Reduction of dynamic problems in game theory: applications for IT project management

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Abstract

In many cases after static consideration of a mathematical model for conflict situation description as a matrix game in the normal representation form, we encounter a need to consider this game already as a dynamic game in the extended recording form. The players interact between each other in time and communicate their decisions on their own behavior depending on the information about the decisions the other players take or will take, which begins to constitute a game model hierarchic structure (sequence of moves). In other words, taking a decision on one's own behavior, each player can have information about the decisions taken by other participants. Should account be taken of this possibility, the game and theory model changes the form of its representation from static to dynamic. As static games have been deeper researched, it becomes reasonable to reduce the solution of an extended dynamic problem to a static matrix game in the normal form of representation.

Keywords
game theory, dynamic problem, reduction, normal representation, matrix game, Nash equilibrium.

1. Introduction

The games are generally to be considered including interaction of players in time. Unlike static games, where the actions are predetermined and analyzed independently, in dynamic games players interact with each other sequentially and adapt their behavior depending on the strategies previously chosen by others [1, 2, 7]. Therefore, a player can select an individual behavior strategy based on available information about the decisions of the other participants. This type of games is normally called dynamic games with perfect information. Such games are typically solved using backward induction methods or the concept of Nash equilibrium, often represented graphically as a game tree [1, 4].

For utilizing the Nash equilibrium concept, it is required to represent a dynamic game in normal form. The description of a normal-form game includes three key components: the set of players, the set of pure strategies available to each player, and the payoff matrices of the players [2, 4]. Thus, both extended and normal forms of games operate with the same participants, but differ in the way strategies and outcomes are structured and interpreted.

In the context of information technologies, these theoretical models gain practical importance. Modern IT project teams often operate in highly dynamic environments, where decisions made by one developer or manager directly influence the choices and productivity of others. For instance, in software engineering projects, the sequence of design, coding, and integration tasks can be seen as a chain of strategic moves with partial information about potential responses of colleagues [8, 9, 11].

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Applying game-theoretic approaches, particularly the reduction of dynamic problems to static matrix games, makes it possible to formalize these interactions, forecast conflicts, and identify optimal collaborative strategies. This not only supports rational decision-making but also enhances the efficiency of IT project management by providing mathematical tools for resolving strategic disagreements, optimizing teamwork, and predicting long-term project outcomes [8, 10, 12].

2. The problem reduction breakdown

The plan of full behavior is to be combinatorially full, i.e. it has to contain information on all possible actions in the game process [3, 4]. The player's actions plan is to be so full to avoid any doubts that the real behavior would be exactly as planned.

In the dynamic variant of game with perfect information, each player communicates the selected behavior strategy before the game beginning. The game process runs and is controlled under this plan, which enables moving to static games [2].

This is the way of moving to the hierarchic variant of a matrix game. Following the game rules, the known chains of the players' behavior strategies are executed step by step. The players make their moves in compliance with these chains. When the sequence of moves brings to the final apex of the chain, the players receive their respective reward. This approach enables interpreting a dynamic game as a static one [3].

The described algorithm will enable considering the extended form of a dynamic game with perfect information as a game in normal form [6].

In the beginning, we solve a model problem in normal form, and then we consider its dynamic variant [5, 6].

Model example

A team created for IT project implementation frequently encounters a situation when two team members have different views of the approaches to creating the final product. In this case, each of them has a significant experience in developing similar software products or services in different teams. For efficient solution of this situation, we propose to use the approaches attributable to matrix games, interpreting each of these team members as a player.

Using the concept of the first team member for creating a software product can be positively evaluated as $r(r>0)$ - units, and the proposal of the second team member is evaluated as $t(t>0)$ - of some conditional units.

Let us adopt that the conditional benefit of using other proposals is zero. The team of developers receives an additional positive benefit $s\ (s>0)$ in case of using the proposal of the first player enabling the usage of the experience acquired by the team of developers who worked at the same company [4]. For taking the optimum decision bringing the biggest benefit, let us use the following calculations.

We have a typical matrix game. The first player A – submitted the first proposal. The second player B - submitted the second proposal. Each of the players has two strategies – to use the first proposal A_1, B_1 or the second one - A_2, B_2. Therefore - $A=[A_1,A_2]$, $B=[B_1,B_2]$.

