Local positioning of the particle swarm

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Abstract
The development of technical systems leads to the complication of control tasks and increased requirements for the efficiency of their solution, which necessitates the need to improve the means of solving problems related to the optimization of interaction of objects and their navigation. The paper considers the task of controlling a group of unmanned aerial vehicles (UAVs) as a swarm motion organization. Considering the specific requirements for the spatial and temporal position of individual UAVs in the swarm, it is proposed to use a model considering elites planning a movement route in accordance with a given task. A general mathematical model has been developed for the description, implementation and application of swarm intelligence algorithms using systems analysis methods to solve navigation problems of UAV systems for the development of an alternative to satellite navigation and the assessment and refinement of UAV swarm positioning using ephemerides and almanacs. Within the framework of the classification of local particle swarm positioning problems, the process of swarm formation with a given positioning is described. The object of the study is the procedure of particle swarm formation in cognitive technologies of metaphorical optimization as a geoinformation object (GIO), namely, objects located on a given surface or locally changing it are considered. The main difference of the conducted analysis is the cognitive-semantic approach based on the definition of information interaction in the intellectual information environment "object-subject" within the framework of certain problems. The task of controlling a group of UAVs to organize swarm movement to ensure the most effective achievement of the flight goal is considered. At the same time, the guidance signal can allow individual UAVs to gather at leader positions and follow their velocity vector to provide UAV-system topology connectivity and enable swarm formation. Also, the peculiarities of UAV group management organization, existing and prospective methods of their interaction organization as part of one grouping when performing various tasks are considered. Mechanisms for deriving a particle swarm genetic optimization model in metaphorical algorithms can be used to create a new generation of artificial intelligence. We consider the procedure of particle swarm optimization (PSO) localization clustering based on determining the similarity measure of hash functions of isoportraits as well as their graphs of the structure of color atlases of images.

Keywords
PSO, positioning, localization, color atlas, geoinformation object, cognitive optimization, mathematical model, similarity measure, diffeomorphism, persistent homologies

1. Introduction

The relevance of PSO-based positioning lies in its applicability to object localization and identification. This study focuses on analyzing the topology of 2D images to identify invariants under diffeomorphic transformations. Camera maneuvers such as yaw, roll, and pitch during UAV-based surface scanning are modeled as diffeomorphisms controlled by onboard gyroscopes. Using a digital image ontology (DIO) based on color space models, we construct a topology of color distribution to detect similarity through a Color Atlas Structure Graph (CASG). This method enables invariant detection across image sequences and offers practical potential in AI-based image analysis, including metaphorical algorithms, by accelerating data processing.

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2. Literature data analysis

Paper [1] provides an overview of transfer learning methods for visual recognition, highlighting both advances and underexplored challenges. In [2], recent deep learning techniques for semantic segmentation, including those applied to hyperspectral images and point clouds, are reviewed with a focus on pixel-level accuracy. The rapid growth of video data [4] and the demand for efficient processing drive the development of less computationally intensive methods [5-7]. Studies [8, 9] also explore AI-based image processing techniques, including inter-frame analysis [10, 11] and steganography [12]. Promising approaches include feature-space video coding (FVC) [13] and topology-based methods [14, 15], which, when combined with fast computation, improve video surveillance efficiency. This underscores the need for rapid, high-performance visual data processing systems.

3. Problem statement

This study uses theoretical methods to identify invariants in inter-frame processing. The source images include a stationary background, random noise, and geo-information objects (GIOs). Background characteristics vary across frames and exhibit anisotropic correlations, while noise is near-normal and stationary. GIO appearance changes with scan position, and frames in a sequence are not aligned to a common coordinate system. Temporal background stability is observed only in short sequences, and noise differs between forward and backward scans. Images may also include glare.

We assume GIOs are localized on a surface with a visible horizon and a bounded light source. Converting images to grayscale reveals light and shadow patterns corresponding to GIO structure. Camera motion (yaw, roll, pitch) during scanning is tracked via gyroscopes and external navigation, making accurate positioning essential. Grayscale transformation further enables detection of brightness gradients, whose collinearity remains invariant under camera motion. Additional invariants include region size ratios and persistence of local luminosity anomalies, distinguishing elevated structures and depressions from the background.

Enhancing UAV navigation performance in dynamic environments requires the use of adaptive swarm intelligence algorithms. Key challenges include:

1. High density of network connectivity;
2. Ensuring high data throughput;
3. Achieving ultra-low communication latency;
4. Maintaining high reliability, including self-recovery and survivability of individual UAVs and the swarm as a whole.

The study aims to address the integration of heterogeneous data through the development of an adaptive ontological framework for swarm-based genetic control, enabling parallel execution and evolutionary adaptability in organizational and technological optimization tasks.

The study applies methods of analysis, synthesis, abstraction, system analysis, and mathematical modeling. Optimization tasks are approached using metaphorical evolutionary swarm algorithms, based on prior work [18–20].

4. Main results

For a particle is set $\bar{x}_0$ – the initial location reference for $P_i(\bar{x}_0, t_0)$ and the following functions:

- $Kg(P_i(\bar{x}, t))$ – cognitive P_i (approximation by self-learning);

- $Sc(P_i(\bar{x}, t))$ – trajectory approximation by exchanging information with particles from the neighborhood $P_i(t)$;

- $Es(P_i(\bar{x}, t))$ – atlas of the surface over which the trajectory $P_i(\bar{x}, t)$ is located (passes);

- $Ss(P_i(\bar{x},t))$ – atlas of the system of distant stars for $P_i(\bar{x},t)$.

