

# A self-organizing control system for deterministic chaotic modes of a nonlinear spacecraft in the class of two-parameter structurally stable maps

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## Abstract

The article presents a study of a self-organizing control system for deterministic chaotic modes of a nonlinear spacecraft in the class of two-parameter structurally stable maps. The study of a self-organizing control system for a nonlinear spacecraft is based on the Lyapunov vector function gradient-velocity method. Spacecraft control systems are dynamic systems. One of the most important discoveries of the late twentieth century in nonlinear dynamical systems is deterministic chaos and the "strange attractor".

Deterministic chaos with the generation of "strange attractors" in the control systems of a nonlinear spacecraft manifests itself in the form of "separation", which leads to "accidents". Accordingly, the problem of controlling deterministic chaotic modes of nonlinear spacecraft control systems arose.

Under conditions of uncertainty, the regime of deterministic chaos in the control systems of a nonlinear spacecraft is generated as instabilities. When the conditions of robust stability in the system are violated by a nonlinear spacecraft, a deterministic chaotic regime with a "strange attractor" arises.

The self-organizing control system for deterministic chaotic modes of a nonlinear spacecraft in the class of two-parameter structurally stable maps has several stationary states. They both do not exist and are not robustly stable. When the conditions of robust stability are violated, other stationary states appear for the main stationary state. They also become robustly stable. Accordingly, instability and deterministic chaos with a "strange attractor" will be absent in the processes of control systems for a nonlinear spacecraft.

## Keywords

deterministic chaos, nonlinear systems, gradient-velocity method, vector of Lyapunov functions, strange attractor, chaotic systems, classification, robust stability, dynamical systems, spacecraft

## 1. Introduction

The most significant scientific discovery of the end of the last century in the field of nonlinear dynamics is deterministic chaos with a "strange attractor".

Deterministic chaos manifests itself in mechanical systems in the form of vibrations, in technical and technological systems in the form of "separation", which leads to accidents, in economic, ecological, biological, medical, social, etc. systems – fluctuations and fluctuations that provoke a "crisis" [1, 2].

The development of the theory of deterministic chaos has shown that in nonlinear dynamical systems, chaotic regimes and "strange attractors" are indeed always generated, sometimes proving useful [2,3]. Accordingly, classes of tasks have arisen when a deterministic chaotic process must be controlled by reducing or, conversely, increasing the degree of its randomness. Methods of controlling such chaotic processes are developing in several directions [2,3]: stabilization of unstable periodic oscillations [4,5], chaoticization [3, 4, 5], controlled synchronization [3, 4, 5, 6], modification

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of attractors [7, 8]. Another particularly relevant area of management of deterministic and chaotic processes is systems of complete suppression of the deterministic chaos regime [9, 10].

It has been found that in a nonlinear system, when deterministic chaos is generated, the trajectories of the system in phase space are globally limited and locally unstable inside the "strange attractor" [1, 2, 8], i.e. the process of deterministic chaos is characterized as instability.

Real control systems are designed and operate under conditions of uncertainty. The ability of the control system to maintain stability in conditions of uncertainty is understood as robust stability [11]. When going beyond the boundaries of the robust stability of uncertain parameters in a nonlinear dynamical system, a regime of deterministic chaos and instability in linear dynamical systems is generated. In conditions of uncertainty, the main factor guaranteeing protection from the regime of deterministic chaos and instability is the construction of a self-organizing control system [9, 10] in the class of structurally stable maps from the theory of catastrophes [12, 13].

Self-organizing control systems built in the class of structurally stable maps have several stationary states.

They both do not exist and are not stable [14]. With changes in uncertain parameters, as a result of "bifurcations", the system switches from the initial robustly stable stationary state to another [9, 10], i.e. the initial state loses stability, and the other stationary state acquires the property of robust stability. Thus, self-organization occurs in the system and unstable and deterministic chaotic regimes are excluded from the scenarios of the development of the process in the system.

Deterministic chaotic modes of a spacecraft can be generated as a result of exposure to cosmic rays or special directional magnetic waves. The polarity of the power supplies may change. This corresponds to indefinite changes in the parameters of the spacecraft's control system, which leads to a loss of robust stability and the creation of a regime of deterministic chaos and loss of controllability and operability of the spacecraft's control system. Other parameters of the spacecraft may also change during operation.

Protection against the regime of deterministic chaos is the construction of a control system for a nonlinear spacecraft in the class of two-parameter structurally stable representations from the theory of catastrophes, i.e. the construction of a control system for a nonlinear spacecraft in the form of self-organizing systems.

The purpose of this study is to show that a self-organizing control system for a nonlinear spacecraft in the class of two-parameter structurally stable maps has several stationary states. They do not simultaneously exist and are not robustly stable [15, 16]. When uncertain parameters change as a result of "bifurcations", the control system of a nonlinear spacecraft switches from the basic robustly stable stationary state to another robustly stable stationary state. The initial stationary state will lose its robust stability, and the other state will acquire the property of robust stability. In the control system of a nonlinear spacecraft, self-organization occurs and deterministic chaotic processes are excluded from the scenarios of the development of processes.

## 2. Methods

Currently, it is generally accepted that real control objects are nonlinear or linearized, high-dimensional and deterministic chaos, and "strange attractors" are the main property of any deterministic dynamical system. They function in conditions of uncertainty [11, 17].

Aperiodic robust stability means that the norm of the state vector decreases to zero without oscillations under parameter uncertainty.

