Subsumption in the Mirror of Ontological and Logical Choices

Fumiaki Toyoshima[1,*], Satoru Niki[2]

[1] Institut de philosophie, Faculté des lettres et sciences humaines, Université de Neuchâtel, Espace Tilo-Frey 1, 2000 Neuchâtel, Switzerland

[2] Department of Applied Systems and Mathematics, Faculty of Informatics, Kanagawa University, 3-27-1 Rokkakubashi, Kangawa-ku, Yokohama-shi, Kanagawa, 221-8686, Japan

Abstract

Concept subsumption and inclusion are a formal relation of paramount importance in ontology development. This paper aims to shed a new light on these notions from a logical perspective. We begin by clarifying that they are traditionally understood within the framework of classical logic, as exemplified by the description logic *ALC*. After presenting some paraconsistent description logic that is a constructive variant of *ALC*, we investigate the notions of concept subsumption and inclusion in this non-classical logical system by showing their potential use cases in ontologies. This study contributes to highlighting and reinforcing the view that logical choices (i.e. choices as to logical representation languages used in computational modelling such as ontology development) not only support but also shape or even could determine ontological choices (i.e. choices as to ontological commitments involved in computational models such as ontologies).

Keywords

concept subsumption, concept inclusion, ontological choice, logical choice, non-classical logic, paraconsistent description logic

1. Introduction

Subsumption is one of the most important relations used in formal ontologies. This is particularly because it plays a vital role for disambiguating the meaning of the commonly used linguistic expression "is a" [1, 2]. To illustrate this point with Johansson's [3] analysis, the term "is a" figuring in the sentence "The cat is a mammal" refers to "subsumption under a genus", whereas the same term figuring in the sentence "Being scarlet is a kind of being red" refers to "subsumption under a determinable". In contemporary ontological parlance, the former subsumption relation is the *is_a* relation, meaning "is a subtype of" (see e.g. [4, p. 22]) and the latter is the determinable-determinate relation [5].

This paper aims to deepen the understanding of the subsumption relation in the field of formal ontology. Our investigation is motivated by an increasing awareness of the importance of "logical choices" for "ontological choices". A logical choice is a choice about logical representation languages that are employed for the purpose of computational modeling (e.g. ontology development). An ontological choice [6] is a choice about "ontological commitments", that is, which fundamental principle(s) is/are involved in computational models (e.g. ontologies).[1]

It is generally acknowledged that appropriate logical choices can help to articulate ontological choices and to have them well reflected in computational models.[2] For example, OWL 2 DL [11, ch. 14] is widely used for ontology development because it deals effectively with the vexed issue of the trade-off between expressivity and computational complexity of logics. Despite its undecidable nature, however, standard (to

Proceedings of the Joint Ontology Workshops (JOWO) - Episode XI: The Sicilian Summer under the Etna, co-located with the 15th International Conference on Formal Ontology in Information Systems (FOIS 2025), September 8–9, 2025, Catania, Italy

*Corresponding author.
fumiaki.toyoshima@unine.ch (F. Toyoshima); satoru-niki@kanagawa-u.ac.jp (S. Niki)
0000-0002-4690-1221 (F. Toyoshima); 0000-0002-0882-806X (S. Niki)

[1] The term "ontological commitment" originates from philosophy, as it is famously construed by Quine [7]. In this paper we employ this term in accordance with its usage in information science (see e.g. [8]), although we do not necessarily espouse Guarino et al.'s [9] logical view of ontological commitments as "intensional first-order structures".

[2] Cf. Kutz et al.'s [10] argument for "logical pluralism" in ontology design.

wit, e.g. non-modal) first-order logic can be suitable for making sufficiently explicit ontological choices made by ontologies, particularly the ones that are made by and inherent in foundational ontologies (cf. [12]), as is witnessed by the recent development of a method to extend OWL ontologies with first-order logical annotations [13].

In foundational ontology research during the last decade, by contrast, there has been a growing consensus that logical choices not only support but also shape or even could determine ontological choices. To take a few examples, Borgo et al. [14] argue that the usage of other logics (e.g. linear logic) than standard and modal first-order logics in ontologies can have potential merits such as reducing *ad hoc* ontological commitments. Fillottrani & Keet [15] analyze ontological commitments embedded in logics (including OWL 2 DL and first-order logic) to develop an engineering approach to ontology language design. Toyoshima & Niki [16] problematize ontological commitments involved in the semantics of logic, such as the possible-world semantics of modal logic.[3] Sacco et al. [18] compare different non-monotonic logics by focusing on their different ontological commitments. For that matter, logical choices can be crucial for conceptual data modeling as well as for ontology development [19].

We think that this burgeoning line of study in formal ontology can be enriched through the investigation into subsumption. More specifically, closer examination of the formal relation of subsumption from the perspective of non-classical logic will contribute to widening the scope of the research on logical and ontological choices. As we will see below, the notion of subsumption and concept inclusion (as its basis) are traditionally understood within the framework of classical logic as exemplified by the description logic *ALC* and a non-classical logical characterization of subsumption can lead to ontological choices that we would be hardly able to make, or even to identify, in classical logic.

The paper is organized as follows. As a preliminary, Section 2 presents a standard view of subsumption in formal ontology, together with the description logic *ALC*. Section 3 explores the ontological choice as to subsumption and concept inclusion in connection with the logical choice as to whether classical or non-classical logic is adopted. More concretely, we will investigate the notion of subsumption and concept inclusion within Odintsov & Wansing's "paraconsistent description logic" [20, 21] that is a constructive variant of *ALC* based on a logic by Almukdad & Nelson [22], by showing its potential use cases in ontologies.[4] Section 4 discusses the implications of our investigation into ontological and logical choices for other foundational topics such as the identity of ontologies. Section 5 concludes the paper with some brief remarks on future work.

2. Preliminaries

2.1. Subsumption in the OntoClean methodology

We said that subsumption is traditionally understood within the framework of classical logic. To see this, let us consider OntoClean, proposed by Guarino & Welty [26] (see also [2]) and further elaborated by Welty & Andersen [27]. It is a widespread methodology for evaluating and validating ontologies in terms of "formal metaproperties" such as essence, rigidity, identity, unity and dependence. Subsumption is one of the most basic notions in the OntoClean methodology. Guarino & Welty [26] explain subsumption in terms of properties, by which they mean: "the *meanings* or (*intensions*) of expressions like *being an apple* or *being a table*, which correspond to unary predicates in first-order logic" ([26, p. 202]). They characterize subsumption as follows:

> A property p subsumes q if and only if, *for every possible state of affairs*, all instances of q are also instances of p. On the syntactic side, this corresponds to what is usually held for description logics, P subsumes Q if and only if there is no model for $Q \wedge \neg P$. [26, p. 202]

Note that they use the terms "possible state of affairs" and "possible world" synonymously, the latter term being sometimes used in modal logic.