Local positioning of swarm particles means:

$$\{P_i\}_{i=\overline{1,n}} \overset{\text{def}}{\Leftrightarrow} \text{define } \Omega,\ T_0,\ \bar{\Delta},\ \gamma \text{ for } cl(\{P_i\}_{i=\overline{1,n}}),\ Sw(\{P_i\}_{i=\overline{1,n}}),\ Sn(\{P_i\}_{i=\overline{1,n}}),.$$

where:
Sn – synchronization of the speed of swarm particles $\{P_i\}_{i=\overline{1,n}}$ at a point in time T_0, Sw – swarm particle localization $\{P_i\}_{i=\overline{1,n}}$ at a point in time T_0 on the interval $[T_0-\Delta t, T_0+\Delta t]$, cl – accumulation, $cl(\{P_i\}_{i=\overline{1,n}}) \equiv \{\{P_i\}_{i=\overline{1,n}}, \Omega, T_0\}$; given Ω – particle swarm localization area, $\bar{\Delta}$ - particle alignment, γ - velocity cone angle.

A swarm as a structured set with respect to the types of behavior of a cluster of particles (by polarization) in dynamics can be represented by the following sequence: free particle – particle in a cluster – particle in a swarm as a polarized cluster.

The structure of the swarm is characterized by the following:

1. For each particle of the swarm there is at least one particle of the swarm, between them there is an information exchange, i.e. Is – adjacent or informationally adjacent, and Ir – adjacent in direction, i.e. $\|grad\, r_1 - grad\, r_2\| \le \varepsilon$.
2. Relations Is and Ir define the adjacency connectivity on the set of swarm particles.
3. $Tk(\bar{r}_i,[t_0,dt]) \equiv \{\bar{r}_i(t)\},\ \forall t \in [t_0,dt]$ – particle trajectory over a given time interval, particle swarm (PS) drift function: $Tk(\bar{r}_i,[t_0,dt]) \subset C[t_0,dt]$, $Tk(\bar{r}_i) \cap Tk(\bar{r}_j) = \varnothing, i \ne j,\ \forall t$.
4. Swarm trend $Tr(t,N) = \sum_{i=1}^{N} \bar{r}_i$, $r(t)$ – radius vector of a swarm particle, $\forall i \in N : \bar{r}_i(t), N < \infty$.
5. For the accumulation (conglomeration) of the swarm the following is performed:

$$cl(X(N,t),\varepsilon) \overset{def}{\Leftrightarrow} \left| \begin{array}{l} 1)\ \bar{X}(N,t) - \text{related to localization} \\ 2)\ adjacent \in R^m \\ 3)\ \forall t, grad\, \bar{r}_i \in [\|\overline{Tr}(N,t)-\bar{\varepsilon}\|, \|\overline{Tr}(N,t)+\bar{\varepsilon}\|] \end{array} \right.$$

Polarization of the motion of swarm particles, i.e. the motion of adjacent PS is co-directed.

- $\forall cl(\{\bar{r}_i\}_{i=1}^{n_i}, t, \varepsilon_{Tr}) \subset X(N,t),\ \forall n_i \le N,\ \forall t$;
- $cl(\{\bar{r}_i\}_{i=1}^{n_i}, t, \varepsilon_1) \subset cl(\{\bar{r}_i\}_{i=1}^{n_i}, t, \varepsilon_2) \Leftrightarrow (\{\bar{r}_i\}_{i=1}^{n_1} \subset \{\bar{r}_i\}_{i=1}^{n_2}) \wedge (\varepsilon_1 \le \varepsilon_2),\ \forall t$;
- $\sum_{j=1}^{N_j} \overline{Tr}(\{\bar{r}_i\}_{i=1}^{N_j}, t) = \bar{T}(N,t),\ \forall$ partitions $T = \sum_j N_j$.

6. $\{\bar{r}_i(t)\}_{i=1}^{N}$ – specifies the vector field of the swarm trend (movement) $X(N,t)$ in R^m.
7. Swarm as a way of mediating through other swarm particles and within swarm information exchange the horizon of information gathering by a particle:

$G(\{\bar{r}_i(t)\}, d\varepsilon^1, d\varepsilon^2) = U_i G(\bar{r}_i(t), \varepsilon^1, d\varepsilon^2)$, horizon of information selection by particle $\bar{r}_i(t)$: $G(\{\bar{r}_i(t)\}, d\varepsilon^1, d\varepsilon^2) \subset R^m | S^1 \wedge S^2 \ne 0$.

