, et al., Advances in MRI-guided adaptive brachytherapy in cervical cancer, Seminars in Radiation Oncology 31(4) (2021) 349–363. doi:10.1016/j.semradonc.2021.05.003

[6] B. Morén, T. Larsson, A. C. Tedgren, Optimization in treatment planning of high dose-rate brachytherapy – Review and analysis of mathematical models, Med. Phys. 48 (2021) 2057–2082. doi:10.1002/mp.14762.
[7] Y. A. M. Yousif, A. F. I. Osman, M. A. Halato, A review of dosimetric impact of implementation of model-based dose calculation algorithms (MBDCAs) for HDR brachytherapy, Phys Eng Sci Med 44 (2021) 871–886. doi:10.1007/s13246-021-01029-8.
[8] A. K. Hu, R. Qiu, H. Liu, et al., THUBrachy: fast Monte Carlo dose calculation tool accelerated by heterogeneous hardware for high-dose-rate brachytherapy. Nucl Sci Tech 32 (2021).
[9] G. Yaskov, A. Chuhai, Y. Yaskova, M. Shcherbyna, Intelligent System and Technology for Optimized Object Placement in Medical and Biological Applications, Proceedings of the eighth international workshop on computer modeling and intelligent systems (CMIS 2025), Zaporizhzhia, Ukraine, May 5, 2025, pp. 241–252, CEUR Workshop Proceedings, Vol. 3988. https://ceur-ws.org/Vol-3988/paper19.pdf.
[10] L. Tagliaferri, B. Fionda, J. Grummet, A. See, G. Kovács, Focal Brachytherapy (Interventional Radiotherapy) and IMRT, in: T. J. Polascik, J. de la Rosette, R. Sanchez-Salas, A. R. Rastinehad (Eds.) Imaging and Focal Therapy of Early Prostate Cancer, Springer, Cham, 2024.
[11] G. Wäscher, H. Haußner, H. Schumann, An improved typology of cutting and packing problems. European Journal of Operational Research 183(3) (2007) 1109–1130. doi:10.1016/j.ejor.2005.12.047.
[12] A.M. Chugay, A.V. Zhuravka, Packing Optimization Problems and Their Application in 3D Printing, Advances in Intelligent Systems and Computing, (2021) vol 1247. Springer, Cham. https://doi.org/10.1007/978-3-030-55506-1_7.
[13] G. Yaskov, A. Chugay, Packing Equal Spheres by Means of the Block Coordinate Descent Method, CEUR Workshop Proceedings 2608 (2020) 150–160. doi:10.32782/cmis/2608-13.
[14] S. Ali, A. G. Ramos, M. A. Carravilla, J. F. Oliveira, On-line three-dimensional packing problems: A review of off-line and on-line solution approaches, Computers & Industrial Engineering 168 (2022) 108122. doi: 10.1016/j.cie.2022.108122.
[15] R. Moqa, J. Zheng, K. He., Online and offline packing cylinders in a cylindrical container. Discrete Applied Mathematics 375 (2025) 85–104. doi:10.1016/j.dam.2025.05.039.
[16] G. Yaskov, T. Romanova, I. Litvinchev, S. Shekhovtsov, Optimal Packing Problems: From Knapsack Problem to Open Dimension Problem, in: P. Vasant, I. Zelinka, G. W. Weber (Eds.), Intelligent Computing and Optimization, ICO 2019, Advances in Intelligent Systems and Computing, vol 1072, Springer, Cham, 2020, pp. 671–678. doi:10.1007/978-3-030-33585-4_65.
[17] A. Wächter, L. T. Biegler, On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming, Mathematical Programming 106(1) (2006) 25–57. doi:10.1007/s10107-004-0559-y.