The existing methods of studying deterministic chaotic processes: the method of point maps, the method of the phase plane [2, 3, 18], etc. are applicable to the study of a nonlinear dynamical system of small dimension and order.

Robust stability research methods are mainly devoted to [11, 17]: the study of the robust stability of polynomials and matrices, i.e. linear systems with parametric uncertainty.

Methods based on the Lyapunov function [19, 20, 21] are core tools in control theory but remain mostly theoretical. Building such functions for nonlinear or large systems is difficult. Classical

methods like the direct Lyapunov, Popov, or circle criteria work only for simple models and ignore uncertainty [22, 23, 24]. The gradient - velocity Lyapunov method solves this. It builds the function from system equations, links stability to system geometry, and checks robustness without linearization. This makes it practical for real nonlinear systems that operate under uncertainty.

The article considers the problem of studying a self-organizing control system for deterministic chaotic processes of a nonlinear spacecraft in the class of two-parameter structurally stable maps. The problem is solved by the gradient - velocity method of the Lyapunov vector function. [9, 10, 25, 26]. The Lyapunov gradient-velocity vector function method was developed based on a geometric interpretation of the stability research process using the Lyapunov direct method [27, 28, 29]. At the same time, it was found that the research process is described by the equation of gradient dynamical systems from disaster theories [9, 10, 12, 13]:

$$\left(\frac{dx_i}{dt}\right)_{x_j} = \frac{-\partial V_i(x)}{\partial x_j}, i=1, \dots, n; j=1, \dots, n$$

Here  $x(t) \in R^n$  - are the state variables of the dynamic control system;  $V_i(x)$  - are the components of the vector of Lyapunov functions (potential functions).

Based on the equations of gradient dynamical systems, the components of the gradient vector from the Lyapunov vector functions are determined from the equations of state of the control system:

$$\frac{\partial V_i(x)}{\partial x_j}, i=1, \dots, n; j=1, \dots, n$$

Components of the decomposition of the velocity vector according to the coordinates of the control system  $(x_1, \dots, x_n)$

$$\left(\frac{dx_i}{dt}\right)_{x_j}, i=1, \dots, n; j=1, \dots, n$$

The total time derivative of the vector of Lyapunov functions  $V(x)$  is calculated as the dot product of the components of the gradient vector from the vector of Lyapunov functions

$$\frac{\partial V_i(x)}{\partial x_j}, i=1, \dots, n; j=1, \dots, n$$

Decomposition of the velocity vector into components according to the coordinates of the control system:

$$\left(\frac{dx_i}{dt}\right)_{x_j}, i=1, \dots, n; j=1, \dots, n$$

$$\frac{dV(x)}{dt} = -\sum_{i=1}^n \sum_{j=1}^n \frac{\partial V_i(x)}{\partial x_j} \left(\frac{dx_i}{dt}\right)_{x_j}$$

The total time derivative of the vector of Lyapunov functions will always be a sign-negative function. Then, using the gradient of the vector of Lyapunov functions, the vector of the Lyapunov function is constructed in scalar form. The conditions of positive certainty of the vector of Lyapunov functions define the region of aperiodic robust stability of the system [25, 26, 27]. The Lyapunov gradient-velocity vector function method distinguishes high-precision systems with good control quality into a class of aperiodically robustly stable systems for which the mathematical norm of solutions to the equation of state of the control system decreases monotonously aperiodically, that is, technically transients are aperiodic in nature. The error in the system tends aperiodically to zero, that is, the main goal of management is always achieved.

### 3. Results and discussion

1. Let the control system of a nonlinear spacecraft, taking into account the dynamics of the actuator (flywheel) and the solar sensor, be described by the equations [18, 29, 30]:

$$\begin{aligned}
 \dot{x}_1 &= x_2 \\
 \dot{x}_2 &= \left( \frac{K_{D_1}}{T_{D_1}^2} - \frac{1}{T_{D_2}^2} \right) x_1 + \left( \frac{K_{D_1}}{T_{D_1}^2} - \frac{2\xi D_1}{T_{D_1}} \right) x_2 + \frac{1}{I_x} x_3 \\
 \dot{x}_3 &= x_4 \\
 \dot{x}_4 &= \frac{-1}{T_{M_1}^2} x_3 - \frac{2\xi_{M_1}}{T_{M_1}} x_4 + \frac{K_{M_1}}{T_{M_1}^2} u_1 + \frac{K_{M_1}}{T_{M_1}^2} u_2 \\
 \dot{x}_5 &= x_6 \\
 \dot{x}_6 &= \frac{1}{I_y} (I_z - I_x) x_2 x_{10} + \left( \frac{K_{D_2}}{T_{D_2}^2} - \frac{1}{T_{D_2}^2} \right) x_5 + \left( \frac{K_{D_2}}{T_{D_2}^2} - \frac{2\xi D_2}{T_{D_2}^2} \right) x_6 + \frac{1}{I_y} x_6 \\
 \dot{x}_7 &= x_8 \\
 \dot{x}_8 &= \frac{-1}{T_{M_2}^2} x_7 - \frac{2\xi_{M_2}}{T_{M_2}} x_8 + \frac{K_{M_2}}{T_{M_2}^2} u_5 + \frac{K_{M_2}}{T_{M_2}^2} u_6 \\
 \dot{x}_9 &= x_{10} \\
 \dot{x}_{10} &= \frac{1}{I_z} (I_x - I_y) x_2 x_6 + \left( \frac{K_{D_3}}{T_{D_3}^2} - \frac{1}{T_{D_3}^2} \right) x_9 + \left( \frac{K_{D_3}}{T_{D_3}^2} - \frac{2\xi D_3}{T_{D_3}^2} \right) x_{10} + \frac{1}{I_z} x_{11} \\
 \dot{x}_{11} &= x_{12} \\
 \dot{x}_{12} &= \frac{-1}{T_{M_3}^2} x_{11} - \frac{2\xi_{M_3}}{T_{M_3}} x_{12} + \frac{K_{M_3}}{T_{M_3}^2} u_9 + \frac{K_{M_3}}{T_{M_3}^2} u_{10}
 \end{aligned} \tag{1}$$