To study the synchronization of the particle swarm motion based on the surface graph, we introduce the following notations:

- $df(t)$ – is a diffeomorphism of the particle motion at time t , where $df(t)$ = (yaw direction, roll, pitch);
- P_i – is a particle in motion, where $P_i \overset{def}{\equiv} (h, |\bar{V}| grad(\bar{V}), df(t), t)$, where $\bar{V}(t)$ – is the velocity of the particle at time t, $h(t)$, $h(t+\Delta t)$ – is the height above the surface S at time t and $t+\Delta t$;

- $I m(t)$, $I m(t+\Delta t)$ – are part of the image of the surface S at time t and $t+\Delta t$ within the viewing cone $Con(t)$ and $Con(t+\Delta t)$;
- $\Delta I m = I m(t) \cap I m(t+\Delta t) \Rightarrow \bar{I}(t, t+\Delta t) = max \Delta I m$ – is the image of the viewing surface, which is defined from the condition of the direction in which $I m(t)$ and $I m(t+\Delta t)$ intersect maximally;
- $\{I m(t_1), \ldots, I m(t_n)\}$ – is the trajectory $\overset{def}{\Leftrightarrow}$ $I m(t_i) \cup I m(t_{i+1}) \neq 0,\ \forall i = \overline{1.n-1}$. If $I m(t_1)$ – is the start and $I m(t_n)$ – is the finish, then the trajectory will be the reference for the targeted movement of the swarm.
- $G(t)$, $G(t+\Delta t)$ – graphs of images $I m(t)$ and $I m(t+\Delta t)$;
- $SG(t)$, $SG(t+\Delta t)$, – corresponding graph schemes;
- $\Delta G(t, \Delta t) \equiv G(t+\Delta t) \acute{\Delta} G(t)$, $\Delta SG(t, \Delta t) \equiv SG(t+\Delta t) \acute{\Delta} SG(t)$ – symmetric difference of graphs and graph schemes, where $\acute{\Delta}$ – is the symmetric difference operation;
- $g G(t, \Delta t) = G(t+\Delta t) \setminus G(t)$; $gSG(t, \Delta t) \equiv SG(t+\Delta t) \setminus SG(t)$ – difference of graphs and graph schemes.

If the inequality $h(t) < h(t+\Delta t)$ is true for the values of the height above the surface, then it follows that $SG(t) \subset SG(t+\Delta t)$, $G(R_{in}, t) \subset G(R_0, t)$, $G(R_{in}, t+\Delta t) \subset G(R_0, t+\Delta t)$, i.e. the subsequent image includes the previous one.

The condition of observability of the continuity of motion: $SG(R_0, t) \cap SG(R_0, t+\Delta t) \neq \varnothing$. From here, the radius of the maximum displacement from the point of location of the particle at time t over time Δt : $R_{max}(t, \Delta t) = \int_0^{\Delta t} |\bar{V}(t+z)| dz$.

The navigation continuity condition is defined as follows:

$$\{G(R_0, t) \cap G(R_0, t+\Delta t) \neq \varnothing\} \cup \{G(R_0, t) \cap G(R_{in}, t+\Delta t) \neq \varnothing\} \cup$$
$$\cup \{G(R_{in}, t) \cap G(R_{in}, t+\Delta t) \neq \varnothing\} \cup \{G(R_{in}, t) \cap G(R_0, t+\Delta t) \neq \varnothing\}$$

We define synchronization Sn and the sticking togetherSl of particles P_1 and P_2:

$$\exists Sn(P_1, P_2, t) \overset{def}{\Leftrightarrow} \exists \varepsilon > 0, grad \bar{V}(P_1, t) \subset Congrad(\bar{V}(P_2, t), \varepsilon)$$

$\exists t \leq T,\ \forall t \in [t, t+\Delta t]$, $Congrad$ –is the gradient cone with vertex angle ε

$$\exists Sl(P_1, P_2, t) \overset{def}{\Leftrightarrow} \begin{cases} 1. \exists Sn(P_1, P_2, t) \\ 2. T = +\infty \\ 3. |loc(P_1, t) - loc(P_2, t)| \leq const,\ \forall t \end{cases}$$

Let us introduce the localization operation $loc()$, the essence of which is the definition of the coordinates of the particles. Let us define the adjacent particles in the region: $\exists Sm(P_i, P_j, t, \varepsilon) \overset{def}{\Leftrightarrow} \exists \varepsilon : \exists O_\varepsilon$ – is the region of R^n, then $loc(P_i, t) \subset O_\varepsilon$, $loc(P_j, t) \subset O_\varepsilon$.

Based on the definition of the coordinates of particles by the operation $loc(\cdot)$ we define particles adjacent in the region: $\exists Sm(P_{i,j}, t, \varepsilon) \overset{def}{\Leftrightarrow} \exists \varepsilon > 0 : \exists O_\varepsilon$ – is a region in R^n, then $loc(P_i, t) \subset O_\varepsilon$, $loc(P_j, t) \subset O_\varepsilon$.

Let us introduce the definitions:

- (P_i, t) – neighborhood:

$Ok(P_i,t,\varepsilon) \overset{def}{\Leftrightarrow} \{\{P_j\}: \forall j, \forall t, P_j: |loc(P_j,t)-loc(P_j,t)|<\varepsilon\}$;

- Sd – neighborhood of particles: $\exists Sd(P_i,\{P_j\},\varepsilon_0,t) \Leftrightarrow$
$\Leftrightarrow \{\forall_j \subset Ok(P_i,t,\varepsilon), \varepsilon>\varepsilon_0\} \wedge \{\forall P_j \not\subset Ok(P_i,t,\varepsilon_0)\}$.

The set $\{P_i\}$ is a swarm on the interval $(t,t+\Delta t)$, $\{P_i\}$ – are adjacent in the region and $\forall i$ there are neighboring particles stuck together with it.

We define the boundary points of the swarm $\{P_i^b\}$ as: $\forall i\, P_i^b \in (\{P_i\},t)$ in $\{P_i \neq P_i^b\} \not\subset Ok(P_i^b)$, i.e. the point $(\cdot)$ is a boundary point, there exists a hyperplane in the space R^3, such that the points from the adjacency regions of a given point lie on one side of the given hyperplane.