[3] See also Loebe & Herre's [17] study on the interconnection between formal semantics and ontologies.

[4] For more on paraconsistent/constructive description logics, see [23, 24, 25] and the references therein.

The same OntoClean treatment of subsumption is presented by Welty & Andersen [27, p. 108] in the following formal way:

$$subsumes(\psi, \phi) \leftrightarrow \Box\forall x(\phi(x) \rightarrow \psi(x))$$

where: "OntoClean was formalized in S5 modal logic with the Barcan Formula, which gives us a *constant domain* (every object exists in every possible world) and *universal accessibility* (every world is accessible from every other world)" [27, p. 108].[5] Informally, this biimplication says that ψ subsumes ϕ if and only if: it is necessarily the case that, for every x, if x is ϕ, then x is ψ.

We will examine a logical choice that is involved in the OntoClean characterization of subsumption. Since OntoClean is formalized in the modal logical system S5, subsumption in OntoClean *prima facie* appears to be intimately related with modal logic. However, Welty & Andersen [27, p. 108] argue that "one does not need modal logic, nor modal logic reasoning, to use OntoClean in ontology-based systems" and "[m]odal logic was chosen for the formalization [of OntoClean] mainly due to the needs of specifying the semantics of rigidity, in particular anti-rigidity." Therefore, modal logic may not constitute a logical choice that is fundamental to the OntoClean notion of subsumption.

Taking a cue from Guarino & Welty's [26] remark (cited above), we will instead think that the choice of description logic can be relevant to the OntoClean analysis of subsumption. As a matter of fact, OWL 2 DL is broadly employed for ontology development and the description logic $\mathcal{SROIQ}$ [28] can offer a logical foundation of OWL 2 DL. Relatedly, the term "subsumption" used in ontologies may sometimes be interpreted as referring to "concept subsumption" in description logic (e.g. [29]). We remark that our focus on description logic is not incompatible with the relevance of necessity to the notion of subsumption as found in OntoClean.

To be more concrete, we will scrutinize subsumption in the description logic $\mathcal{ALC}$ (see e.g. [11, ch. 2]). This is mainly because $\mathcal{ALC}$ is one of the most basic and simple systems of description logic and many descriptions logics used in ontologies characterize (concept) subsumption in the same way as $\mathcal{ALC}$ does. For example, $\mathcal{SROIQ}$ can be considered as an extension of $\mathcal{ALC}$ that is enriched with e.g. nominal concepts [28].

2.2. The description logic $\mathcal{ALC}$

To specify the language of $\mathcal{ALC}$, let N_C be a non-empty set of *concept names* and N_R be a non-empty set of *role names*. For any arbitrary $A \in N_C$ and $R \in N_R$, the language of $\mathcal{ALC}$ is defined by the following clauses:

$$C ::= A \mid \top \mid \bot \mid (\neg C) \mid (C \sqcap C) \mid (C \sqcup C) \mid \forall R.C \mid \exists R.C.$$

Parentheses will be abbreviated when there is no fear of confusion. Intuitively, C stands for a concept or a set of individuals, A an atomic concept, $\neg C$ the complement of a concept, $C \sqcap C$ (respectively: $C \sqcup C$) the intersection (respectively: union) of two concepts — in particular, the top and bottom concepts $\top$ and $\bot$ can be seen as abbreviations of $C \sqcup \neg C$ and $C \sqcap \neg C$, respectively — and $\forall R.C$ (respectively: $\exists R.C$) the universal (respectively: existential) restriction of a concept by a role. For example, the concept $\mathsf{Person} \sqcap \neg\mathsf{Male} \sqcap (\forall\mathsf{hasAnimal}.\mathsf{Male} \sqcup \exists\mathsf{hasChild}.\top)$ can be read as "those non-male persons either all of whose animals are male or who has at least one child."

As in first-order logic, the semantics for $\mathcal{ALC}$ can be provided through interpretations $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$, where $\Delta^{\mathcal{I}}$ is a non-empty set (the domain of the interpretation $\mathcal{I}$) and the interpretation function $\cdot^{\mathcal{I}}$ maps every atomic concept name $A \in N_C$ to a set $A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$ and every role name $R \in N_R$ to a binary relation $R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. The interpretation function $\cdot^{\mathcal{I}}$ is extended to arbitrary concept descriptions as follows:

[5] More precisely, we can say that the S5 modal logic gives us universal accessibility and the Barcan formula (formally: $\forall\Box x\phi(x) \rightarrow \Box\forall x\phi(x)$) gives us a constant domain.

$\top^I = \Delta^I$ | $\bot^I = \emptyset$ | $(\neg C)^I = \Delta^I \setminus C^I$ | $(C \sqcap D)^I = C^I \cap D^I$ | $(C \sqcup D)^I = C^I \cup D^I$ | $(\forall R.C)^I = \{x \in \Delta^I \mid \forall y.\,(x,y) \in R^I \to y \in C^I\}$ | $(\exists R.C)^I = \{x \in \Delta^I \mid \exists y.\,(x,y) \in R^I \wedge\ y \in C^I\}$.

We can now define concept inclusion and subsumption in *ALC* as follows [20, Definition 1]:

> **Definition 1** (concept inclusion in *ALC*)
> Let C and D be concepts. Let the *concept inclusion* ($C \sqsubseteq D$) be defined as ($\neg C \sqcup D$).
> *C is subsumed by D* in *ALC* (symbolically: $C \vDash_{ALC} D$, or equivalently, $\vDash_{ALC} C \sqsubseteq D$) if and only if: for every interpretation I, $(C \sqsubseteq D)^I = \Delta^I$, i.e. $C^I \subseteq D^I$.

3. Subsumption: An ontological and logical analysis

3.1. Subsumption and inclusion in *ALC* and their classical logical characters

One way to understand the classical logical characteristic of subsumption in *ALC* is to highlight the fact that: "subsumption of concepts $C \vDash D$ is defined using material implication $\neg C \sqcup D$" [20, p. 305] in this logical system. Indeed, the relationship between (concept) subsumption, (material) implication [i.e. concept inclusion] and classical logic has been previously explored in formal ontology. For instance, Borgo et al.'s [14] proposal to use linear logic in ontologies is motivated by the observation that "classical implication $A \to B$ is not apt to model processes, transformations, causal entailments, etc." (ibid., p. 26) and "by representing knowledge via classical logic, we are abstracting away from use, context, quantities of formulas occurring when reasoning" (ibid., p. 27).[6]

For another example, Fillottrani & Keet [15] compare several logical representation languages used in ontologies, including first-order logic and OWL 2 DL, by the criterion which they call "concept subsumption as primitive". According to their analysis, this feature is "possible" in first-order logic in the sense that: "there is a direct and simple reconstruction of the meaning of the feature in the language without altering the essential properties of the language" (ibid., p. 55).