where:

$T_{D_i} (i=1, 2, 3)$  - are the time constants of the channels measuring the deflection angle and angular velocity of the solar sensor;

$\xi_{D_i} (i=1, 2, 3)$  - coefficients of relative damping of oscillations;

$K_{D_i} (i=1, 2, 3)$  - respectively, the channel gain coefficients for measuring the deflection angle and angular velocity of the solar sensor;

$x_1, x_2$  and  $x_9$  are, respectively, the angles of course  $\psi$ , pitch  $\theta$ , and roll  $\gamma$ , which determine the orientation of the spacecraft relative to the coordinate system  $Ox_g, y_g, z_g, x_2, x_6$  and  $x_{10}$ , respectively, the projections of the angular velocity vector of the spacecraft  $\omega_x, \omega_y$  and  $\omega_z$  on the axes of the coordinate system  $O_{xyz}, I_x, I_y$  and  $I_z$  are, respectively, the main moments of inertia relative to the axes  $O_x, O_y, O_z$ ;

$x_3, x_4; x_7, x_8; x_{11}, x_{12}$  - respectively variables characterizing the states of the actuator (flywheel) 1, 2 and 3;

$K_{M_i} (i=1, 2, 3)$  - respectively, the transmission coefficients of the actuators;

$T_{M_i} (i=1, 2, 3)$  is, respectively, the time constant of the actuator (flywheel);

$\xi_{M_i} (i=1, 2, 3)$  are the relative damping coefficients of the actuators ( $1 \leq \xi_M < 0$ ).

$u_1(t), u_2(t), u_5(t), u_6(t), u_9(t), u_{10}(t)$  are, respectively, the deflection angle control and the angular velocity of the spacecraft by the angle of course  $\psi$ , pitch  $\theta$  and roll  $\gamma$ , which determine the orientation of the spacecraft.

Designations are introduced:

$$\begin{aligned}
a_{21} &= \frac{K_{D_1}}{T_{D_1}^2} - \frac{1}{T_{D_1}}, a_{22} = \frac{K_{D_1}}{T_{D_1}^2} - \frac{2\xi_{D_1}}{T_{D_1}}, a_{23} = \frac{1}{I_x}, \\
a_{43} &= \frac{-1}{T_{M_1}^2}, a_{44} = \frac{-2\xi_{M_1}}{T_{M_1}}, b_{41} = \frac{K_{M_1}}{T_{M_1}^2}, b_{42} = \frac{K_{M_1}}{T_{M_1}^2}, d_6 = \frac{1}{I_y} (I_z - I_x) \\
a_{65} &= \frac{K_{D_2}}{T_{D_2}^2} - \frac{1}{T_{D_2}}, a_{66} = \frac{K_{D_2}}{T_{D_2}^2} - \frac{2\xi_{D_2}}{T_{D_2}}, a_{67} = \frac{1}{I_y}, \\
a_{87} &= \frac{-1}{T_{M_2}^2}, a_{88} = \frac{-2\xi_{M_2}}{T_{M_2}}, b_{85} = \frac{K_{M_2}}{T_{M_2}^2}, b_{86} = \frac{K_{M_2}}{T_{M_2}^2}, \\
d_{10} &= \frac{1}{I_z} (I_x - I_y), a_{10,9} = \frac{K_{D_3}}{T_{D_3}^2} - \frac{1}{T_{D_2}}, a_{10,10} = \frac{K_{D_3}}{T_{D_3}^2} - \frac{2\xi_{D_3}}{T_{D_3}}, \\
a_{10,11} &= \frac{1}{I_z}, a_{12,9} = \frac{-1}{T_{M_3}^2}, a_{12,10} = \frac{-2\xi_{M_3}}{T_{M_3}^2}, \\
b_{12,9} &= \frac{K_{M_3}}{T_{M_3}^2}, b_{12,10} = \frac{K_{M_3}}{T_{M_3}^2}
\end{aligned}$$