In a swarm of particles, we select a group of leader particles $\{L_k\} \subset \{P_i\}_{i=\overline{1,n}}$. The leader particles $\{L_k\}$ are those particles for which the following conditions are met:

1. $grad\{L_k\}$ is oriented toward the external environment;
2. the middle part of the swarm is oriented to the set $\{L_k\}$ i.e $\forall l, \forall M_l, \exists k, \exists L_k\, grad\, M_l \subset Ok(grad\, L_k)$;
3. the direction of movement of the swarm $\{L_k\}$ is a bundle of directions for which the following condition is met $\forall k\, grad\, L_k: \cap Ok(grad\, L_k) \neq \emptyset$, $\forall Ok(grad\, L_k)\, \exists Ok(grad^*\{L_k\})$;
4. for the structure of the neighborhood of the swarm direction vector $Ok(grad\, L_k)$, $\forall L_k$ the following conditions are satisfied:

- $grad\, L_k+\bar{\varepsilon}_k$, where $\bar{\varepsilon}_k$ – is a vector in the parameter space $\{L_k\}$, $|\bar{\varepsilon}_k|<\varepsilon_0$ – is the admissible scatter of particles;
- $(\text{grad}\, L_k, \bar{\varepsilon}_k) \geq 0$, where $\alpha(L_{i1})$ – is the scatter angle of the direction change:

$\alpha(L_k)=\arccos\left(\frac{(grad\, L_k,\bar{\varepsilon}_k)}{|grad\, L_k|\cdot|\bar{\varepsilon}_k|}\right)$;

- $\alpha^*=\max\limits_{L_k}(\alpha(L_k))$ – is the swarm maneuver angle.

The direction $\bar{e}(t)$ is admissible for the swarm $(\{P_i\},t)$ if $\forall i (grad\, P_i, \bar{e})>0$ at time t.

By the direction of swarm motion $(\{P_i\},t)$ we will mean the direction $\bar{e}^*(t)$ such that $\max\limits_{\bar{e}} \sum\limits_i (grad\, P_i,\bar{e}) \to \bar{e}^*(t)$.

For the swarm motion $(\{P_i\},t)$ it is true that:

- $\{\forall(\{P_i\},t)\, \exists \bar{e}^*(t)\} \wedge \{V_j(P_i,t)=V_k(P_i,t), \forall j,k\}$ – in the case of translational motion;
- $\{\forall(\{P_i\},t)\, \exists \bar{e}^*(t)\} \wedge \{V_j(P_i,t)\neq V_k(P_i,t), \forall j,k\}$ – in the case of translational circular motion.

Consider a particle swarm, and specifically its part $\{L_k\}$ in the process of determining the direction of motion as an optimization procedure within some opt criterion.

Based on the opt criterion and the exchange of information between particles $\{L_k\}$, the correction of the swarm motion vector is determined. It should be noted that the set $\{L_k\}$ is co-directed with respect to the particle swarm motion vector.

For the set $\{L_k\}$ we consider $\{L_k^b\}$ – boundary points $\{L_k\}$, adjacent to the direction of motion within the swarm motion cone. Then the procedure of grouping of particles $\{L_k\}$ into a swarm is the synchronization of $grad\{L_k\}$ along the direction $grad\{L_k^b\}$, $\{L_i^b\}<\{L_i\}$, $\forall t$ taking into account the adjacency conditions.

Next, consider for direction correction a nonuniform mesh obtained as projections of the motion $\{L_k^b\}$ onto a plane frontal to the swarm motion. Also, consider the family of planes $\{PL_k\}$ passing through the axis of the viewing cone relative to the direction of movement of the swarm and perpendicular to the frontal plane. With respect to each such plane, two families $\{L_i^b\}^+$ and $\{L_i^b\}^-$ – can be defined – particles with the best values of the *opt* criterion relative to the current direction of movement of the swarm and other particles. To compare individual planes from the bundle, we introduce two indicators:

$$p^+=\frac{\left|\{L_i^b\}^+\right|}{\left|\{L_i^b\}^+\right|+\left|\{L_i^b\}^-\right|},\ p^-=\frac{\left|\{L_i^b\}^-\right|}{\left|\{L_i^b\}^+\right|+\left|\{L_i^b\}^-\right|},\ p^+,p^-\geq 0,\ p^++p^-=1,$$

$$\forall k, PL_k\subset\{PL_i\}, \exists\phi: PL_k\rightarrow\left(p_k^+,p_k^-\right).$$

Selecting the direction correction, the following options are possible:

1. if $p_k^+\approx p_k^-$ – the current direction is optimal;
2. if $p_k^+\approx 1$ – there is a new optimal direction;
3. if $p_k^-\approx 1$ – defines the worst direction to which the perpendicular plane sets the initial approximation of the optimization direction by its normal;
4. dividing the swarm due to the polymodality of *opt* by $\{L_i\}$, i.e. each extremum of *opt* by $\{L_i\}$ will make its own correction in the direction of movement of the sub-swarm belonging to this swarm. In particular, this is how the situation of the swarm flowing around an obstacle or the process of dividing into parts is modeled.