In OWL 2 DL, by contrast, the same feature is "partial" in the sense that "the feature is not completely represented in the language" (ibid.). To borrow their example, consider the "DL axiom $\mathsf{Cat} \sqsubseteq \mathsf{Animal}$" and the counterpart formula $\forall x(\mathsf{Cat}(x) \to \mathsf{Animal}(x))$ translated in first-order logic.[7] They say that, unlike the latter, "the former has the embedded notion of *property inheritance along the taxonomy* where the properties of Animal are inherited by Cat" (ibid., p. 53) but "there are no different symbols for properties of classes [in OWL terms, or concepts in DL terms]" (ibid., p. 55) such as Cat and Animal.

In comparison with these preceding works, we will consider the ontological and logical choices that may remain arguably uninvestigated thus far: the ontological choice as to subsumption in view of the ontology of concepts and the corresponding logical choice that is motivated by the limitations of classical logic.[8] The guiding idea is that, given the nature of concepts, concept subsumption and inclusion in ontologies may sometimes be better represented within a specific kind of non-classical logic, and at the same time, that the logical choice of such a non-classical logic shapes or even could determine the ontological choice as to subsumption.

We will illustrate this idea with the example of gender. Gender is one of the most elemental attributes of people and its formal representation is useful in developing various ontologies — such as biomedical ones, because gender is an important social determinant of health [35]. As the notion of gender may be elusive from an ontological point of view [36], we will make the simple assumption that "man" and "woman" are gender terms in contrast with the sex terms "male" and "female".

Let us first consider the following concept subsumption in *ALC*:

---

[6] To this context, we may add that Sacco et al. [18] do not discuss (material) implication explicitly but their study of non-monotonic logics in ontologies may be partially driven by "the monotonicity of the entailment of classical logic" [14, p. 26].

[7] It is a well-established logical finding that every concept C in *ALC* can be translated into a formula $\phi_x(C)$ with one free variable x. See Baader et al. [11, ch. 2] for details.

[8] It lies outside the purview of this paper to analyze fully the ontological notion of concept. See [30] for a general overview of theories of concepts, [31] for "formal concept analysis" and [32, 33, 34] for works on concepts in formal ontology.

$$\models_{ALC} (\mathsf{Man} \sqcap \neg\mathsf{Man}) \sqsubseteq \mathsf{Woman} \tag{1}$$

Recall Fillottrani & Keet's [15] analysis that the "DL axiom $\mathsf{Cat} \sqsubseteq \mathsf{Animal}$" has the "embedded notion of property inheritance along the taxonomy". We may interpret subsumption (1) as the case in which the properties of Woman are *not* inherited by ($\mathsf{Man} \sqcap \neg\mathsf{Man}$) owing to lack of the (appropriate) taxonomy involving these two concepts.[9] One way to articulate this statement may be to appeal to the methodology for ontology development that is called "ontological realism": roughly, the view that (scientific) ontologies should represent, at least primarily, the actual entities (notably "universals") as described by science ([37]; see Section 4.3 for details). Assuming the realist methodology, one may insist that, although (1) is valid in *ALC*, it should not be taken with ontological seriousness because in no way does $\mathsf{Man} \sqcap \neg\mathsf{Man}$ correspond to any universals in reality.

However, this realism-based understanding of *ALC* may underestimate the significance of logical choices for ontological choices. For one thing, as *ALC* can be seen as a decidable fragment of first-order logic [11, ch. 3], a "realist understanding of first-order logic" [38, 39] could fail to enjoy some merits (e.g. reducing *ad hoc* ontological commitments) that the usage of other logical systems would be able to (cf. [14]).[10] For another, as we will explain below, it is possible to design a description logic that does not allow such problematic subsumptions as (1).

We will turn to another example, this time of concept inclusion in *ALC*:

$$\mathsf{Man} \sqsubseteq \mathsf{Male} \tag{2}$$

$$\mathsf{Man} \not\sqsubseteq \mathsf{Male} \tag{3}$$

Which property should be true in the interpretation that reflects an adequate understanding of the concepts, (2) or (3)? On the one hand, (2) may be supported by the pre-modern view of gender as being identical to sex.[11] On the other hand, the presently prevailing view of gender as being distinct from sex leads naturally to adopting (3). The point is that (2) may have been (considered as) true eternally or atemporally (cf. Borgo et al.'s [14] analysis of classical logic) — or, according to our preferred informational interpretation (cf. [20]), at a certain *informational state* (to be elucidated below) — but it turns out to be (considered as) true at a "richer" informational state[12], hence the truth of (3); and that this may be attributed to the evolution of the concept Man (or both the concepts Man and Male).[13]

More generally, we can think that concept inclusion in *ALC* holds at a certain informational state, but it may no longer hold at another informational state during the evolution of the concept(s) under consideration. In that respect, the *ALC* conception of concept inclusion may not be well-suited for accommodating concept evolution in the context of ontology development. This observation can motivate seeking a non-classical logical framework for ontologies in which alternative notions of concept inclusion and subsumption can hold.

---

[9] (1) can be seen as an example of the principle of *ex falso quodlibet* or *ex contradictione quodlibet* ("anything follows from contradiction") holding in classical logic — which represents one feature of "radical exclusion" criticized by Plumwood [42] from the perspective of "feminist logic".

[10] See also Kless & Jansen's [40] argument that "OWL is not a perfect language for describing realist ontologies" because there is "no one-to-one mapping between syntactical categories in OWL and the elements of realist ontologies" (ibid., p. 1863).

[11] The pre-modern view of gender may be found in anthropology, for example, as Chui et al. [41] state that "[w]ithin kinship systems in anthropology, a strict binary gender system of male and female is adopted", while their ontology for kinship "does not have any inherent bias towards any gender ontology" (ibid., p. 99).

[12] Scheele [24, p. 3] describes similar examples to motivate constructivity in description logic.

[13] By the term "evolution of concepts" or "concept evolution", we mean roughly that the intension of a given concept changes or can change over time. See [43, 44] for thoughts on concept evolution. Also relatedly, [21] explains informational states as being *states of a data base*. When compared to an explicitly conceptual interpretation, this seemingly more syntactic interpretation enables one to eschew a commitment about questions concerning intensions.

