Order in the Mereology of Slots

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Abstract

In this paper, we extend the mereology of slots with an order relation, of particular importance for characterizing the structure and identity of informational entities such as texts. We propose a theory for linear informational entities. We show the theory consistent using Alloy, a model finder. We discuss under which condition two informational entities that share mereological and order structure should be considerd as identical or not.

Keywords

Slot Mereology, Structural universals, Informational entities, Order, Time, Identity

1. Introduction

The ontology of informational entities is currently receiving attention [1, 2, 3]. It has been shown in such work that classical mereology is not adequate for accounting for their parthood structure, and that one rather needs to resort to slot mereology [4], initially developed for structural universals [5]. In this paper, we intend to supplement this work by proposing an extension involving order relations, of particular importance for two kinds of informational entities, texts (which will be the main focus here) and procedures. Structural universals are less prominent in applied ontology, which usually considers only domains of particulars. However, the ontology of structural universals also crucially requires additional relations on top of mereology to characterize their identity.

In classical extensional mereology [6], adequate in particular for material entities, two entities are identical iff they share exactly the same proper parts—this is extensionality. As argued in [7], the identity of structural universals and informational entities cannot be grasped by mereology alone and extensionality doesn't apply. For instance two different texts can have exactly the same words but arranged differently: 'Lea loves Leo' is not the same sentence as 'Leo loves Lea' while they are composed of exactly the same three words. Additional structural relations are needed to characterize the identity of structural universals and informational entities; in the case of texts, we need word order. In this paper, we propose to extend the mereology of slots proposed in [7] with order or sequence relations, building on Van Benthem's classical theory of time based on periods, parthood and temporal precedence [8].

We first briefly present the mereology of slots, and then discuss and motivate choices for new primitives and axioms to extend this mereology of slots with order.

2. Mereology of slots

Slot mereology was introduced by Bennett in 2013 [4] to address the issue of multiple parthood, which is relevant among structural universals. Structural universals are universals with an internal structure, that is, having other universals as parts [5]. A standard example is a universal of molecule, say H_2O, having universals of atoms as parts, here H and O. Notably, one can argue that the (unique) universal H is part twice in the H_2O universal. Similar situations occur with informational entities, for instance texts often have a same word as part several times (e.g., the sentence 'Structural universals are universals' has the word 'universals' twice), and some written words have the same letter as part several times (e.g., the word 'structural' has the letter 't' twice). Blueprints, designs, procedures and plans are other

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kinds of informational entities that can have several times the same parts, sub-designs or element types (say, a given screw type) and sub-plans or action types (say, move-forward).

Standard mereology cannot represent such facts. Slot mereology addresses this by introducing new entities—called "slots" of a whole—to account for cases where the same part occurs multiple times within that whole. Each slot is filled by a unique "filler" and identifies one of the possibly multiple specific contexts in which this filler appears as a part in the whole. A same part, then, can fill different slots of a given whole. The domain is thus partitioned into fillers, which could be, e.g., structural universals or informational entities, and slots, that characterize the mereological structure of such fillers.

Bennett's slot mereology was recently thoroughly investigated from a theoretical point of view, leading to a revised and expanded theory named "mereology of slots" [7]. This new theory enables a proper counting of parts of fillers and provides a full mereological account by including a form of supplementation as well as sum and fusion operators at the level of slots. It develops a non-classical mereology among fillers, as expected, but crucially strengthens it by defining parthood between slots, leading to a classical extensional mereology among slots. Extending Bennett's theory with some form of supplementation is desirable to make it a significant mereology [9], but in addition, the new proposal fixes a flaw in Bennett's theory, which doesn't count properly parts when parts of parts are involved.