The division of swarm particles into a group of leaders $\{L_k\}$ and the remaining particles $\{P_i\}$ is dynamic in the sense that the following transformations are possible:

- $P_i\rightarrow L_k$ – in this case, the particle becomes a boundary particle and a leader in the direction of the swarm movement;
- $L_k\rightarrow P_i$ – in this case, the particle acquires an internal neighborhood of the swarm and ceases to be a boundary particle;
- $P_i\rightarrow L_k\rightarrow P_i$ – the transformation is based on the two transformations described above;
- $L_k\rightarrow P_i\rightarrow L_k$ – the transformation is described similarly to the previous point.

In the transformations described above, the delayed response value Δr In the transformations described above, the delayed response value $\{P_i\}$ to:

- $\{P_i\}\leftrightarrow\{P_j\}$ – describes the convergence and divergence of particle P_i relative to particles P_j in terms of neighborhood or adjacency;
- P_j describes the reaction of P_i to the presence of an obstacle or restriction in the direction of movement relative to the direction of movement of the particle P_i within the swarm;
- external control signals.

The factor Δr is crucial for ensuring the dynamic stability of the elite swarm $\{L_k\}$ in general. In particular, for the problems:

- definition (identification) of the situation of loss of a particle P_i discretely falling out of the swarm $\{L_k\}$ due to loss of dependence on it;
- determining the shape of the swarm neighborhood;
- definition of the codependency function as the degree of connectivity of the swarm $\{L_k\}$, i.e. when P_i is removed or distanced, other P_j change the direction of movement to the direction of movement of P_i and to what extent;
- determination of the conditions for the adhesion of a free particle to a swarm or the departure of a particle P_i from the swarm;
- definition of the procedure for restoring the integrity of the swarm $\{P_j\}$ at loss of particle P_i.

In addition to the factor Δr the goal-setting of the swarm movement $\{P_j\}$, namely how it is carried out, plays an essential role:

- based on the correction of the given direction of movement relative to the given reference trajectory;
- at given start and end points of the trajectory;
- adaptively due to correction of swarm movement from outside.

We will represent the division of the swarm into parts based on the graph G as a forest on SG. We will represent the division of the swarm with subsequent restoration of the initial swarm (flow) based on the graph G as SG having a cycle.

We will represent the flow of the swarm around obstacles based on the graph G as the absence of a change in the structure of SG .

We will represent the merging of swarms based on their schemas-graphs G_i as the formation of a metaschema over the schemas SG_i.

The restoration of swarm integrity depends on the position of the failed particle:

- If a boundary particle drops out, the following particle shifts forward, and others adjust accordingly.
- If an internal particle drops out, a neighboring particle with the closest velocity vector to the swarm direction replaces it with minimal disturbance.

This mechanism ensures the swarm's self-healing and survivability.

Furthermore, due to the discrete structure and reaction delay Δr, elite particles can overtake the swarm tail and bypass obstacles. Swarm structure is maintained through stratification, elite preservation, and selection based on maximum utility, overcoming evolutionary constraints.

The swarm structure is formed through stratification, elite particle generation, and utility-based selection, ensuring elite preservation beyond evolutionary constraints.

The swarm exhibits emergent properties similar to a neural network, demonstrating cognitive behavior via neuroplasticity and adaptive activation.

Let us define a "spider" object for a swarm of particles $\{P_i\}_{i=\overline{1,n}}$:

$$Sp(P_0(t)) = \left\{ P_i(t) \left| \begin{array}{c} \|P_i(t)-P_0(t)\|_{f_1} \le \varepsilon_1 \wedge \|P_i(t)-P_0(t)\|_{f_1} \ge \varepsilon_0 \\ P_i(t) - neighbours\, P_0(t), \exists\, O_{\varepsilon 3}(P_0(t)) \vee P_i(t) \in O_{\varepsilon 3}(P_0(t)), \\ V(P_i(t)) \uparrow \vee V(P_{0\Delta}(t)) \end{array} \right. \right\}$$

Characteristics of spider topology (homogeneous/heterogeneous particles):

1. Number of elements or weight of the spider – $n(Sp)$;
2. center of gravity of the spider – $W(Sp)$;

3. boundary of the minimal region encompassing the spider – $\partial(Sp)$;
4. particles forming the spider and not belonging to the boundary – $Int(p)$;
5. the spider is flat if all its points belong to the hyperplane $Pl(Sp)$;
6. asymmetry of the spider with respect to the geometric center ($As(Sp)$) and with respect to the center of gravity $As_W(Sp)$;
7. spider velocity – $V(Sp)$;
8. divergence of spider particle velocity – $\mathrm{div}(Sp)$;
9. cone of particle velocities P_i – $Con(P_i)$
10. consistency of particle velocities within the cone (distribution law) – $F(P_i, Con)$
11. swarm particle loadings (utility function) – $Wet(P_i)$
12. swarm loadings – $Wet(\{P_i\}_{i=\overline{1,n}})$

The following has been determined:

1. $\forall P_i(t) \exists Sp^1(P_i(t))$ – "spider" of particle $P_i(t)$, a discrete local neighborhood of the particle consisting of the swarm particles $\{P_i(t)\}_{i=\overline{1,n}}$, closest to particle $P_i(t)$ and defined by a radially expanding sphere centered at the coordinates of particle $P_i(t)$;
2. $\{P_i(t)\}_{i=\overline{1,n}} = U_i Sp^1(P_i(t))$ – swarm coverage;
3. $U_j Sp^1(P_i(t)) = Sp^2(P_i(t))$ – “web” of the swarm $\{P_i(t)\}_{i=\overline{1,n}}$,