3.2. Subsumption and inclusion in the paraconsistent description logic $CALC^C$

We examined the ontological choice as to subsumption and concept inclusion in terms of the logical choice of the description logic ALC, or more specifically, with the illustrative examples of ALC subsumptions (1) and (2)/(3). To further analyze these notions, we will turn our attention to the constructive and paraconsistent description logic $CALC^C$ [20, 21].[14]

Let us present the informal idea behind $CALC^C$ before specifying its syntax and semantics. The key idea is that a given element of Δ^I may be seen as an individual *simpliciter* in the semantics for ALC, whereas it will be interpreted as an individual *at an informational state* in the semantics for $CALC^C$. We can also think that, for given $x, y \in \Delta^I$ in the semantics for $CALC^C$, y is *informationally accessible* from x if (i) y is identical to x (in the sense that they represent the same individual at different informational states) and (ii) y is at an informational state that is a "possible expansion" of the informational state of x. Moreover, these new notions enable us to *restrict* concept inclusion in ALC (namely: $C \sqsubseteq D$) to informationally accessible individuals — in such a semi-formal way that, for every I, every $x \in \Delta^I$ and every $y \in \Delta^I$ that is informationally accessible from x, $y \in D$ if $y \in C$.[15]

Suppose for the sake of illustration that the non-male person Sam was not (known as) a man in the pre-modern period but is presently (known as) a man. Let s_1 refer to Sam in the pre-modern period and s_2 refer to Sam in the present. We can think that s_2 is informationally accessible from s_1, as s_2 is identical to s_1 and the informational state (involving e.g. being a male) of s_2 lies within a possible expansion of the informational state of s_1. However, it is not the case that $s_2 \in \mathsf{Male}$ if $s_2 \in \mathsf{Man}$. Therefore, $\mathsf{Man} \sqsubseteq \mathsf{Male}$ does not hold in an interpretation representing such a state of affairs in the semantics for $CALC^C$.

For any arbitrary $A \in N_C$ and $R \in N_R$, the language of $CALC^C$ is defined by the following clauses:

$$C ::= A \mid \top \mid \bot \mid (\sim C) \mid (C \sqcap C) \mid (C \sqcup C) \mid (C \sqsubseteq C) \mid \forall R.C \mid \exists R.C.$$

Note that, in ALC, classical negation ("$\neg$") is used and concept inclusion ("$\sqsubseteq$") is defined in terms of material implication (i.e. $(C \sqsubseteq D) \triangleq (\neg C \sqcup D)$); whereas in $CALC^C$, constructive negation ("$\sim$") is used instead of classical one and concept subsumption is treated as a *primitive* (i.e. undefined) constructor.

The semantics for $CALC^C$ can be provided through interpretations $I = \langle \Delta^I, \preccurlyeq, \cdot^I \rangle$, where Δ^I is a non-empty set, $\preccurlyeq \subseteq \Delta^I \times \Delta^I$ is a reflexive and transitive relation ("informational accessibility"[16]) and the interpretation function $\cdot^I$ maps every atomic concept name $A \in N_C$ to a set $A^I \subseteq \Delta^I$, every "atomic constructive negation" $\sim A$ with $A \in N_C$ to a set $\sim A^I \subseteq \Delta^I$ and every role name $R \in N_R$ to a binary relation $R^I \subseteq \Delta^I \times \Delta^I$. The interpretation function $\cdot^I$ in this semantics is extended to arbitrary concept descriptions exactly as in the semantics for ALC, except for the following constructors:

- $(\sim (C \sqcap D))^I = \sim C^I \cup \sim D^I$
- $(\sim (C \sqcup D))^I = \sim C^I \cap \sim D^I$
- $(C \sqsubseteq D)^I = \{x \in \Delta^I \mid \forall y.\ x \preccurlyeq y \to (y \in C^I \to y \in D^I)\}$
- $(\sim (C \sqsubseteq D))^I = C^I \cap \sim D^I$
- $(\sim (\forall R.C))^I = \{x \in \Delta^I \mid \exists y.\ (x, y) \in R^I \land y \in \sim C^I\}$
- $(\sim (\exists R.C))^I = \{x \in \Delta^I \mid \forall y.\ (x, y) \in R^I \to y \in \sim C^I\}$

[14] Previous studies [10, 45] on the usage of paraconsistent logic in ontologies focus primarily on the capacity of paraconsistent reasoning to deal with inconsistent data, as well as Belnap's [46] four-valued logic. In contrast, we will consider the formal relation of subsumption in a different kind of paraconsistent logic. It is also interesting to note that Odintsov & Wansing [20] state: "Belnap's four-valued logic has no genuine implication satisfying the Deduction Theorem" (ibid., p. 301, fn. 1).

[15] The ontological import of this restriction on concept inclusion is relatively underemphasized in Odintsov & Wansing's [20] original presentation. While the authors explicitly discuss the stricter nature of subsumption in their Example 2 (ibid., p. 306), they do not sufficiently clarify why such restriction would be desirable. Furthermore, their motivating examples are concerned mainly with handling inconsistent/incomplete information, but they in fact do not rely on the non-classical notion of concept inclusion; we can find interpretations for their Examples 5 and 6 (ibid., pp. 309-310) wherein the preorder is an identity — i.e. the notion is classical.

[16] Since the contrast between atemporal (static) and temporal (dynamic) resulting from the accessibility relation can be reinterpreted as the one between extensional and intensional notions within possible world semantics, our enquiry might be viewed as that of the relationship between subsumption and intension as well.

The clauses for the constructive negation may appear unusual in that they are defined negandwise: this is however the standard way to treat the type of negation. We also make the following conditions on the binary relation $\preccurlyeq$:

- For every $x, y \in \Delta^I$, $A \in N_C$: if $x \preccurlyeq y$ and $x \in A^I$, then $y \in A^I$.
- For every $x, y \in \Delta^I$, $A \in N_C$: if $x \preccurlyeq y$ and $x \in\sim A^I$, then $y \in\sim A^I$.
- For every $R \in N_R$: $\preccurlyeq \circ R^I \subseteq R^I \circ \preccurlyeq$.
- For every $R \in N_R$: $\preccurlyeq^{-1} \circ R^I \subseteq R^I \circ \preccurlyeq^{-1}$.

where $R \circ S = \{(x,z) : \exists y(xRy \text{ and } yRz)\}$ and $xR^{-1}y$ iff yRx. In particular, the last two conditions are used to prove the "persistence of concepts":

Proposition 1 (persistence of concepts)
Let $I = \langle \Delta^I, \preccurlyeq, \cdot^I \rangle$ be an interpretation. For every concept C and $x, y \in \Delta^I$:
if $x \preccurlyeq y$ and $x \in C^I$, then $y \in C^I$.
Proof. Omitted (see [20, p. 309]).