The control laws are given in the form of two-parameter structurally stable maps [12, 13]:

$$u_i(t) = -x_i^4 - k_i^1 x_i^2 + k_i x_i, i = 1, 2, 5, 6, 9, 10. \quad (2)$$

The system of equations (1), taking into account the notation and the control law (2), is written in an expanded form:

$$\left\{ \begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_{21} x_1 + a_{22} x_2 + a_{23} x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= a_{43} x_3 + a_{44} x_4 - b_{41} (x_1^4 + k_1^1 x_1^2 - k_1 x_1) - b_{42} (x_2^4 + k_2^1 x_2^2 - k_2 x_2) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= d_6 x_2 x_{10} + a_{65} x_5 + a_{66} x_6 + a_{67} x_7 \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= a_{87} x_7 + a_{88} x_8 - b_{85} (x_5^4 + k_5^1 x_5^2 - k_5 x_5) - b_{86} (x_6^4 + k_6^1 x_6^2 - k_6 x_6) \\
\dot{x}_9 &= x_{10} \\
\dot{x}_{10} &= d_{10} x_2 x_6 + a_{10,9} x_9 + a_{10,10} x_{10} + a_{10,11} x_{11} \\
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= a_{12,11} x_{11} + a_{12,12} x_{12} - b_{12,9} (x_9^4 + k_9^1 x_9^2 - k_9 x_9) - b_{12,10} (x_{10}^4 + k_{10}^1 x_{10}^2 - k_{10} x_{10})
\end{aligned} \right. \quad (3)$$

The system (3) has a basic stationary state [9, 12]:

$$x_{1S}^1 = 0, x_{2S}^1 = 0, \dots, x_{12S}^1 = 0 \quad (4)$$

Other stationary states of the system (3) [10, 13]:

$$x_{iS}^{2,3,4} = \sqrt[3]{k_i}, k_i^1 = \sqrt{k_i}, i = 1, 2, 5, 6, 9, 10 \quad (5)$$

The self-organizing control system of a nonlinear spacecraft (3) has stationary states (4) and (5). They do not simultaneously exist and at the same time do not allow the robust stability of these stationary states [14].

When the coefficient  $k_i < 0, i=1,2,5,6,9,10$  respectively, the stationary state (5) does not exist, but the stationary state (4) exists and is aperiodically robustly stable.

When the coefficient  $k_i > 0, i=1,2,5,6,9,10$  a stationary state (5) appears and will be aperiodic robust stable. In this case, the stationary state (4) will lose its aperiodic robust stability.

When the value of the regulator coefficient  $k_i > 0, i=1,2,5,6,9,10$  as a result of "bifurcations", the control system of a nonlinear spacecraft (3) switches from the initial stationary state (4) to another aperiodically robustly stable stationary state (5). It is obvious that the initial stationary state (4) of the control system of a nonlinear spacecraft (3) loses its aperiodic robust stability, and the other stationary state (5) acquires the property of aperiodic robust stability. Self-organization occurs in the control system of a nonlinear spacecraft (3), unstable and deterministic chaotic processes are excluded from the scenarios for the development of the process in the control system of a nonlinear spacecraft.

2. Investigation of Lyapunov vector functions using the gradient velocity method.

2.1. The periodic robust stability of the stationary state (4) of the control system of a nonlinear spacecraft (3) is investigated using the Lyapunov vector function gradient velocity method.

From (3), the components of gradients from the vector of Lyapunov functions are determined [15, 28]:

$$\begin{aligned}
\frac{\partial V_1(x)}{\partial x_2} &= -x_2, \frac{\partial V_2(x)}{\partial x_1} = -a_{21}x_1, \frac{\partial V_2(x)}{\partial x_2} = -a_{22}x_2, \frac{\partial V_2(x)}{\partial x_3} = -a_{23}x_3 \\
\frac{\partial V_3(x)}{\partial x_4} &= -x_4, \frac{\partial V_4(x)}{\partial x_1} = b_{41}(x_1^4 + k_1^1 x_1^2 - k_1 x_1), \frac{\partial V_4(x)}{\partial x_2} = b_{42}(x_2^4 + k_2^1 x_2^2 - k_2 x_2), \\
\frac{\partial V_4(x)}{\partial x_3} &= -a_{43}x_3, \frac{\partial V_4(x)}{\partial x_4} = -a_{44}x_4, \frac{\partial V_5(x)}{\partial x_6} = -x_6, \frac{\partial V_6(x)}{\partial x_2} = -d_6 x_2 x_{10}, \\
\frac{\partial V_6(x)}{\partial x_5} &= -a_{65}x_5, \frac{\partial V_6(x)}{\partial x_5} = -a_{66}x_6, \frac{\partial V_6(x)}{\partial x_7} = -a_{67}x_7, \frac{\partial V_7(x)}{\partial x_8} = -x_8, \\
\frac{\partial V_8(x)}{\partial x_5} &= b_{85}(x_5^4 + k_5^1 x_5^2 - k_5 x_5), \frac{\partial V_8(x)}{\partial x_6} = b_{86}(x_6^4 + k_6^1 x_6^2 - k_6 x_6), \\
\frac{\partial V_8(x)}{\partial x_7} &= -a_{87}x_7, \frac{\partial V_8(x)}{\partial x_8} = -a_{88}x_8, \frac{\partial V_9(x)}{\partial x_{10}} = -x_{10}, \frac{\partial V_{10}(x)}{\partial x_2} = -d_{10} x_2 x_6, \\
\frac{\partial V_{10}(x)}{\partial x_9} &= -a_{10,9}x_9, \frac{\partial V_{10}(x)}{\partial x_{10}} = -a_{10,10}x_{10}, \frac{\partial V_{10}(x)}{\partial x_{11}} = -a_{10,11}x_{11}, \\
\frac{\partial V_{11}(x)}{\partial x_{12}} &= -x_{12}, \frac{\partial V_{12}(x)}{\partial x_9} = b_{12,9}(x_9^4 + k_9^1 x_9^2 - k_9 x_9), \frac{\partial V_{12}(x)}{\partial x_{10}} = b_{12,10}(x_{10}^4 + k_{10}^1 x_{10}^2 - k_{10} x_{10}), \\
\frac{\partial V_{12}(x)}{\partial x_{11}} &= -a_{12,11}x_{11}, \frac{\partial V_{12}(x)}{\partial x_{12}} = -a_{12,12}x_{12}
\end{aligned} \tag{6}$$