As per the game statement, the payment matrixes for the players:

$$C_A=\begin{bmatrix} r & r+s \\ s & 0 \end{bmatrix},\quad C_B=\begin{bmatrix} t & s \\ t+s & 0 \end{bmatrix}.$$

Static game

In case of multiple repetition of the game with the same statement, player A shall use own strategies $\{A_1,A_2,\dots,A_n\}$ with respective frequencies $\{p_1,p_2,\dots,p_n\}$, $\sum_{i=1}^{n} p_i=1$. Transition to mixed strategies is effected via determination of frequencies of using pure strategies for the first player $X_A = [p_1, p_2, \ldots, p_n]$. Player B shall use the frequencies $X_B = [q_1, q_2, \ldots, q_n]$, $\sum_{i=1}^{n} q_i = 1$ of own strategies $\{B_1, B_2, \ldots, B_n\}$.

The mixed strategies enabled extending the problems of matrix games and finding possibilities of more substantiated distribution of wins in the game as average values – mathematic expectations set by matrix distributions C_A and C_B, and calculating under the formulas:

$$M_A(X_A, X_B) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} p_i q_j,$$

$$M_B(X_A, X_B) = \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} p_i q_j.$$

The status (X_A^*, X_B^*) in the matrix game system given by frequencies $X_A^* = [p_1^*, p_2^*, \ldots, p_n^*]$ and $X_B^* = [q_1^*, q_2^*, \ldots, q_n^*]$ is called Nash equilibrium in mixed strategies if the following conditions are met for any of X_A and X_B:

$$M_A(X_A^*, X_B^*) \geq M_A(X_A, X_B^*),$$

$$M_B(X_A^*, X_B^*) \geq M_B(X_A^*, X_B).$$

These conditions of a matrix game equilibrium status are interpreted as follows: a deviation from the system status (X_A^*, X_B^*) by one of the players, subject to keeping own selection by the other, does not provide the said player with the possibility of increasing the win. In other words, an alteration of the equilibrium status in the matrix game system is not reasonable for either of the players.

Let the player A have pure strategies $\{A_1, A_2, \ldots, A_n\}$. Each of them is correlated with the vector of frequencies X_{A_i}, $i = 1, 2, \ldots, n$. Thus, the pure strategy A_1 corresponds to the frequency vector $X_{A_1} = [1, 0, \ldots, 0]$. Similarly for the second player B - $X_{B_1} = [1, 0, \ldots, 0]$. Therefore, each of the players' pure strategies corresponds to the frequency vectors X_{A_i} and X_{B_i}, $i = 1, 2, \ldots, n$. In this case, the system of conditions describing the point (X_A^*, X_B^*) of the matrix game equilibrium can be recorded as follows:

$$\begin{cases} M_A(X_A^*, X_B^*) - M_A(X_{A_1}, X_B^*) \geq 0, \\ M_A(X_A^*, X_B^*) - M_A(X_{A_2}, X_B^*) \geq 0, \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ M_A(X_A^*, X_B^*) - M_A(X_{A_n}, X_B^*) \geq 0, \\ M_B(X_A^*, X_B^*) - M_B(X_A^*, X_{B_1}) \geq 0, \\ M_B(X_A^*, X_B^*) - M_B(X_A^*, X_{B_2}) \geq 0, \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ M_B(X_A^*, X_B^*) - M_B(X_A^*, X_{B_n}) \geq 0. \end{cases} \tag{1}$$

It is worth mentioning that the signs of ratios between the parts of the system conditions (1) can be clarified following the principle of ultimates. Thus, if the component $p_i^* > 0$ at the equilibrium point $\left(X_A^*, X_B^*\right)$, the respective inequality is executed as an equation (so-called active correlation)

$$M_A\left(X_A^*, X_B^*\right) - M_A\left(X_{A_i}, X_B^*\right) = 0,$$

and in case when $q_j^* > 0$ - we have the equality

$$M_B\left(X_A^*, X_B^*\right) - M_B\left(X_A^*, X_{B_j}\right) = 0.$$

Let $\left(X_A^*, X_B^*\right)$ be the equilibrium point in mixed strategies of a matrix game. In this case, the statement of the so-called passive correlation

$$M_A\left(X_A^*, X_B^*\right) - M_A\left(X_{A_i}, X_B^*\right) > 0$$

results in $p_i^* = 0$, and the statement

$$M_B\left(X_A^*, X_B^*\right) - M_B\left(X_A^*, X_{B_j}\right) > 0$$

results in $q_j^* = 0$.