For lack of space, we will not reproduce the whole theory here, which has 3 primitive relations: F, P_s and a ternary primitive of "contextualization" (omitted here), 18 axioms, 18 definitions and more than 60 theorems. We'll use only some of its vocabulary, namely:

- F — $F(x, s)$ reads "filler x fills slot s"
- P_s — $P_s(s, x)$ reads "slot s is a parthood slot of filler x" or "filler x owns slot s"
- S — $S(s)$ reads "s is a slot". This is defined as $\exists x P_s(s, x)$
- P — $P(x, y)$ reads "filler x is part of filler y". This is defined as $\exists s(P_s(s, y) \wedge F(x, s))$
- SO — $SO(s, t)$ reads "slots s and t share the same owner".
- PoS — $PoS(s, t)$ reads "slot s is part of slot t"
- $PPoS$ — $PPoS(s, t)$ reads "slot s is a proper part of slot t"
- OoS — $OoS(s, t)$ reads "slot s overlaps slot t"
- $+$ — $s = t + u$ reads "slot s is the sum of slots t and u"
- FoS_ϕ — $FoS_\phi(x)$ reads "x is the fusion of the slots having property ϕ"

We'll also use relevant axioms and theorems, in particular that parthood between slots PoS is transitive, antisymmetric and (conditionally) reflexive (T18, T19, T20),[1] that SO is an equivalence relation and that sums and fusions (conditionally) exist. Other important axioms and theorems state that each slot is filled by a unique filler (BA7) and that each slot is owned by a unique filler (A1). In addition, each filler has a unique improper slot, filled by itself (A2, A3). The transitivity of parthood between fillers P (T13) is guaranteed by a mechanism of duplication of slots between levels, based on the ternary primitive of contextualization. What is important for the remainder of the paper is that only slots belonging to a same owner can be mereologically related: the mereology of slots is local. Each filler generates a separate extensional mereology of its slots, whose universe is the improper slot of the filler (T62).

Let's illustrate this theory with a simple example: the word w 'ada' has 2 atomic parts (assuming letters are considered as atoms), the letter l_1 'a' being part of w twice. Thus w has 3 slots, s_1 to s_3, filled by these 2 letters: s_1 and s_3 by the letter l_1 'a' and s_2 by the letter l_2 'd'. All sum combinations of these atomic slots give rise to three additional slots filled by a combination of two letters. In addition, w has the improper slot s_0, filled by itself. This can be (partially) described by the following formulas:

$P_s(s_0, w)$
$P_s(s_1, w)$
$P_s(s_2, w)$

[1]All numbered axioms, definitions, lemmas and theorems not explicitly introduced in this paper refer to those of [7].

$P_s(s_3, w)$
$F(w, s_0)$
$F(l_1, s_1)$
$F(l_2, s_2)$
$F(l_1, s_3)$
$s_{12} = s_1 + s_2$
$s_{23} = s_2 + s_3$
$s_{13} = s_1 + s_3$

Notice that the slot s_{13}, sum of s_1 and s_3, is filled by something having as part l_1 twice, and this thing is therefore part of w. However, it can be argued that the string 'aa' isn't a part of the word 'ada' and that the filler of s_{13} is not a string. In fact, words and strings are (convex) *sequences* of letters, a feature impossible to grasp with mereology alone. This is one motivation to add order to the theory.

No order relation can be introduced between fillers since they may repeatedly appear in a word and moreover appear in many words with different orderings: in the above example of the word 'ada', the letter 'a' would be both before and after the letter 'd', which is inconsistent with the asymmetry of strict order. But order between slots can be introduced, as they stand for specific contexts in which the letters appear (and are not shared between words). We will address the interplay between this order relation and the mereological relations and operators on slots.

3. Order and mereology

Order considerations are especially prominent in the ontology of time. In the '80s and '90s of last century, a number of first-order theories of time have been proposed. Some of them consider intervals (aka periods) as basic elements of the domain, and therefore also handle notions of sub-interval, that is, mereological relations.

One such proposal is the theory of Allen and Hayes [10], which is an axiomatization of Allen's well-known algebra of 13 relations between intervals [11]. We will rather exploit Van Benthem's theory of periods [8] and subsequent work by Hajnicz [12]. In fact, Allen and Hayes's theory used the unique primitive relation "meets", capturing the relation between two ordered adjacent convex intervals. That is, they axiomatized in an interdependent way relations that pertain to mereology and to order. On the other hand, Van Benthem's theory is based on two primitive relations, a mereological parthood and an order, with axioms characterizing each separately, and axioms linking them. We will adapt this approach, adding to the mereology of slots an order primitive $<$ on slots, with axioms on $<$ and mixed axioms on both $<$ and mereological relations and operators. We will show that, in this extended theory, the slot parthood of the mereology of slots behaves as Van Benthem's mereological parthood. We will also recover some elements from Allen and Hayes's theory.