From (3), the components of the decomposition of the components of the velocity vector along the coordinates of the system  $(x_1, \dots, x_{12})$  are determined [15, 28]:

$$\begin{aligned}
\left(\frac{dx_1}{dt}\right)_{x_2} &= x_2, \left(\frac{dx_1}{dt}\right)_{x_1} = a_{21}x_1, \left(\frac{dx_2}{dt}\right)_{x_2} = a_{22}x_2, \left(\frac{dx_2}{dt}\right)_{x_3} = a_{23}x_3, \left(\frac{dx_3}{dt}\right)_{x_4} = x_4, \\
\left(\frac{dx_4}{dt}\right)_{x_1} &= -b_{41}(x_1^4 + k_1^1x_1^2 - k_1x_1), \left(\frac{dx_4}{dt}\right)_{x_2} = -b_{42}(x_2^4 + k_2^1x_2^2 - k_2x_2), \\
\left(\frac{dx_4}{dt}\right)_{x_3} &= a_{43}x_3, \left(\frac{dx_4}{dt}\right)_{x_4} = a_{44}x_4, \left(\frac{dx_5}{dt}\right)_{x_6} = x_6, \left(\frac{dx_6}{dt}\right)_{x_2} = d_6x_2x_{10}, \\
\left(\frac{dx_6}{dt}\right)_{x_5} &= a_{65}x_5, \left(\frac{dx_6}{dt}\right)_{x_6} = a_{66}x_6, \left(\frac{dx_6}{dt}\right)_{x_7} = a_{67}x_7, \left(\frac{dx_7}{dt}\right)_{x_8} = x_8 \\
\left(\frac{dx_8}{dt}\right)_{x_5} &= -b_{85}(x_5^4 + k_5^1x_5^2 - k_5x_5), \left(\frac{dx_8}{dt}\right)_{x_6} = -b_{86}(x_6^4 + k_6^1x_6^2 - k_6x_6) \quad (7) \\
\left(\frac{dx_8}{dt}\right)_{x_7} &= a_{87}x_7, \left(\frac{dx_8}{dt}\right)_{x_8} = a_{88}x_8, \left(\frac{dx_9}{dt}\right)_{x_{10}} = \left(\frac{dx_{10}}{dt}\right)_{x_2} = d_{10}x_2x_6 \\
\left(\frac{dx_{10}}{dt}\right)_{x_9} &= a_{10,9}x_9, \left(\frac{dx_{10}}{dt}\right)_{x_{10}} = a_{10,10}x_{10}, \left(\frac{dx_{10}}{dt}\right)_{x_{11}} = a_{10,11}x_{11}, \left(\frac{dx_{11}}{dt}\right)_{x_{12}} = x_{12}, \\
\left(\frac{dx_{12}}{dt}\right)_{x_9} &= -b_{12,9}(x_9^4 + k_9^1x_9^2 - k_9x_9), \left(\frac{dx_{12}}{dt}\right)_{x_{10}} = -b_{12,10}(x_{10}^4 + k_{10}^1x_{10}^2 - k_{10}x_{10}), \\
\left(\frac{dx_{12}}{dt}\right)_{x_{11}} &= a_{12,11}x_{11}, \left(\frac{dx_{12}}{dt}\right)_{x_{12}} = -a_{12,12}x_{12}
\end{aligned}$$

The total time derivative of the vector of Lyapunov functions  $V(x)$  is calculated as the scalar product of the component vectors of the gradient vector from the vector of Lyapunov functions (6) by the vector components decomposition of the components of the velocity vector in coordinates (7):

$$\begin{aligned}
\frac{dV(x)}{dt} &= -x_2^2 - (a_{21}x_1)^2 - (a_{22}x_2)^2 - (a_{23}x_3)^2 - x_4^2 - (a_{43}x_3)^2 - (a_{44}x_4)^2 \\
&\quad - b_{41}^2(x_1^4 + k_1^1x_1^2 - k_1x_1)^2 - b_{42}^2(x_2^4 + k_2^1x_2^2 - k_2x_2)^2 - x_6^2 - (d_6x_2x_{10})^2 \\
&\quad - (a_{65}x_5)^2 - (a_{66}x_6)^2 - (a_{67}x_7)^2 - x_8^2 - (a_{87}x_7)^2 - (a_{88}x_8)^2 \\
&\quad - b_{85}^2(x_5^4 + k_5^1x_5^2 - k_5x_5)^2 - b_{86}^2(x_6^4 + k_6^1x_6^2 - k_6x_6)^2 - x_{10}^2 - (d_{10}x_2x_6)^2 \\
&\quad - (a_{10,9}x_9)^2 - (a_{10,10}x_{10})^2 - (a_{10,11}x_{11})^2 - x_{12}^2 - (a_{12,11}x_{11})^2 \\
&\quad - (a_{12,12}x_{12})^2 - b_{12,9}^2(x_9^4 + k_9^1x_9^2 - k_9x_9)^2 \\
&\quad - b_{12,10}^2(x_{10}^4 + k_{10}^1x_{10}^2 - k_{10}x_{10})^2, \quad (8)
\end{aligned}$$