Note that this proposition is plausible, given our informational construal of $CALC^C$. To illustrate it with the example of Sam, $s_2 \in \mathsf{Male}$ follows from $s_1 \preccurlyeq s_2$ and $s_1 \in \mathsf{Male}$.[17]

We can now introduce the notions of validity and concept subsumption in the semantics for $CALC^C$:

Definition 2 (validity in $CALC^C$)
A concept C is *$CALC^C$-valid* (symbolically: $\models_{CALC^C} C$) if and only if:
for every interpretation $I = \langle \Delta^I, \preccurlyeq, \cdot^I \rangle$, $C^I = \Delta^I$.

Definition 3 (concept subsumption in $CALC^C$)
A concept C *is subsumed by* a concept D (symbolically: $C \models_{CALC^C} D$) if and only if:
$\models_{CALC^C} C \sqsubseteq D$.

In light of the primitive constructor $\sqsubseteq$ and Definition 3, we can think that Fillottrani & Keet's [15] feature "concept subsumption as primitive" would be "yes" for $CALC^C$, as distinct from "possible" for first-order logic and "partial" for ALC (see Section 3.1).

More concretely, let us recall the subsumption (1) in ALC. We can prove that its straightforwardly translated counterpart in the language of $CALC^C$, namely $(\mathsf{Man} \sqcap \sim \mathsf{Man}) \sqsubseteq \mathsf{Woman}$, does *not* hold in $CALC^C$ (cf. [20, Example 5] for an analogous example). That is:

$$\not\models_{CALC^C} (\mathsf{Man} \sqcap \sim \mathsf{Man}) \sqsubseteq \mathsf{Woman} \tag{4}$$

To show the invalidity of (4), it will suffice to use an interpretation where $\mathsf{Man}^I \cap \sim \mathsf{Man}^I \neq \emptyset$ but $\mathsf{Woman}^I = \emptyset$.

Let us also recall the question of which property is true, (2) or (3). We can ask the parallel question of which of the following should hold in an intended interpretation of $CALC^C$:

$$\mathsf{Man} \sqsubseteq \mathsf{Male} \tag{5}$$

$$\mathsf{Man} \not\sqsubseteq \mathsf{Male} \tag{6}$$

This question as to (5) and (6) comes down to the question of whether, for such an interpretation $I = \langle \Delta^I, \preccurlyeq, \cdot^I \rangle$, $(\mathsf{Man} \sqsubseteq \mathsf{Male})^I = \Delta^I$ or not, i.e. $\{x \in \Delta^I \mid \forall y.\ x \preccurlyeq y \rightarrow (y \in \mathsf{Man}^I \rightarrow y \in \mathsf{Male}^I)\} = \Delta^I$ or not. This additional consideration of the aspect of informational expansion stands in striking contrast

[17] At the same time, it is an idealization that is perhaps not applicable in circumstances where known information may be lost or revised. One way to reject persistence is to go *subintuitionistic*; see [47, 48] for more details.

with the preceding question as to (2) and (3) in $\mathcal{ALC}$ — that is, whether, for every interpretation $\mathcal{I}$, $(\mathsf{Man} \sqsubseteq \mathsf{Male})^{\mathcal{I}} = \Delta^{\mathcal{I}}$ or not, i.e. $\mathsf{Man}^{\mathcal{I}} \subseteq \mathsf{Male}^{\mathcal{I}}$ or not, disregarding completely a prospect e.g. of a future change in conceptualization, which however may well happen given how our understanding of the concepts have been evolving.

We submit that an ontological analysis of the logical difference between (1) (2)/(3) in $\mathcal{ALC}$ and (4) and (5)/(6) in $CALC^C$, respectively, can lend plausibility to the statement that logical choices shape or even could determine ontological choices. Being formalized in $CALC^C$ — unlike in $\mathcal{ALC}$ —, concepts are in nature relative to the informational accessibility between individuals and hence to the informational states of individuals. Consequently, concept subsumption and inclusion in $CALC^C$ may be interpreted as "dynamic" (rather than "static" as in $\mathcal{ALC}$) and the evolutionary dimension of concepts (e.g. gender) may be better accommodated in $CALC^C$ than in $\mathcal{ALC}$. The logical choice between $\mathcal{ALC}$ and $CALC^C$ can be a deciding factor in the ontological choice as to subsumption.

4. Discussion

4.1. The identity of ontologies

The identity of ontologies is a thorny issue in foundational ontology research, or especially in the discussion on the definition of the term "ontology".[18] Consider for instance the upper ontology "a Descriptive Ontology for Linguistic and Cognitive Engineering" (DOLCE) [6, 50]. Is a modal first-order logical formalization of DOLCE [51] the same as (to wit, numerically identical to) an OWL 2 formalization thereof [52]? Some people would say yes, as "the same ontology may be maintained in different languages (e.g., OWL and [first-order logic])" [49, p. 4] and it may also be realized in different "ontology versions" [49] formulated in these different logics. Others would say no, as "even minor changes in the formalisation may result in significant semantic discrepancies and consequently in negotiation failure" and "each ontology is expressed exactly in one language" [53, p. 6].

Although we did not directly address the topic of the identity of ontologies, our study in this paper may provide one consideration to favor the negative answer to this identity question. This is because we have been emphasizing that logical choices not only support but also shape or even could determine ontological choices, and different ontologies can be individuated, at least partially, according to their different ontological choices.[19] For example, $\mathcal{ALC}$ and $CALC^C$ are both variants of description logic, as their languages are very similar. The choice between these logics can nonetheless yield ontologically different conceptions of concept subsumption and inclusion and hence different ontologies involving them.

One may wonder what "the upper *ontology* DOLCE" amounts to, if modal first-order logical and OWL 2 versions thereof are different ontologies. Although answering this question fully goes beyond the scope of this paper, we may hypothesize that the term "DOLCE" may sometimes be taken to refer (albeit loosely) to an initial, core set (say S_{DOLCE}) of ontological choices.[20] A modal first-order logical ontology of DOLCE involves S_{DOLCE} and other ontological choices, including the ones (say c_{MFOL}) shaped by the logical choice of modal first-order logic. Being devoid of c_{MFOL}, by contrast, an OWL 2 ontology thereof involves S_{DOLCE} and new ontological choices, including the ones shaped by the logical choice of OWL 2. These two different ontologies can be grouped under the heading of "DOLCE" because they commonly involve S_{DOLCE}.

[18] See [29, ch. 1.2.1] and [49] for a quick overview of existing definitions of the term "ontology".