3.1. From time to sequences in informational entities

There are important differences between axiomatizing the temporal order between temporal intervals and the sequence between slots of an informational entity in the context of the mereology of slots.

First, the explicit or implicit assumption of Van Benthem and Hajnicz, as well as Allen & Hayes, is that intervals or periods are "one-piece", i.e., convex, following the mathematical definition of intervals in $\mathcal{Z}$, $\mathcal{Q}$ or $\mathcal{R}$ (any point situated in between two points of the interval belongs to the interval). On the other hand, as seen in the example above with slot s_{13}, the sum operator between slots in the mereology of slots makes it impossible to adopt this assumption. Such slots can be compared to non-convex intervals [12] or "generalized intervals" [13]. So, here, the convexity of slots cannot be assumed.

Second, time is generally assumed to be unbounded, and this is reflected in axioms in Van Benthem's, Hajnicz's and Allen & Hayes's works. For informational entities, for instance words as sequences of letters and texts as sequences of words, it is clearly impossible to assume that all of them are infinite, so such axioms have to be dropped. Assuming that, on the contrary, there always is a beginning and an end is a step ahead that we shall leave open. Similarly, time is often seen as dense, as with intervals in $\mathcal{Q}$ or $\mathcal{R}$, but this is questionable here, especially with words and texts. Atomicity, discussed by Van Benthem, was not discussed in the mereology of slots. Again, this is an option that will be proposed.

Finally, linearity is adopted by Van Benthem, whose aim is to axiomatize the structures of intervals in $\mathcal{Q}$ or $\mathcal{R}$. The same assumption is made by Allen & Hayes, but this is discussed by Hajnicz. Linearity is not always adopted in theories of time and action, as time is sometimes seen as branching (to the right, to the left, or both—as illustrated e.g. by the work of Broersen [14]). Words and texts are linear, and this is what we assume here. This might be questionable for other informational entities, especially procedures for which parallelism or alternatives following a test-action is often needed. We will leave this possibility for future work and simply impose linearity in this paper.

3.2. Axiomatizing order between slots

The order relation $<$ is intended to hold when a slot fully precedes another one, including adjacency (i.e., "meets" in Allen's algebra). For instance, in the case above of slots of the word w ('ada'), $s_1 < s_2$, $s_2 < s_3$ and $s_1 < s_3$, but also $s_{12} < s_3$ and $s_1 < s_{23}$.

Just as mereological relations, the order $<$ holds only on slots belonging to the same owner:

OA 1. $x < y \rightarrow (S(x) \wedge S(y) \wedge SO(x, y))$

The order $<$ standardly is transitive and irreflexive (and thus asymmetric):

OA 2. $(x < y \wedge y < z) \rightarrow x < z$

OA 3. $\neg x < x$

Since the order $<$ covers adjacency, it makes no sense to assume a form of density, there is not necessarily a third slot between two slots preceding one another. But we do assume that the order has no "holes", there always is (at least) an adjacent slot to the right when there is a slot to the right (and similarly to the left). To do so, we first introduce the definition of Allen's meets relation, and adopt Allen & Hayes's axiom guaranteeing the unicity of meeting points:[2]

OD 1. $M(x, y) \equiv_{def} x < y \wedge \forall z\, \neg(x < z \wedge z < y)$

OA 4. $(M(x, y) \wedge M(x, u) \wedge M(v, y)) \rightarrow M(v, u)$

Note that, trivially, two slots that meet have the same owner. Then, the equivalent of Van Benthem's "neighbour" conditions are adopted:

OA 5. $x < y \rightarrow \exists u\, M(x, u)$

OA 6. $y < x \rightarrow \exists u\, M(u, x)$

As discussed above, we leave it open to add further axioms constraining the order relation, depending on the domain at hand. For informational entities, having an unbound order does not seem to make sense; on the contrary, one could assume the existence of initial and ending slots. We first define initial and ending slots. Note that among the slots of a same filler, there can be several such slots (in our example above, s_1 and s_{12} are both initial slots).