It follows from (6) that the total time derivative of the vector of Lyapunov functions is always a nonpositive function ( $\leq 0$ ). The Lyapunov function decreases along the trajectories of the system. This ensures the sufficient condition for aperiodic robust stability of the stationary state (4).

$$\begin{aligned}
V(x) = & \frac{1}{5}b_{41}x_1^5 + \frac{1}{3}b_{41}k_1^1x_1^3 - \frac{1}{2}(b_{41}k_1 + a_{21})x_1^2 + \frac{1}{5}b_{42}x_2^5 + \frac{1}{3}b_{42}k_2^1x_2^3 \\
& - \frac{1}{2}(b_{42}k_2 + a_{22} + 1)x_2^2 - \frac{1}{2}d_6x_2^2x_{10} - \frac{1}{2}(a_{23} + a_{43})x_3^2 - \frac{1}{2}(a_{44} + 1)x_4^2 + \frac{1}{5}b_{85}x_5^5 \\
& + \frac{1}{3}b_{85}k_5^1x_5^3 - \frac{1}{2}(b_{85}k_5 + a_{65})x_5^2 + \frac{1}{5}b_{86}x_6^5 + \frac{1}{3}b_{86}k_6^1x_6^3 - \frac{1}{2}(b_{86}k_6 + a_{66} + 1)x_6^2 \\
& - \frac{1}{2}d_{10}x_2x_6^2 - \frac{1}{2}(a_{67} + a_{87})x_7^2 - \frac{1}{2}(a_{88} + 1)x_8^2 + \frac{1}{5}b_{12,9}x_9^5 + \frac{1}{3}b_{12,9}k_9^1x_9^3 \\
& - \frac{1}{2}(b_{12,9}k_9 + a_{10,9})x_9^2 + \frac{1}{5}b_{12,10}x_{10}^5 + \frac{1}{3}b_{12,10}k_{10}^1x_{10}^3 - \frac{1}{2}(b_{12,10}k_{10} + a_{10,10} + 1)x_{10}^2 \\
& - \frac{1}{2}(a_{10,11} + a_{12,11})x_{11}^2 - \frac{1}{2}(a_{12,12} + 1)x_{12}^2 \tag{9}
\end{aligned}$$

From (9), the condition of positive definiteness of the Lyapunov function is not immediately evident [12, 13]. However,  $V(x)$  vanishes at the origin and is continuously differentiable. To apply the Morse lemma, it is necessary that the equilibrium state (4) be a nondegenerate critical point of  $V(x)$ , that is,

$$\nabla V(x)=0, \det[Hess(V(0))] \neq 0.$$

The Hessian matrix of  $V(x)$  at the origin was verified to be nonsingular, which confirms that the equilibrium state (4) is indeed a nondegenerate critical point. Therefore, all conditions of the Morse lemma are satisfied, and the function  $V(x)$  can be locally reduced to a quadratic form in the neighborhood of the origin:

$$V(x) \approx V(0) + \frac{1}{2}x^T H_V(0)x.$$

$$\begin{aligned}
V(x) = & -\frac{1}{2}(b_{41}k_1 + a_{21})x_1^2 - \frac{1}{2}(b_{42}k_2 + a_{22} + 1)x_2^2 - \frac{1}{2}(a_{23} + a_{43})x_3^2 - \frac{1}{2}(a_{44} + 1)x_4^2 \\
& - \frac{1}{2}(b_{85}k_5 + a_{65})x_5^2 - \frac{1}{2}(b_{86}k_6 + a_{66} + 1)x_6^2 - \frac{1}{2}(a_{67} + a_{87})x_7^2 \\
& - \frac{1}{2}(a_{88} + 1)x_8^2 - \frac{1}{2}(b_{12,9}k_9 + a_{10,9})x_9^2 - \frac{1}{2}(b_{12,10}k_{10} + a_{10,10} + 1)x_{10}^2 \\
& - \frac{1}{2}(a_{10,11} + a_{12,10})x_{11}^2 - \frac{1}{2}(a_{12,12} + 1)x_{12}^2, \tag{10}
\end{aligned}$$

From (10) the condition of positive certainty, the vector of Lyapunov functions (9), i.e., the necessary condition for aperiodic robust stability is written:

$$\begin{aligned}
& b_{41} > 0, b_{42} > 0, b_{85} > 0, b_{86} > 0, b_{12,9} > 0, b_{12,10} > 0 \\
& -(b_{41}k_1 + a_{21}) \geq 0 \\
& -(b_{42}k_2 + a_{22} + 1) \geq 0 \\
& -(a_{23} + a_{43}) \geq 0 \\
& -(a_{44} + 1) \geq 0 \\
& -(b_{85}k_5 + a_{65}) \geq 0 \\
& -(b_{86}k_6 + a_{66} + 1) \geq 0 \\
& -(a_{67} + a_{87}) \geq 0 \\
& -(a_{88} + 1) \geq 0 \\
& -(b_{12,9}k_9 + a_{10,9}) \geq 0 \\
& -(b_{12,10}k_{10} + a_{10,10} + 1) \geq 0 \\
& -(a_{10,11} + a_{12,10}) \geq 0 \\
& -(a_{12,12} + 1) \geq 0
\end{aligned} \tag{11}$$

The domain of aperiodic robust stability of the stationary state (4) is determined by the system of inequalities (11) and, when performed, characterizes the absence of instability and a deterministic chaotic process in the control system of a nonlinear spacecraft.