The system of statements (1) enables the matrix game analysis for finding the equilibrium points in mixed strategies in all cases, as the following statement is being confirmed: any matrix game always has at least one equilibrium point in mixed strategies.

Let $X_A = [p_1, p_2] = [p, 1-p]$, $X_B = [q_1, q_2] = [q, 1-q]$ be the mixed strategies of the players' behavior.

In this case, the system of statements describing the point $\left(X_A^*, X_B^*\right)$ of the matrix game equilibrium can be recorded in the form (1). The average values of the players' wins will be:

$$M_A\left(X_A, X_B\right) = \sum_{i=1}^{2}\sum_{j=1}^{2} a_{ij} p_i q_j = (r + s - 2qs)p + sq,$$

$$M_B\left(X_A, X_B\right) = \sum_{i=1}^{2}\sum_{j=1}^{2} b_{ij} p_i q_j = (s + t - 2ps)q + sp.$$

The system (1) of the game equilibrium description looks as follows:

$$\begin{cases} (r + s - 2qs)(1 - p) \le 0, \\ (2qs - r - s)p \le 0, \\ (s + t - 2ps)(1 - q) \le 0, \\ (2ps - t - s)q \le 0. \end{cases} \tag{2}$$

Solving the system, we obtain the frequencies of strategies utilization by the players and their respective wins:

$$X_A = \left[\frac{t+s}{2s}, \frac{s-t}{2s}\right], \qquad \upsilon_x^A = M_A\left(X_A, X_B\right) = \frac{r+s}{2},$$

$$X_B = \left[\frac{r+s}{2s}, \frac{s-r}{2s}\right], \quad \upsilon_x^B = M_B(X_A, X_B) = \frac{s+t}{2}.$$

The existence of equilibrium in pure strategies of a matrix game shall not exclude the existence of equilibrium in mixed strategies. Let the additional benefit from joint utilization of the players' concepts be bigger than the benefit from utilization of one's own concept only, i.e. $s > r$ and $s > t$.

In this case, we have the first player's feedback

$$p = \begin{cases} 0, & q < \frac{t+s}{2s}, \\ [0,1], & q = \frac{t+s}{2s}, \\ 1, & q > \frac{t+s}{2s}. \end{cases}$$

The second player will have the response

$$q = \begin{cases} 0, & p < \frac{r+s}{2s}, \\ [0,1], & p = \frac{r+s}{2s}, \\ 1, & p > \frac{r+s}{2s}. \end{cases}$$

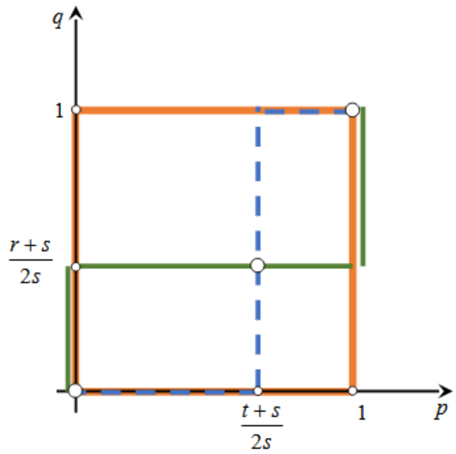

Figure 1: Three equilibrium points: two in pure strategies and one in mixed ones.

Let us graphically represent the players' feedback on the square $\{p, q \mid p \in [0,1], q \in [0,1]\}$ (Fig. 1.). Fig. 1 shows the presence of three equilibrium points two if which are implemented in pure strategies (A_1, B_2) with game prices $\upsilon_1^A = r+s$ and $\upsilon_1^B = s$ and (A_2, B_1) with prices $\upsilon_2^A = s$ and $\upsilon_2^B = t+s$ plus one point $\left(\frac{t+s}{2s}, \frac{r+s}{2s}\right)$ in mixed strategies for the first player with frequency vector $X_A = \left[\frac{t+s}{2s}, \frac{s-t}{2s}\right]$ and price $\upsilon_x^A = \frac{r+s}{2}$ and for the second player - $X_B = \left[\frac{r+s}{2s}, \frac{s-r}{2s}\right]$, $\upsilon_x^B = \frac{s+t}{2}$.