OD 2. $INIT(x) \equiv_{def} S(x) \wedge \forall y (SO(x, y) \rightarrow \neg y < x)$

OD 3. $END(x) \equiv_{def} S(x) \wedge \forall y (SO(x, y) \rightarrow \neg x < y)$

Then, the following definition can be used to characterize the boundedness of the order among the slots of a given filler, and used to impose an axiom claiming that every filler is bounded, if required.

OD 4. $BOUND(x) \equiv_{def} \exists y, z\, (P_s(y, x) \wedge P_s(z, x) \wedge INIT(y) \wedge END(z))$

[2] Axiom M1 in [10]

3.3. Integrating order into the mereology of slots

We now need to link mereology and order. As proposed by Van Benthem, we adopt axioms of monotonicity. Any part of a slot x preceding a slot y also precedes y, and vice versa:

OA 7. $x < y \rightarrow \forall z(PoS(z, x) \rightarrow z < y)$

OA 8. $x < y \rightarrow \forall z(PoS(z, y) \rightarrow x < z)$

As discussed above, slots can be non-convex, but of course there are convex slots. A slot is convex iff for any two parts of it that are ordered, every slot in between is also a part of it:

OD 5. $CONV(x) \equiv_{def} S(x) \wedge \forall yz((PoS(y, x) \wedge PoS(z, x)) \rightarrow \forall u((y < u \wedge u < z) \rightarrow PoS(u, x)))$

Actually, such a definition makes sense only if we assume a form of linearity of the order (the sum of two slots on different branches would be trivially convex, which does not capture the intuition of convexity). We do adopt linearity here, as discussed above. Any two slots of a same owner always are "comparable" in this mixture of order and mereology: they cannot be on separate branches of the order, as may happen in a branching time. A simple way of adopting linearity in the presence of non-convex slots is to exploit the underlying order of implicit meeting "points" of adjacent slots:[3]

OA 9. $(M(x, y) \wedge M(u, v) \wedge SO(x, u)) \rightarrow (x < v \vee u < y)$

That is, if x meets y, u meets v and x and u have the same owner, then either x is before v or u is before y. For example, in our former example, s_{12} meets s_3, s_1 meets s_{23} and as it happens, s_1 is before s_3.

This is not enough to exclude branching at the boundaries of the whole order (if present). We need to impose that any two initial slots are related by parthood, and the same constraint for any two ending slots:

OA 10. $(SO(x, y) \wedge INIT(x) \wedge INIT(y)) \rightarrow (PoS(x, y) \vee PoS(y, x))$

OA 11. $(SO(x, y) \wedge END(x) \wedge END(y)) \rightarrow (PoS(x, y) \vee PoS(y, x))$

In our example, s_1 and s_{12} are initial slots, and indeed, s_1 is a part of s_{12}.

Finally, to guarantee linearity, we need to add the constraint that two convex slots being situated after the same slots and before the same slots are identical.[4] This avoids diamond structures in which x is before both y and z, which are both before t, with no order relation between y and z.

OA 12. $(SO(x, y) \wedge CONV(x) \wedge CONV(y) \wedge \forall z(z < x \leftrightarrow z < y) \wedge \forall z(x < z \leftrightarrow y < z)) \rightarrow x = y$

As discussed above, it makes sense in certain cases to add the optional axiom of atomicity in the mereology of slots, something not considered in [7].

OD 6. $AT(x) \equiv_{def} S(x) \wedge \forall y(PoS(y, x) \rightarrow x = y)$

ATOM $S(x) \rightarrow \exists y\ (PoS(y, x) \wedge AT(y))$

3.4. Validation with a model finder

The mereology of slots (without sum and fusion axioms which generate too many entities to remain tractable) conjoined with the definitions OD1, OD2, OD3, OD5 and the axioms OA1-OA12 was validated with Alloy, a model finder. The model presented on Figure 1 was found (where $<$ is coded 'INF'). Alloy was also used as a heuristic tool to check whether some unwanted models were found and add axioms to exclude those when needed (axioms OA10 and OA11 were added in that respect).