2.2. To study the aperiodic robust stability of the stationary state (5) of control systems of a nonlinear spacecraft (3), it is represented in deviations relative to the stationary state (5) [9, 10, 19]:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_{21}k_1 + a_{22}x_2 + a_{23}x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= a_{43}x_3 + a_{44}x_4 - b_{41}\left(x_1^4 + 4\sqrt[3]{k_1}x_1^3 + 3\sqrt[3]{(k_1)^2}x_1^2 + k_1x_1\right) \\
& \quad - b_{42}\left(x_2^4 + 4\sqrt[3]{k_2}x_2^3 + k_2x_2\right), \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= d_6x_2x_{10} + a_{65}x_5 + a_{66}x_6 + a_{67}x_7, \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= a_{67}x_7 + a_{88}x_8 - b_{85}\left(x_5^4 + 4\sqrt[3]{k_5}x_5^3 + 3\sqrt[3]{(k_5)^2}x_5^2 + k_5x_5\right) \\
& \quad - b_{86}\left(x_6^4 + 4\sqrt[3]{k_6}x_6^3 + 3\sqrt[3]{(k_6)^2}k_6^2 + k_6x_6\right), \\
\dot{x}_9 &= x_{10} \\
\dot{x}_{10} &= d_{10}x_2x_6 + a_{10,9}x_6 + a_{10,10}x_{10} + a_{10,11}x_{11} \\
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= a_{12,11}x_{11} + a_{12,12}x_{12} - b_{12,9}\left(x_9^4 + 4\sqrt[3]{k_9}x_9^3 + 3\sqrt[3]{(k_9)^2}x_9^2 + k_9x_9\right) \\
& \quad - b_{12,10}\left(x_{10}^4 + 4\sqrt[3]{k_{10}}x_{10}^3 + 3\sqrt[3]{(k_{10})^2}x_{10}^2 + k_{10}x_{10}\right),
\end{aligned} \tag{12}$$

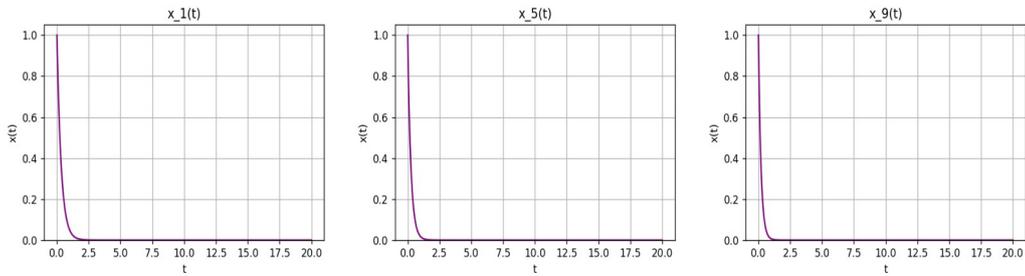
The periodic robust stability of the control system of a nonlinear spacecraft in deviations (12) is investigated using the Lyapunov vector function gradient-velocity method [9, 10].

The conditions of aperiodic robust stability are obtained as

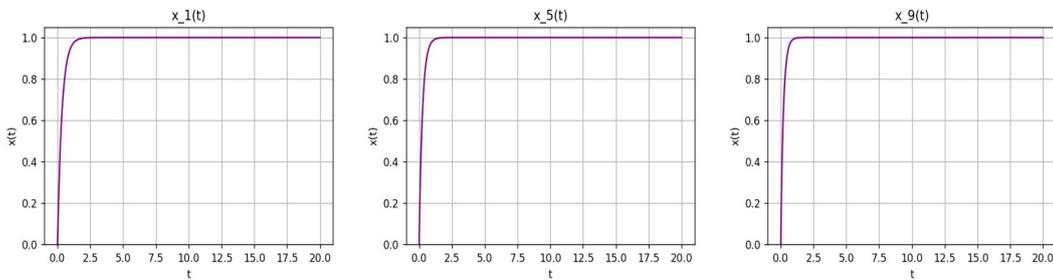
$$\left. \begin{aligned}
& b_{41} > 0, b_{42} > 0, b_{85} > 0, b_{86} > 0, b_{12,9} > 0, b_{12,10} > 0 \\
& b_{41} k_1 - a_{21} > 0 \\
& b_{42} k_2 - a_{22} - d_6 \sqrt[3]{k_{10}} - 1 > 0 \\
& -(a_{23} + a_{43}) > 0 \\
& -(a_{44} + 1) > 0 \\
& b_{85} k_5 - a_{65} > 0 \\
& b_{86} k_6 - a_{66} - d_{10} \sqrt[3]{k_2} - 1 > 0 \\
& -(a_{67} + a_{87}) > 0 \\
& -(a_{88} + 1) > 0 \\
& b_{12,9} k_9 - a_{10,9} > 0 \\
& b_{12,10} k_{10} - a_{10,10} - d_{10} \sqrt[3]{k_2} - d_6 \sqrt[3]{k_2} - 1 > 0 \\
& -(a_{10,11} + a_{12,10}) > 0 \\
& -(a_{12,12} + 1) > 0
\end{aligned} \right\} \quad (13)$$

The region of aperiodic robust stability of the stationary state (5) of the control system of a nonlinear spacecraft is determined by a system of inequalities (13). The fulfillment of inequalities (13) guarantees the absence of fluctuations, instability and deterministic chaotic processes in the control system of a nonlinear spacecraft.