Dynamic variant of a model game

A dynamic variant of model problem contains a prerequisite – the first player A makes a move first.

The dynamic game variant tree is shown on Fig. 2.

The tree apexes are numbered for suitability of pure strategies designation. The first player A has two strategies in this game $A=[(1:A_1),(1:A_2)]$ at apex 1. The second player B has four strategies at two apexes 2 and 3. Each of the second player's strategy already defines action at two apexes. The second player has the following strategies

$$B=[(2:B_1,3:B_1),(2:B_1,3:B_2),(2:B_2,3:B_1),(2:B_2,3:B_2)]$$

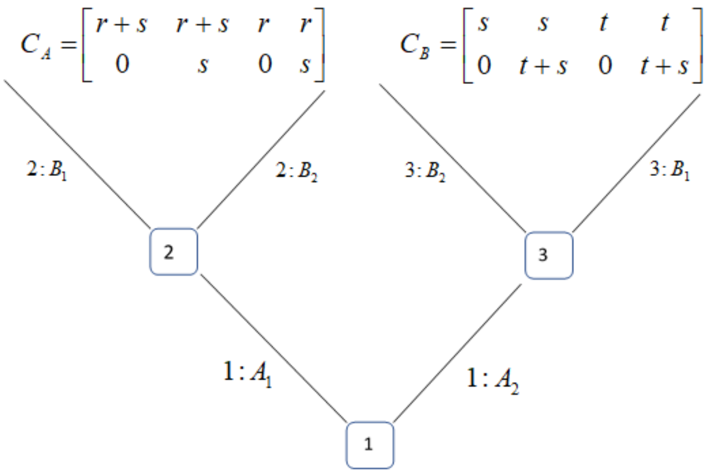

Figure 2: Dynamic game variant tree.

In the normal form of the dynamic game variant representation, the players' payment matrixes look as follows:

$$C_A=\begin{bmatrix} r+s & r+s & r & r \\ 0 & s & 0 & s \end{bmatrix}, \quad C_B=\begin{bmatrix} s & s & t & t \\ 0 & t+s & 0 & t+s \end{bmatrix}.$$

The normal form of the dynamic game variant is more complicated than the normal form of a static game. In a game with four pure strategies, the second player will have sixteen pure strategies.

Let us solve the game subject to $r<s$ and $t<s$. We check the new matrix game for Nash equilibrium. For payment matrix C_A, we select the maximum values in the columns (outlined with ovals)

$$C_A=\begin{bmatrix} r+s & r+s & r & r \\ 0 & s & 0 & s \end{bmatrix}.$$

For payment matrix C_B, we select the maximum values in the rows (outlined with ovals)

$$C_B=\begin{bmatrix} s & s & t & t \\ 0 & t+s & 0 & t+s \end{bmatrix}.$$

The agreement of maximum values separates the Nash equilibrium points. Therefore, we have three Nash equilibrium points: (1,1); (1,2); (2,4).

Let us analyze the correspondence of the solution obtained and the backward induction method.

As per the backward induction procedure, the second player *B* selects the move $2:B_1$ at the tree apex 2 following the game statement as $s<r$, and the said player will select $3:B_2$ at the apex 3 as $r+s>0$. In the dynamic game variant, the first player is to make a choice between the wins $r+s>0$ and s. The first player selects the bigger one - $1:A_1$. Therefore, the backward induction renders the following game solution: The first player selects the optimum strategy $1:A_1$; The second player selects the optimum strategy $(2:B_1,3:B_2)$.

Previously, it was ascertained that the game has three Nash equilibrium points: (1,1); (1,2); (2,4). Only one of them matches the solution rendered by the backward induction method. Solutions obtained by the backward induction always constitute Nash equilibrium statuses.

3. Research methods and results obtained

In a game with perfect information and a finite number of moves, the solution obtained by the backward induction method is always the Nash equilibrium status.

Let us have a certain set of strategies, in fact. If we make our moves based on these strategies, each apex would correspond to one common trajectory or chain of moves connecting it to one of the finite apexes. An arbitrary apex can be compared with the single set of wins selecting it from that finite apex, which terminates its respective trajectory.