[3]This is akin to Allen & Hayes's linearity axiom M2 in [10], and much simpler than the LIN* axiom proposed in [12].
[4]This is akin to Allen & Hayes's M4 axiom in [10].

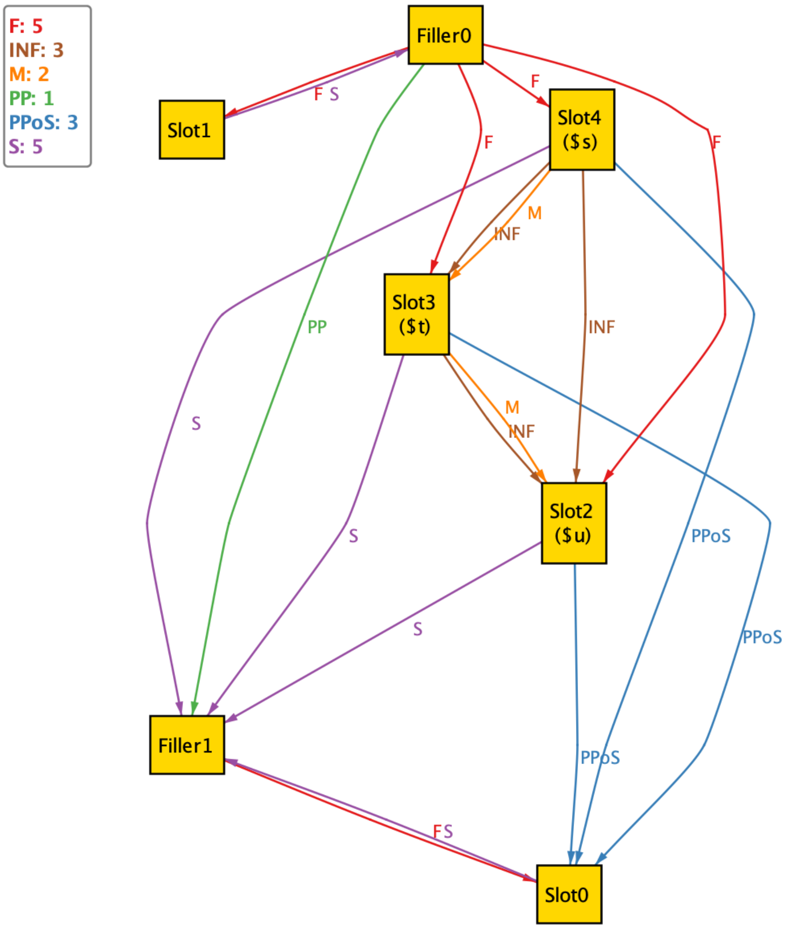

Figure 1: Model of the theory

3.5. The Mereology of Slots entails Van Benthem's axioms on period parthood

Some of Van Benthem's axioms on period parthood were already discussed in Sec.2: in the mereology of slots, the transitivity, reflexivity and anti-symmetry of PoS are theorems (with domain restriction to slots for reflexivity). We'll now show that Van Benthem's axioms FREE, CONJ, DISJ and DIR on period parthood are theorems in the mereology of slots (with suitable restrictions to slots of a same owner where necessary).

FREE. FREE says that if all parts of x overlap y, then x is part of y. This is a standard theorem in classical extensional mereology (see SCT13 in [15]).

OT 1. $\forall z(PoS(z,x) \rightarrow OoS(z,y)) \rightarrow PoS(x,y)$

Proof: suppose $\forall z(PoS(z,x) \rightarrow OoS(z,y))$ and $\neg PoS(x,y)$. The slot strong supplementation axiom A10 with $\neg PoS(x,y)$ implies that $\exists z(PoS(z,x) \wedge \neg OoS(z,y))$ which contradicts the premise.