$Sp_j^2(\{P_i(t)\}_{i=\overline{1,n}})$ – j -th subnet of the swarm $\{P_i(t)\}_{i=\overline{1,n}}$;p

$Sp^2(\{P_i(t)\}_{i=\overline{1,n}})$ – swarm network $\{P_i(t)\}_{i=\overline{1,n}} \overset{\text{def}}{\Leftrightarrow} Sp^2(\{P_i(t)\}_{i=\overline{1,n}}) = U_j Sp_j^2(\{P_j(t)\})$;

$Sp_j^2(\{P_i(t)\}_{i=\overline{1,n}}) \cap Sp_k^2(\{P_i(t)\}_{i=\overline{1,n}}) = \varnothing, j \neq k$;

4. $Sp^3(\{P_i(t)\}_{i=\overline{1,n}})$ – swarm network $\{P_i(t)\}_{i=\overline{1,n}}$;

$\overset{\text{def}}{\Leftrightarrow} \forall i, \forall Sp_i^3 \exists P_i(t) \in \{P_i(t)\} | Sp_i^3(t) = \Pi\mathrm{p}(P_i(t))$ – projection $Es(t)$.

For a set of spiders, the following operations are defined: formation, fission, growth, compression, intersection, merging, swarm creation, and web formation.

The web is characterized by:

- tensile strength (information integrity),
- compressive strength (maximum swarm density),
- permissible deformations (growth, shrinkage, rupture),
- swarm and spider density.

UAV swarm control is implemented as a decentralized system using collective strategies, where each UAV adjusts its actions via shared communication to achieve a common goal based on environmental feedback.

Decentralized swarms offer high reliability and adaptability, compensating for UAV loss, communication failures, and environmental challenges. Their emergent behavior results from cooperative interactions and spatial synchronization, leading to a synergistic, coherent system.

The use of persistent homology in graph-based image analysis enhances object shape recognition, supports diffeomorphic mappings with structural variations, and provides robustness against image modifications.In a swarm of particles, we can distinguish a group of particle leaders $\{L_k\} \subset \{P_i\}$ and a group of particles forming the middle part of the genome $\{M_l\} \subset \{P_i\}$, and the following is defined:

- at the leading edge of the swarm, the swarm leaders;
- instantaneous directionality of swarm motion;

- coefficients of considering the coordination of the directions of motion of particles in the swarm;
- movement of the rearguard in the swarm;
- "compression" of the swarm during the motion;
- increasing utility function or fitness functions for internal particles of the swarm;
- non-increasing penalty functions for border particles of the swarm.

To the leader particles $\{L_k\}$ we will refer those particles for which the following conditions are satisfied:

1. $grad\{L_k\}$ is externally oriented;
2. The middle part $\{M_l\}$ of the genome, is oriented on the population $\{L_k\}$ i.e. $\forall l, \forall M_l, \exists k, \exists L_k \, grad\, M_l \subset Ok(grad\, L_k)$;
3. Swarm motion direction $\{L_k\}$ is a bundle of directions for which the following condition is satisfied:
   $\forall k\, grad\, L_k : \cap Ok(grad\, L_k) \neq \emptyset,\ \forall Ok(grad\, L_k) \exists Ok(gra\, d^*\{L_k\})$;
4. For the structure of the neighborhood structure of the swarm direction vector $Ok(grad\, L_k)$, $\forall L_k$ the following conditions are satisfied:

- grad $L_k + \bar{\varepsilon}_k$, where $\bar{\varepsilon}_k$ – s a vector in the parameter space $\{L_k\}$, $|\bar{\varepsilon}_k| < \varepsilon_0$ – is the allowed particle spread;
- $(grad\, L_k, \bar{\varepsilon}_k) \geq 0$, where $\alpha(L_k)$ – is the scatter angle of directional change: $\alpha(L_k) = \arccos\left(\frac{(grad\, L_k, \bar{\varepsilon}_k)}{|grad\, L_k| \cdot |\bar{\varepsilon}_k|}\right)$;
- $\alpha^* = \max_{L_k}(\alpha(L_k))$ – swarm maneuver angle.

Exist:

- function $fit(P_i)$ and $fit(\{P_i\})$ – are fitness functions (measures) $fit(P_i(t)) = \begin{cases} \geq 0, t \in [0, t_i], \\ 0, t \in (t_i, \infty] \end{cases}$;
- function $plt(P_i)$ and $plt(\{P_i\})$ – are penalty functions (measures) $plt(P_i(t)) = \begin{cases} \geq 0, t \in [0, t_i], \\ 0, t \in (t_i, \infty] \end{cases}$;
- function $ext(P_i)$ and $ext(\{P_i\})$ – are existential functions (switches) $ext(P_i(t)) = \begin{cases} 1, t \in [0, t_i) \\ 0, t \in [t_i, \infty) \end{cases}$;
- swarm functioning quality structure: $(fit(\{P_i\}), plt(\{P_i\}), ext(\{P_i\}), n(t))$.