Let us assume that the set of strategies obtained by the backward induction method is not the Nash equilibrium. This means that there is a strategy for a certain player giving a bigger win. This new status corresponds to a new alternate game trajectory beginning from the initial apex. Let us consider this trajectory commencing from the final apex. At some of the apexes of this trajectory, the player's win is to be smaller than the win of the other strategy. The move at this apex belongs to an arbitrary player that has to make a choice. This contradicts the principle of player's reasonable behavior, which actually confirms the previous statement.

The backward induction method cannot be used for obtaining all the Nash equilibriums. Let us consider the equilibrium $1:A_2$ and $(2:B_2,3:B_2)$. It can be interpreted as follows: the second player B creates a "danger" for the first player A as *B* would use own concepts in the case *A* does the same. Under the effect of this "danger", the first player selects the concept of the second player. However, this situation in the game contradicts the concept of player's reasonable behavior constituting the basis for the backward induction method. Actually, in case the second player gets to apex 2, this player would prefer the first player's concept. As the first player knows that the second is reasonable, the first would not trust the empty "danger". Therefore, the considered set of strategies is not the solution of the game. The situation $1:A_1$ and $(2:B_1,3:B_1)$ is examined in the same way.

Not all Nash equilibrium points in dynamic games can be derived through the method of backward induction. This indicates that some Nash equilibria are inconsistent with the assumption that players behave rationally at every stage of the game. The main difficulty arises from representing a dynamic game in normal form: the information about the sequence of moves is lost, and the players' strategies are assumed to be chosen once at the beginning of the game and remain unchanged afterward. Because of this simplification, the Nash equilibrium may describe strategy combinations that players would not realistically follow during the actual course of play.

Thus, the classical Nash equilibrium is not always sufficient for accurately predicting the outcome of dynamic games. To address this limitation, it becomes necessary to refine and strengthen the concept. If the game has already progressed through several steps, the remaining part of the game can be viewed as an independent subgame. In this new situation, players are free to revise their strategies and select different actions if the initial strategies turn out to be unprofitable. Their goal is to find new equilibrium strategies that none of the players would want to deviate from in the subsequent play.

A subgame is defined as a portion of the original game that begins at any non-terminal node and includes everything that follows from that point. If the subgame begins at the initial node of the original game, it is called a proper subgame.

There are three subgames in the model example: the first subgame is an initial game with three apexes 1, 2, 3 and two own subgames beginning at apexes 2 and 3.

Let us introduce the strengthened Nash equilibrium based on the dynamic agreement foundation.

Such a set of strategies is called perfect Nash equilibrium, which is a Nash equilibrium in the initial game, and the respective parts of this set of strategies are Nash equilibriums in own subgames of the game.

Let us analyze the execution of perfect Nash equilibrium in the model example. For this purpose, we separate the subgame beginning at apex 2 of the normal game representation form. Player A makes no choices in this subgame. Player B has two strategies B_1 and B_2. The payment matrixes for this subgame are:

$$C_A = \begin{bmatrix} r+s & r \end{bmatrix},\; C_B = \begin{bmatrix} s & t \end{bmatrix}.$$

We have a single Nash equilibrium at location (1,1). Therefore, to make the Nash equilibrium perfect in the initial game, it is required to select B_1 strategy at apex 2.

The other set of strategies $(1:A_1),(2:B_1,3:B_2)$ does not meet this requirement. That is why it cannot be the perfect equilibrium in the game. In the second own subgame beginning at apex 3, the second player B selects B_2 strategy in the Nash equilibrium. Thus, the set of strategies $(1:A_1),(2:B_1,3:B_1)$ is not the perfect equilibrium in subgames.

At the same time, the set of strategies $(1:A_1),(2:B_1,3:B_2)$ is a Nash equilibrium in the initial game and meets the equilibrium requirements in each own game. Therefore, this set of strategies is the perfect equilibrium.

4. Conclusions

The solution meets the solution obtained by the backward induction method. This agreement is not incidental because the set of solutions obtained by the backward induction method meets the set of perfect Nash equilibrium situations in the game with perfect information and finite number of moves.

As the normal form of the game can be too big in terms of the form of recording, utilization of this agreement essentially simplifies the search for the perfect equilibrium in subgames – there is no need for recording the game in normal form and finding any Nash equilibrium points.

Declaration on Generative AI

During the preparation of this work, the authors used Grammarly in order to: Grammar and spelling check. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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