[19] See Guarino et al.'s [9] characterization of ontologies in terms of ontological commitments (see Footnote 2 for the notion of ontological commitment); although we do not necessarily agree with their view that ontologies *are* a kind of "logical theories" (cf. Section 4.2).

[20] See [6] for examples of such core ontological choices of DOLCE.

4.2. Meaning in ontologies

The notion of meaning is of vital importance for ontologies, as Guarino et al. [9, p. 11] consider an ontology "as a logical theory designed to account for the intended meaning of the vocabulary used by a logical language." Given our key statement that logical choices not only support but also shape or even could determine ontological choices, it may be reasonable to think that the notion of meaning in an ontology can be substantially conditional on which logical representation language is used in that ontology.

To explore this issue, we will examine Barton et al.'s [54] study of meaning holism and the related phenomenon of indeterminacy of reference in ontologies. Meaning holism roughly says that the meanings of all the terms in a language are so interconnected with one another that a change in the meaning of any single term can bring about changing the meanings of the other terms. They investigate ways of limiting meaning holism in ontologies because it could severely hinder the development of a set of semantically interoperable ontologies.

Barton et al. articulate two ways of limiting meaning holism in ontologies. They take the second way — "(MEAN_2)" in their terms — to be a more effectively restricted theory of meaning in ontologies. The underlying idea behind (MEAN_2) is that the meaning of a class term is determined by "the general analytic claims concerning it — claims that apply to any instances of that class" [54, p. 249] and it is thus determined by its *necessary* conditions (including its necessary and sufficient conditions) but not by its sufficient conditions. They formulate (MEAN_2) and its operationalized version in OWL — "$(\text{MEAN}_2)^{\text{OWL}}$" in their terms — as follows (ibid., p. 250, with some notational modifications for readability):

> (MEAN_2) The formal meanings$_2$ of a class term in an ontology O is the collection of non-tautological axioms expressing necessary conditions (including necessary and sufficient conditions) on this term within the deductive closure of O's analytic theory.[21]

> $(\text{MEAN}_2)^{\text{OWL}}$ The formal meaning of a class term A in an OWL ontology O is the collection of non-tautological axioms of the form 'A SubClassOf $Expr$' and 'A EquivalentTo $Expr$' (where $Expr$ is a named or anonymous class) within the deductive closure of O's analytic theory.

We will focus on the notion of "non-tautological axiom" figuring in (MEAN_2) and $(\text{MEAN}_2)^{\text{OWL}}$. According to Barton et al.'s definition, non-tautological axioms are "axioms that are equivalent to a tautology in the deductive closure of the ontology" [54, p. 250]. Here it is important to point out the general logical fact that which axiom counts as a tautology can vary according to which logical system is adopted. For instance, they say: "adding tautologies such as [...] 'A SubClassOf (B or not-B)' [...] should not change the meaning of 'A' " (ibid., italicized for readability). Certainly, the axiom $A \sqsubseteq (B \sqcup \neg B)$ holds in ALC, as it corresponds to the OWL axiom 'A SubClassOf (B or not-B)'. However, the counterpart axiom $A \sqsubseteq (B \sqcup \sim B)$ does not hold in $CALC^C$ and it *is* a non-tautological axiom in this non-classical logic.

Moreover, (MEAN_2) is motivated by the idea that the meaning of a class term is determined by "claims that apply to any instances of that class". This idea may apply straightforwardly to the SubClassOf relation in OWL, as concept subsumption in ALC is "unrestricted" in the sense of being characterized by $C^I \subseteq D^I$ with respect to every interpretation I. In contrast, concept subsumption in $CALC^C$ is restricted, as it needs to take into account informationally accessible individuals. There may be room for interpretation as to whether the basal idea of (MEAN_2) can be *mutatis mutandis* applied to meanings in ontologies formalized in $CALC^C$ or not. In addition, $CALC^C$ offers multiple notions of equivalence [20], and a choice among them could determine the meaning of a class term.

[21] We make two clarificatory remarks. Firstly, the term "meaning$_2$" refers to a meaning defined in (MEAN_2). Secondly, as for the term "theory", Barton et al. define the term "formal theory of an ontology" as the "collection of formal statements explicitly formulated within this ontology" [54, p. 245].

All these discussions can show that, despite its purported generality as compared to $(\text{MEAN}_2)^{\text{OWL}}$, (MEAN_2) may be significantly shaped by the focus on OWL or, in our terms, by the logical choice of OWL. Indeed, Barton et al. [54, p. 252] state: "The analysis presented here should be operationalized in ontologies written in other languages than OWL, such as [first-order logic] or CLIF [i.e. Common Logic Interchange Format]." In light of our theorizing upon subsumption as well as ontological and logical choices, it would be nevertheless suggested that the foundational topic of meaning in ontologies should be investigated in a non-classical logic, in addition to practically used classical logics such as OWL, first-order logic and CLIF.

4.3. The fundamental methodology for ontology development

There is a long-standing controversy over the fundamental methodology for ontology development, especially between (ontological) realism and conceptualism (see e.g. [37, 55, 56]). On the one hand, the "fundamental principle of ontological realism" is "to view ontologies as representations of the reality that is described by science" [37, p. 139, de-italicized for readability] (cf. [4, ch. 3]). On the other hand, conceptualism is typically construed as implying that ontologies are, in some way, inextricably linked with the mental act of "conceptualization" (see e.g. [49]).[22]

However, it may not always be clear what the methodological theses of realism and conceptualism are like. For instance, realism is often combined with the view that ontologies should represent (at least primarily) universals, which can be denoted by general terms used in science (see e.g. [40]). But this view has been criticized from theoretical and practical viewpoints ([55, 56]; but see [37]) and realism may be characterized without reference to universals (see e.g. [49]). For another example, DOLCE is sometimes described as conceptualist, as "it looks at reality from the mesoscopic and conceptual level" [6, p. 280]. Nonetheless, its core modular part (as illustrated by "DOLCE-CORE" [6]) may be agnostic as to whether realism or conceptualism is adopted.

It will be an interesting line of research to explore the ontological choices of realism and conceptualism in terms of logical choices in order to further elucidate these methodological theses. Here we will briefly consider the relationship between realism and the logical choice of $\mathcal{ALC}$, as well as between realism and the logical choice of $CALC^C$. Concerning realism and $\mathcal{ALC}$, we argued that the valid $\mathcal{ALC}$ subsumption (1) might have a potentially undesirable ontological import for realism, which could be avoided but at the risk of the realist's making some *ad hoc* ontological commitment (see Section 3.1). This can suggest that the logical choice of $\mathcal{ALC}$ may not align with the ontological choice of realism.