Functions $fit(\cdot)$, $plt(\cdot)$, $ext(\cdot) \in C[0, t_i]$ – to the space of integrable functions

grd – is an estimate of the state of the particle:

- $grd(P_i(t)) = \frac{fit(P_i(t))}{a + plt(P_i(t))}$ – at time t;
- $grdt(P_i(t)) \overset{\text{def}}{=} \int_0^t grd(P_i(\tau)) d\tau,\ t \leq t_i$ – on the interval $[0, t]$ (accounting for prehistory);
- $grd(\{P_i\}_{i=\overline{1,n}}) = \sum_{i=\overline{1,n(t)}} grd(P_i(t))$, where $n(t)$ – is the number of particles for which $ext(P_i(t)) \neq 0$ – is at time t;
- $grdt(\{P_i\}_{i=\overline{1,n}}) = grd(\{P_i(\tau)\}_{i=\overline{1,n}}) d\tau,\ t \leq \min t_i$

UAV-based image localization is achieved by comparing hash-functions derived from graph-structured color atlas representations (GSCA).

The temporal knowledge base (TKB) for swarm particles is built upon discrete geometry and photogrammetry, incorporating hierarchical concepts and semantic relations. It is refined through compositional classifications and object–relation diagrams, forming a structured methodology for developing and maintaining the swarm's TKB.

Let's denote the image as IMG, then the partition

$IMG_j \equiv \bigoplus_{i=1}^{n_j} Ob_{ji} \, IMG_j \equiv \bigoplus_{i=1}^{n_j} Ob_{ji}$ – direct algebraic sum of areas.

Intersection of areas $IMG_{jm} \cap IMG_{jl} = \Delta IMG \neq \emptyset \; (m \neq l) \Leftrightarrow$

1. $\forall Ob_k \in \Delta IMG \Rightarrow \{Ob_k \in \{Ob_i\}_l\} \wedge \{Ob_k \in \{Ob_i\}_m\}$
2. $\exists S_n$ – repainting of IMGl or IMGm vertices on the color wheel with preservation of color and area contiguity;
3. in the presence of motion (d) we define the shift of IMGl relative to the recorder to IMGm as ∆IMG = d(IMGl) and GSCA from ∆IMG ∈ { GSCA (IMGl) ∧ GSCA (IMGm)}.

Then ΔIMG_l^T – is a monitoring invariant with respect to the trajectory T, where

$$\forall i = \overline{0, n_l^T} : \{\Delta IMG_l^T \neq \emptyset\} \wedge \{\Delta IMG_l^T \in d_{li}(\ldots d_l(IMG_l)\ldots)\}.$$

Image alignment along a UAV trajectory is achieved by comparing the viewing angles of the camera's optical axis, using diffeomorphisms of the color atlas. Pitch, yaw, and roll transformations lead to image scaling, potentially removing or adding fragments to the camera's field of view.

Graph-based comparison of GSCA provides a structural similarity measure between images. Frame overlaps maintain monitoring continuity, enabling topographic reconstruction similar to lidar through tracked camera motion.

The color atlas enables modeling camera displacement as a collinear motion of region centers and supports the identification of invariant area features across image sets, preserving proportional relationships despite rotation or scale changes.Let us introduce the notations:

- T(t_i) - image transformation due to pitch at time t_i;
- P(t_i) - image transformation due to yaw;
- K(t_i) - image transformation due to roll of the webcam frame;
- {t_i }_(i=¯(0,N)), t_i<t_j, i<j - moments of image fixation, and the exposure time ∆t is determined from the conditions of image clarity for a given web-camera and linear speed of its movement in R^3, and ensuring commutativity of T, P, K transformation for all eight possible combinations of 3-term sequences.

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- $\{t_i\}_{i=\overline{0,N}}$, $t_i < t_j, i < j$ – moments of image fixation, and the exposure time Δt is determined from the conditions of image clarity for a given web-camera and linear speed of its movement in R^3, and ensuring commutativity of T, P, K transformation for all eight possible combinations of 3-term sequences. Let us denote by $Dr_j, j = \overline{1,8}$ these transformation triples, $\mathrm{Or}(t_i)$ – the orientation of the webcam at time t_i. Then: $\forall i, \exists j : Or(t_{i+1}) = Dr_j(Or(t_i))$.

Topological relationships between image regions are invariant to transformations and do not depend on specific coordinate systems, making them robust under diffeomorphisms. The pHash algorithm applies this robustness by comparing images through perceptual hashing within the GSCA framework, enabling similarity detection based on non-geometric characteristics.

Perceptual hashing is widely used in biometric authentication and image-based information retrieval, particularly for systems handling fuzzy or graphical data. Incorporating persistent homology into GSCA enhances shape analysis by identifying topological invariants across large datasets. This enables diffeomorphic mapping with structural variation, computes similarity measures, and provides resilience against image modifications.Next, we classify the problems of local positioning of a swarm of particles. Let us introduce the following definitions at the expense of diagrams (Fig.1):