A numerical experiment based on a model of self-organizing control systems for deterministic chaotic modes of nonlinear spacecraft in the class of two-parameter structurally stable maps confirms the theoretical results. Figure 1 shows the results of a numerical experiment – graphs of transients:  $x_1(t), x_5(t), x_9(t)$  under conditions  $k_1 = k_5 = k_9 = -5$ , and  $x_1(t), x_5(t), x_9(t)$  under conditions  $k_1 = k_5 = k_9 = 5$ .



**Figure 1:** The results of numerical experiments at  $k_1 = k_5 = k_9 = -5$ .



**Figure 2:** The results of numerical experiments at  $k_1 = k_5 = k_9 = 5$ .

3. The study shows that the stationary state (4) exists and is aperiodically robustly stable when the uncertain parameters of the self-organizing system (3) lie in the region defined by (9). The stationary state (5) arises as a result of "bifurcation" when the aperiodic robust stability of the stationary state (4)

is lost. This state (5) also becomes aperiodically stable when the system of inequalities (11) is satisfied. Consequently, there are no oscillatory, unstable, or deterministic chaotic processes in the system. Lyapunov's universal gradient-velocity vector function method makes it possible to study both linear and nonlinear dynamical systems of high dimension and order. It also makes it possible to classify control systems with good transients into the class of aperiodically robustly stable systems. For these systems, the mathematical norm of solutions to the equations of state of the control system of a nonlinear spacecraft decreases monotonously aperiodically, which indicates the technical aperiodicity of transients (Fig. 1). The control system of a nonlinear spacecraft, built in the class of two-parameter structurally stable maps (3), indeed has multiple stationary states. These stationary states (4), (5) do not exist simultaneously and are not aperiodically robustly stable. When the uncertain coefficients of the control system of a nonlinear spacecraft change due to "bifurcations", the system (3) transitions from the initial aperiodically robustly stable stationary state (4) to another aperiodically robustly stable stationary state (5). The initial stationary state (4) of the control system of a nonlinear spacecraft in the class of two-parameter structurally stable maps (3) loses its aperiodic robust stability, and the other stationary state (5) acquires the property of aperiodic robust stability. This means that in the control system of a nonlinear spacecraft, built in the class of two-parameter structurally stable maps, self-organization occurs, instability and deterministic chaotic processes are excluded from the scenario of the development of the processes of the control system of a nonlinear spacecraft.

#### **4. Discussion**

The results of this study open up new opportunities for the construction, research and application of self-organizing automatic control systems in engineering and technology. Automatic control systems are widely used in almost all areas of production and technology: in mechanical engineering, energy, microelectronic industry, transport, robotic and space systems, metallurgical industry, modern military equipment and technologies, etc.

Modern control objects are nonlinear or linearized, of large dimension and order, and instability and deterministic chaos are fundamental properties of a deterministic dynamical system. Instability and deterministic chaos in automatic control systems generate uncontrollability of the object and provokes "separation" and "accident".

In conditions of uncertainty, the main factor of protection against instability and deterministic chaos with a "strange attractor" is the construction of a self-organizing automatic control system in the class of structurally stable representations from disaster theories. There are six classified structurally stable maps in disaster theories.

In engineering and technology, linear and nonlinear control objects are considered. They are controlled by the state vector and by the output. The tasks of analysis, synthesis and synthesis of adaptive self-organizing automatic control systems are solved. All this requires the development of a theoretical basis for self-organizing automatic control systems in the class of structurally stable maps. The tasks of analyzing and synthesizing self-organizing automatic control systems in the class of structurally stable maps are solved by a new, universal gradient-velocity method of Lyapunov vector functions. It is necessary to develop, research and create self-organizing automatic control systems in the class of structurally stable mappings in various industries and engineering.

#### **5. Conclusion**

Currently, it is generally accepted that real control objects are nonlinear or linearized, multidimensional in large dimensions, and they operate under conditions of uncertainty. Instabilities and deterministic chaos are the basic properties of any deterministic dynamical system. Instabilities and deterministic chaotic regimes mainly have harmful effects and objects will lose their "controllability", i.e. they can provoke "separation" and "accident". Deterministic chaos in dynamic control systems is generated when the conditions of robust stability are violated. In conditions of

uncertainty, the main factors guaranteeing protection against the regime of deterministic chaos and instability are the construction of self-organizing control systems in the class of structurally stable maps. The control system of a nonlinear spacecraft in the class of two-parameter structurally stable mappings has several stationary states. They both do not exist and are not robustly stable. When uncertain parameters change as a result of "bifurcation," the system transitions from its initial robustly stable stationary state to another robust stable state, and oscillatory, unstable, and deterministic chaotic modes are excluded from scenarios for the development of processes in the system.

The problem of studying a self-organizing control system for unstable and deterministic chaotic processes is solved by the gradient-velocity method of the Lyapunov vector function. The gradient-velocity method of Lyapunov functions allows us to distinguish control systems with good control quality indicators into a class of aperiodically robustly stable, transients in the control system have an aperiodic character of a given (desired) control quality. The class of the control system is determined by the domain of aperiodic robust stability.

## Declaration on Generative AI

The authors have not employed any Generative AI tools.

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