Concerning realism and $CALC^C$, by contrast, there are at least two interpretations of their compatibility depending on how the notion of "individual at an information state" is understood within the ontological framework of realism. One interpretation is that the ontological commitments involved in the semantics of $CALC^C$ are incompatible with realism because informational states are conceptualizations of reality and they can be represented in conceptualist — but not realist — ontologies.

The other interpretation is that individuals at an informational state would be congruent with realism because they can be seen as part of reality. This construal may be justified on the grounds that the notion of individual at an informational state (e.g. Sam in the pre-modern period and Sam in the present) may be understood as "informationally qualified continuant" by analogy with the existing notion of "temporally qualified continuant" [57, 58] (e.g. caterpillars and butterflies). Further investigation into this hypothesis left for future work.

5. Conclusion

We provided an ontological and logical analysis of the arguably underexplored notions of concept subsumption and inclusion. More specifically, we showed that these notions are traditionally understood within the framework of classical logic, as exemplified by the description logic $\mathcal{ALC}$ and that it can receive a considerably different ontological interpretation in a non-classical logical system such as the

[22] See [9] for a formal analysis of the notion of conceptualization.

paraconsistent description logic $CALC^C$. This study is motivated by, and contributes to highlighting and strengthening, the view that logical choices not only support but also shape or even could determine ontological choices.

There are several future directions of inquiry in which to further pursue the present work. Regarding specific directions, it will be valuable to extend $CALC^C$ to paraconsistent description logics the expressivity of which would be comparable to that of $SROIQ$ and to investigate their ontological and logical properties (e.g. decidability), e.g. by building upon a sound and complete tableau calculus for $CALC^C$ (presented in [20, 21]). Regarding general directions, a systematic analysis of ontological and logical choices will require careful scrutiny of multifarious logical systems, notably non-classical logical ones — part of which our endeavour in this paper intends to be. It will also be worthwhile to connect such a systematic analysis with the notion of "ontological unpacking" [8], i.e. an explanation of symbolic domain descriptions by revealing their ontological commitments.

Acknowledgments

We thank Nicola Guarino, Daniele Porello and Heinrich Wansing (in alphabetic order) for their helpful comments on early drafts of the paper. We also thank anonymous reviewers for their valuable reviews on this paper and its early versions. FT is financially supported by the Swiss National Science Foundation (SNSF). FT's participation in the relevant workshop is made possible by funding from the Swiss Academy of Humanities and Social Sciences (SAHSS). This research has been conducted in cooperation with the Swiss Center for Ontological Research (SCOR).

Declaration on Generative AI

The authors have not employed any Generative AI tools.

References

[1] Guarino N. The role of identity conditions in ontology design. In: Proceedings of COSIT 1999. LNCS vol. 1661. Berlin, Heidelberg: Springer; 1999. p. 221-34

[2] Guarino N, Welty C. Identity and subsumption. In: Green R, Bean CA, Myaeng SH, editors. The semantics of relationships. Information science and knowledge management, vol 3. 2002; Springer, Dordrecht. p. 111-26.

[3] Johansson I. Four kinds of is_a relation. In: Munn K, Smith B, editors. Applied ontology: an introduction. Frankfurt: Ontos Verlag; 2008. p. 235-53.

[4] Arp R, Smith B, Spear AD. Building ontologies with Basic Formal Ontology. MIT Press; 2015.

[5] Wilson J. Determinables and determinates. In: Zalta EN, Nodelman U, editors. The Stanford encyclopedia of philosophy. Spring 2023 ed. Available from: https://plato.stanford.edu/archives/spr2023/entries/determinate-determinables/.

[6] Borgo S, Masolo C. Ontological foundations of DOLCE. In: Poli R, Healy M, Kameas A, editors. Theory and applications of ontology: computer applications. Springer; 2010. p. 279-95.

[7] Quine WV. On what there is. Rev. Metaphys. 1948 Sept;2(1):21-38. Reprinted in: Quine WV. From a logical point of view. New York: Harper; 1953. p. 1-19.

[8] Guizzardi G, Guarino N. Explanation, semantics, and ontology. Data Knowl. Eng. 2024 Sep;153:102325.

[9] Guarino N, Oberle D, Staab S. What is an ontology? In: Staab S, Studer R, editors. Handbook on ontologies. Springer; 2009. p. 1-17.

[10] Kutz O, Mossakowski T, Lücke D. Carnap, Goguen, and the hyperontologies: logical pluralism and heterogeneous structuring in ontology design. Log. Univers. 2010 Nov;4(2):255-333.

[11] Baader F, Calvanese D, McGuinness DL, Nardi D, Patel-Schneider PF, editors. The description logic handbook: theory, implementation and applications. 2nd ed. Cambridge University Press; 2007.

[12] Borgo S, Galton A, Kutz O. Foundational ontologies in action: understanding foundational ontology through examples. Appl. Ontol. 2022 Mar;17(1):1-16.

[13] Flügel S, Glauer M, Neuhaus F, Hastings J. When one logic is not enough: integrating first-order annotations in OWL ontologies. Semant. Web. 2024 Mar; pre-press. doi:10.3233/SW-243440.

[14] Borgo S, Porello D, Troquard N. Logical operators for ontological modeling. In: Proceedings of FOIS 2014. Amsterdam: IOS Press; 2014. p. 23-36.

[15] Fillottrani PR, Keet CM. An analysis of commitments in ontology language design. In: Proceedings of FOIS 2020. Amsterdam: IOS Press; 2020. p. 46-60.

[16] Toyoshima F, Niki S. Semantics matters: a new light on ontological commitments of logics. In: Proceedings of JOWO 2021. CEUR Workshop Proceedings, vol. 2969; 2021. p. 1-16.

[17] F. Loebe, H. Herre. Formal semantics and ontologies - towards an ontological account of formal semantics. In: Proceedings of FOIS 2008. Amsterdam: IOS Press; 2008. p. 49-62.

[18] Sacco G, Bozzato L, Kutz O. Know your exceptions: towards an ontology of exceptions in knowledge representation. In: Proceedings of FOIS 2024. Amsterdam: IOS Press; 2024. p. 254-67.

[19] Fillottrani PR, Keet CM. Logics for conceptual data modelling: a review. Transactions on Graph Data and Knowledge (TGDK). 2024 Feb;2(1): 4:1-4:30.

[20] Odintsov SP, Wansing H. Inconsistency-tolerant description logic: motivation and basic systems. In: Hendricks VF, Malinowski J, editors. Trends in logic, vol. 21. Springer, Dordrecht; 2003. p. 301-35.

[21] Odintsov SP, Wansing H. Inconsistency-tolerant description logic. Part II: a tableau algorithm for $CALC^C$. J. Appl. Log. 2008 Sep:6(3):343-60.