CONJ. CONJ is the existence of the "intersection" of two overlapping slots—that is, two overlapping slots x and y have a largest common slot-part z.

OT 2. $OoS(x,y) \rightarrow \exists z(PoS(z,x) \wedge PoS(z,y) \wedge \forall u((PoS(u,x) \wedge PoS(u,y)) \rightarrow PoS(u,z)))$

This is standardly obtained in general extensional mereology through the fusion operator applied to the property ϕ of overlapping both these slots. Axiom AS12 in [7] guarantees the existence of the fusion of all common parts of two overlapping slots x and y, that is $FoS_{\phi}(z)$, with $\phi(t) \equiv (PoS(t,x) \wedge PoS(t,y))$. Note that theorem T60 guarantees its unicity. Proof: by definition of fusion DS13, any part of x and y is part of the fusion z.

DISJ. DISJ is the existence of the sum of the slots x and y provided they "underlap" (two slots underlap iff there is a slot of which they are both part).

OT 3. $\exists z(PoS(x,z) \wedge PoS(y,z)) \rightarrow \exists z(PoS(x,z) \wedge PoS(y,z) \wedge \forall u((PoS(x,u) \wedge PoS(y,u)) \rightarrow PoS(z,u)))$

Proof: Axiom A11 asserts the existence of the sum, with the premise that x and y have the same owner. Since PoS implies SO (T21), which by definition D3 and the unicity of the owner A1 is an equivalence relation, from the premise of OT3 we get the premise of A11. Then we can apply Lemma 49 to obtain, considering z as the sum, the first two terms of the conclusion of OT3 (the operands are part of the sum), and Lemma L57 to obtain the last part of the conclusion of OT3 (the sum is part of any slot of which the operands are both part).

DIR. DIR is the assumption that any two slots underlap. Here we need to alter Van Benthem's formula: two slots of a same owner underlap.

OT 4. $SO(x,y) \rightarrow \exists z(PoS(x,z) \wedge PoS(y,z))$

Proof: OT4 is implied by the sum existence axiom A11 and Lemma L49 saying that the operands are part of the sum.

SEP. Finally, Van Benthem's SEP principle, which says that order and overlap are separated, is a theorem:

OT 5. $x < y \rightarrow \neg OoS(x,y)$

Proof: Suppose $x < y$ and $OoS(x,y)$, that is $\exists z(PoS(z,x) \wedge PoS(z,y))$. With axiom OA8, $x < y$ and $PoS(z,y)$ entail $x < z$. Applying now OA7 to $x < z$ and $PoS(z,x)$ we obtain $z < z$ which contradicts irreflexivity axiom OA3.

4. Structural equivalence and identity

We can now come back to the discussion on identity that was raised in the introduction. We'll consider whether, having added order to the mereology of slots, we can characterize the identity of (linear) informational entities. As seen in the introduction, having the same parts, even in the same numbers, is not enough to characterize identity of words as sequences of letters: 'ada' and 'daa' have the same parts, but in a different order.

First, let's note that if two entities have the exact same slots, they have the same improper slot and are thus identical (and two entities cannot have different improper slots that would have the exact same slot-parts, because of slot extensionality).

The issue then regards comparing two fillers without assuming they share the same slots. If the two fillers have the same *structure*, both as far as mereology is concerned (and thus have the same parts) and as far as order is concerned, are they identical or not? Of course, if we conclude they are identical, then, they do have the same slots.

Checking isomorphism between structures cannot be done in FOL unless we assume the domain is finite, but this assumption is not first-order expressible. Such an assumption remains here implicit, but as evoked before, it really makes sense when textual informational entities are concerned. We will adopt a similar approach as for the notion of "mirroring" between an informational entity and a template (an informational entity used to express structural constraints on other informational entities, e.g., a form or a document model) [16, 17], axiomatizing a new primitive of equivalence EQ between slots. By extension, two fillers will be considered equivalent if their improper slots (their unique slot filled by themselves) are equivalent. Here, we will merely characterize very broadly this relation of equivalence with a few axioms that should be completed in future work.

The relation EQ holds between slots. It is a symmetric relation.