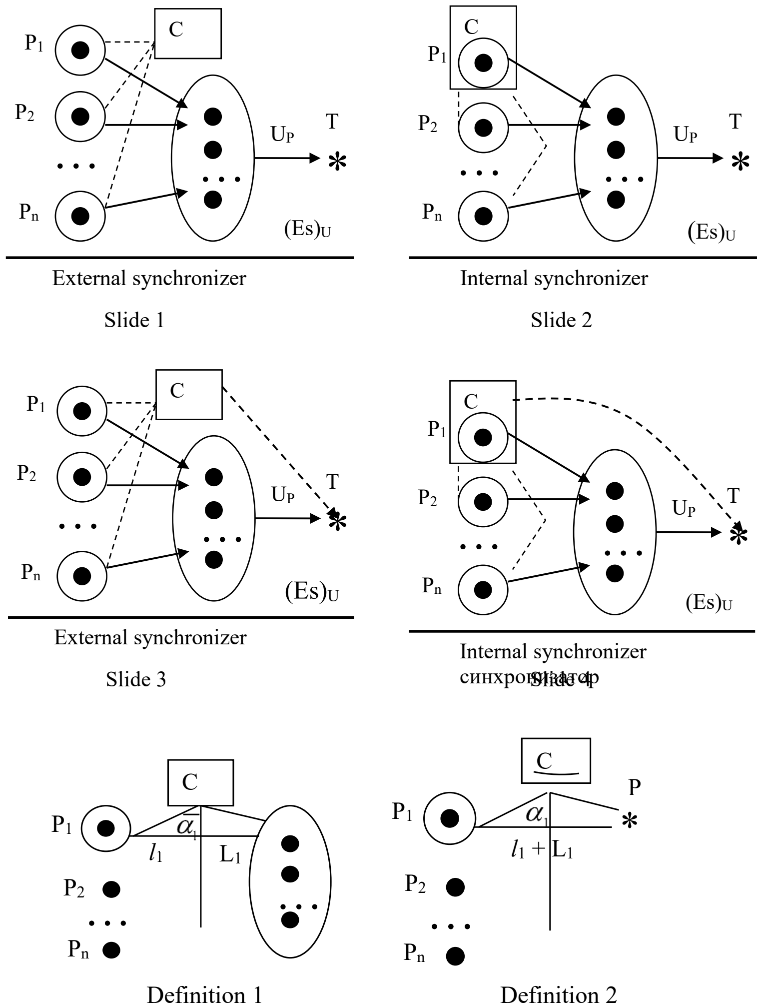

Figure 1: Definitions introduced using diagrams.

The classification of local particle swarm positioning problems contains two problems:

1) creating a group
2) guiding the swarm to the target

Let t^f – be the moment of reaching the goal, t_i^f – be the initial moment of observation.

$\angle\alpha_i, l_i, \bar{v}_i, t_i^f$ are known. The following options are possible:

1.1 $\forall i,\ \exists \lim_{t\to\infty} (\angle\alpha_i, L_i) = (\angle\alpha_0, L_0)$;

1.2 $\forall i,\ \exists \lim_{t\to t_0} (\angle\alpha_i, L_i) = (\angle\alpha_0, L_0)$;

1.3 $\forall i,\ \exists \lim_{t\to\infty} (\angle\alpha_i, L_i) = (\angle\alpha_0, L_0),\ \forall i,\ t_i^f \neq t^f$;

1.4 $\forall i,\ \exists \lim_{t\to t_0} (\angle\alpha_i, L_i) = (\angle\alpha_0, L_0),\ \forall i,\ t_i^f \neq t^f$;

2.1 $\angle\alpha_i \to 0$; $t_i \to \infty$; $\forall i,\ t_i^f = t^f$;

2.2 $\angle\alpha_i \to 0$; $t_i \to [t_0-\varepsilon, t_0+\varepsilon]$; $\forall i,\ t_i^f = t^f$;

2.3 $\angle\alpha_i \to 0$; $t_i \to \infty$; $\forall i,\ t_i^f \neq t^f$;

2.4 $\angle\alpha_i \to 0$; $t_i \to [t_0-\varepsilon, t_0+\varepsilon]$; $\forall i,\ t_i^f \neq t^f$.

Classification of local positioning problems of a particle swarm by taking into account connections:

- tasks due to inertia of swarm particles (accounting of prehistory);
- tasks due to cognitive (local maxima of trends of average particles);
- tasks at the expense of swarm socialization (trends of averages).

Generalized tasks of local positioning of a swarm of particles:

- swarm $\{P_i(t)\}_{i=\overline{1,n}} \to \text{web } Sp^2(\{P_i(t)\}_{i=\overline{1,n}})$;
- web $Sp^2(\{P_i(t)\}_{i=\overline{1,n}}) \to Sp^3(\{P_i(t)\}_{i=\overline{1,n}})$.

5. Conclusions

This study addresses coordinated swarm control of UAVs using a leader-based spatial motion model. A method for processing UAV-acquired image sequences via GSCA ensures robust and efficient extraction of invariant features for dynamic scene analysis.

Leader guidance enables UAVs to align with swarm trajectories, maintaining formation and coordinated motion. The paper reviews current and prospective UAV interaction methods for task execution within a swarm.

Scientific novelty:

1. Developed a comprehensive mathematical model for swarm particle local positioning with genetic modification during operation.
2. Demonstrated the effectiveness of RSO-based swarm algorithms with adaptive heuristic parameters for diverse optimization tasks.

Theoretical significance: A formalized model for applying swarm intelligence algorithms in UAV navigation has been introduced.

Practical significance: Results may support:

1. Alternative satellite-independent navigation systems;
2. Enhanced UAV swarm positioning using ephemeris and almanac data;
3. Integration of logical and heuristic rules into GSCA-based search procedures for improved reliability.

Discussion: The proposed topological, color-atlas-based digital image processing significantly reduces computational load compared to geometric methods, while retaining essential structural information.

Declaration on Generative AI

During the preparation of this work, the authors used GPT-5 in order to Grammar and Spelling check. After using this service, the authors reviewed and edited the content as needed and take full responsibility for the publication’s content.

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