[22] Almukdad A, Nelson D. Constructible falsity and inexact predicates. J. Symb. Log. 1984 Mar;49(1):231-33.

[23] de Paiva V. Constructive description logics: what, why and how. In: Context, representation and reasoning. Riva del Garda; 2006.

[24] Scheele S. Model and proof theory of constructive ALC: constructive description logics. University of Bamberg Press; 2015.

[25] Kamide N. An extended description logic for inconsistency-tolerant ontological reasoning with sequential information. In: Proceedings of ICAART 2020, vol.2. Science and Technology Publications; 2022. p. 313-21.

[26] Guarino N, Welty CA. An overview of OntoClean. In: Staab S, Studer R, editors. Handbook on ontologies. International Handbooks on Information Systems. Springer, Berlin, Heidelberg; 2004. p. 151-71.

[27] Welty C, Andersen W. Towards OntoClean 2.0: a framework for rigidity. Appl. Ontol. 2005 Nov;1(1):107-16.

[28] Horrocks I, Kutz O, Sattler U. The even more irresistible SROIQ. In: Proceedings of KR2006. AAAI Press; 2006. p. 452-57.

[29] Keet CM. An introduction to ontology engineering. v 1.5; 2020. Available from: https://people.cs.uct.ac.za/ mkeet/OEbook/.

[30] Margolis E, Laurence S, editors. Concepts: core readings. MIT Press; 1999.

[31] Ganter B, Wille R. Formal concept analysis: mathematical foundations. 2nd ed. Springer Cham; 2024.

[32] Giunchiglia F, Fumagalli M. Concepts as (recognition) abilities. In: Proceedings of FOIS 2016. Amsterdam: IOS Press; 2016. p. 153-166.

[33] Masolo C, Porello D. Representing concepts by weighted formulas. In: Proceedings of FOIS 2018. Amsterdam: IOS Press; 2018. p. 55-68.

[34] Fumagalli M, Ferrario R. Representation of concepts in AI: towards a teleological explanation. In: Proceedings of JOWO 2019. CEUR Workshop Proceedings, vol. 2518; 2021. p. 1-12.

[35] Toyoshima F, Barton A, Fabry P, Ethier JF. Representing gender in ontologies: a dispositional perspective. In: Proceedings of ICBO 2021. CEUR Workshop Proceedings, vol. 3073; 2019. p.

1-12.
[36] Rosario E. Sex and gender. Koslicki K, Raven MJ, editors. The Routledge handbook of essence in philosophy. New York, NY: Routledge; 2024. p. 375-87.
[37] Smith B, Ceusters W. Ontological realism: a methodology for coordinated evolution of scientific ontologies. Appl. Ontol. 2010 Nov;5(3-4):139-88.
[38] Smith B. Against fantology. In: Marek JC, Reicher ME, editors. Experience and analysis. Vienna: HPT&ÖBV; 2005. p. 153-70.
[39] Smith B. The benefits of realism: a realist logic with applications. In: Munn K, Smith B, editors. Applied ontology: an introduction. Frankfurt: Ontos Verlag; 2008. p. 109-24.
[40] Kless D, Jansen L. How fit is OWL to represent realist ontologies? The semantics of representational units in realist ontologies and the Web Ontology Language. In: Proceedings of INFORMATIK 2013. Gesellschaft für Informatik; 2013. p. 1851-65.
[41] Chui C, Grüninger M, Wong J. An ontology for formal models of kinship. In: Proceedings of FOIS 2020. Amsterdam: IOS Press; 2020. p. 92-106.
[42] Plumwood V. The politics of reason: towards a feminist logic. Australas. J. Philos. 1993 June;71(4):436-62.
[43] Masolo C, Sanfilippo EM, Lamé M, Pittet P. Modeling concept drift for historical research in the digital humanities. In: Proceedings of JOWO 2019. CEUR Workshop Proceedings, vol. 2518; 2019. p. 1-12.
[44] Righetti G. Concepts as variable embodiments. In: Proceedings of FOIS 2025. Amsterdam: IOS Press; 2025. To Appear.
[45] Ma Y, Hitzler P. Paraconsistent reasoning for OWL 2. In: Proceedings of RR 2009. LNCS, vol. 5837. Springer, Berlin, Heidelberg; 2009. p. 197-211.
[46] Belnap ND. A useful four-valued logic. In: Dunn J, Epstein G, editors. Modern uses of multiple-valued logics. Reidel, Dordrecht; 1977. p. 8-37.
[47] Corsi G. Weak logics with strict implication. Zeitschr. f. math. Log und Grundlagen d. Math. 1987;(33)5:389-406.
[48] Restall G. Subintuitionistic logics. Notre Dame J. Form. Log. 1994 Dec;35(1):116-29.
[49] Neuhaus F. What is an ontology? 2018. arXiv:1810.09171v1 [cs.AI]
[50] Borgo S, Ferrario R, Gangemi A, Guarino N, Masolo C, Porello D, Sanfilippo E, Vieu L. DOLCE: a descriptive ontology for linguistic and cognitive engineering. Appl. Ontol. 2022 Mar;17(1):45-69.
[51] Masolo C, Borgo S, Gangemi A, Guarino N, Oltramari A. Wonderweb deliverable D18 - ontology library (final). LOA-NCR-ISTC; 2003. Available from: http://wonderweb.man.ac.uk/deliverables/D18.shtml.
[52] Porello D, Vieu L, Terkaj W, Borgo S, Compagno F, Sanfilippo EM. DOLCE in OWL: the core theory. In: Proceedings of JOWO 2024. CEUR Workshop Proceedings, vol. 3882; 2024. p. 1-14.
[53] Garbacz P, Trypuz R. A metaontology for applied ontology. Appl. Ontol. 2013 Jan;8(1):1-30.
[54] Barton A, Fabry P, Ethier JF. Meaning holism and indeterminacy of reference in ontologies. In: Proceedings of FOIS 2024. Amsterdam: IOS Press; 2024. p. 239-53.
[55] Merrill GH. Ontological realism: methodology or misdirection? Appl. Ontol. 2010 Jan;5(2):79-108.
[56] Merrill GH. Realism and reference ontologies: considerations, reflections and problems. Appl. Ontol. 2010 Aug;5(3-4):189-221.
[57] Jansen L, Grewe N. Butterflies and embryos: The ontology of temporally qualified continuants. In: Ontologies and Data in Life Sciences (ODLS 2014): Freiburg, Germany, Oct 7–8; 2014. p. E1–5.
[58] Grewe N, Jansen L, Smith B. Permanent generic relatedness and silent change. In: Proceedings of JOWO 2016. CEUR Workshop Proceedings, vol. 1660; 2016. p. 1-5.