OA 13. $EQ(x, y) \rightarrow (S(x) \wedge S(y))$

OA 14. $EQ(x, y) \rightarrow EQ(y, x)$

If two slots are equivalent, then each of their proper parts must be equivalent to a single proper part of the other, and these proper parts have the same fillers:

OA 15. $(EQ(x, y) \wedge PPoS(z, x)) \rightarrow \exists! t(PPoS(t, y) \wedge EQ(z, t))$

OA 16. $(EQ(x, y) \wedge PPoS(z, x) \wedge PPoS(t, y) \wedge EQ(z, t) \wedge F(u, z)) \rightarrow F(u, t)$

If two slots are ordered, their equivalent slots of a same owner are similarly ordered:

OA 17. $(x < y \wedge EQ(x, x') \wedge EQ(y, y') \wedge SO(x', y')) \rightarrow x' < y'$

For example, 'ada' and 'daa' have similar mereological structures (two atomic slots filled by 'a' and one atomic slot filled by 'd') but with different orders and so are not equivalent. On the other hand, if we have two strings w and w' written 'ada' with their respective improper slots s and s', each constituted by the three slots $s_1 < s_2 < s_3$ and $s'_1 < s'_2 < s'_3$, such that $EQ(s_1, s'_1)$, $EQ(s_2, s'_2)$ and $EQ(s_3, s'_3)$ and such that the filler 'a' fills s_1, s_3, s'_1 and s'_3 and the filler 'd' fills s_2 and s'_2, then $EQ(s, s')$ is compatible with OA13-17. Assuming that s and s' are equivalent but different would require to introduce two strings of characters (w and w') which share all characteristics but would be different, which might be considered as an unwanted violation of a principle of identity of indiscernible. So for words as sequences of letters and for texts as sequences of words, it seems appropriate to assume that EQ implies identity, and that $w = w'$ and $s = s'$. However, this is not the case for all linear informational entities.

Consider now the case where two different entities are composed by different slots that have exactly similar slot-structures in terms of slot-parthood, slot-order and fillers. This could be useful to represent a situation where two agents utter the same sentence through different speech-acts, e.g. Mary and John both uttering 'I love pasta'. As it happens, we may have to represent those two utterances as different entities even if they share the same slot structure and filler proper parts. First, because they have different authors (which, for example, is relevant for the identity of informational content entities according to the IAO ontology—note though that other ontologies might have different commitments on this matter). Second, because of indexicals : 'I' refers to different persons in both sentences (respectively Mary and John). Third, because of pragmatic inferences. If Mary is at a buffet when uttering this sentence, she might want to imply that she would love to get now a serving of pasta, whereas if John is discussing food preferences with a friend, it might not lead to the same implicature[5]. In case authoring, indexical and pragmatic considerations would contribute to the identity of a utterance, a standard assumption in linguistics (see, e.g., [19]), this might be a reason to accept two utterances or discourses with equivalent mereological and order slot structures and identical proper part fillers, but different identities. According to this conception, in Borges's famous example, Cervantes and Pierre Menard could author two different novels, both named *Don Quixote* and with exactly the same words and the same structure. So, one may hold that there is a unique text, as a sequence of words, but two different novels, as authored discourses in specific contexts.

[5]See [18] for a proposal of accounting for an illocutionary dimension of informational entities through slots—in that case, accounting for their directive dimension through directive slots.

5. Conclusion

We have proposed an axiomatic system that added order to the formerly developed mereology of slots, concentrating on entities such as words, sentences or texts that are linear.

This work would need to be extended to informational artifacts where branching is possible, such as procedures or flowcharts which can branch to the right (depending on a condition, or because of starting parallel processes) but also to the left (as branches might be merged after developing in parallel). We need to leave this possibility for future work, as it would imply assessing whether or not changes in the mereology of slots are needed. Indeed, one may question the relevance of summing slots belonging to different branches. In addition, future considerations could investigate how order on elements of a template can constrain the order of elements of an informational entity compliant with this template [17].

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Declaration on Generative AI

During the preparation of this work, the authors occasionally used DeepL in order to check the phrasing. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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