A mixed-integer optimization model for robotic warehouse SKU allocation and order picking efficiency*

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Abstract

Efficient order picking in robotic warehouses is critical for fast and accurate fulfillment. It is strongly influenced by how stock keeping units (SKU) are allocated across picking zones. This paper proposes a mixed-integer linear programming model for structured SKU allocation in robotic warehouses. The model enforces a SKU family allocation in the picking zones. Allocation decisions respect picking zone capacities, allowable storage configurations, contiguity constraints, and supply limits, while maximizing overall picking utility. We demonstrate that the problem generalizes classical storage assignment and knapsack problems, proving its NP-hardness. The model provides a formal framework for optimizing warehouse layouts in intelligent robotic systems, enabling reduced travel distances, improved load balancing, and increased picking efficiency. The model enforces structural properties that are consistent with robotic routing efficiency principles.

Keywords

warehouse, robotics, order picking, SKU allocation, storage assignment, mixed-integer linear programming, combinatorial optimization

1. Introduction

The rapid growth of e-commerce and omnichannel distribution has significantly increased the operational complexity of modern automated warehouses [1]. Robotic storage and retrieval systems, autonomous mobile robots, and intelligent picking stations are now widely deployed to improve order fulfillment speed and accuracy. In such environments, the efficiency of order picking operations depends on routing algorithms, robot coordination, and spatial allocation of stock-keeping units (SKUs) within picking zones.

The storage assignment problem in robotic warehouses directly affects travel distance, congestion levels, workload balance, and order batching efficiency. Allocation of SKUs across picking zones is the main factor of warehousing.

The evolution of Automated Storage and Retrieval Systems (AS/RSs), which have been widely used in distribution and production environments since the 1950s. Typically, an AS/RS consists of storage racks served by cranes or shuttles that move along aisles between the racks, allowing automated storage and retrieval of products. AS/RSs improve efficiency and accuracy in warehousing operations by reducing manual handling and travel times [2].

In automated AS/RSs, purchase and inventory holding costs are independent of the storage location, and external replenishment processes are typically managed separately. Therefore, when assigning items to locations, the key factors are the labor and time savings during retrieval compared to manual procedures, and the cost to replenish stock from central storage, rather than purchase or holding costs [3].

Frequently co-picked items should be located in spatial proximity, while high-demand SKUs must be positioned in zones that minimize access time. Storage zones have strict capacity

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constraints in terms of horizontal space, depth, vertical stacking, and admissible storage configurations [4–6].

Storage allocation strategy should simultaneously account for operational constraints, demand structure, and system performance objectives. Therefore, SKU allocation in the warehouses remains a challenging combinatorial optimization problem.

Product allocation in warehouses can generally be divided into two categories: assigning products to different warehouse zones and assigning products to specific storage locations within those zones [7]. The first category encompasses studies on the forward-reserve problem [3,8] as well as broader product allocation models [9]. Facility location selection is considered a multi-criteria decision-making problem, where the goal is to determine the placement of one or more new facilities among a set of potential sites. The number of candidate locations must at least match the number of facilities to be located [9]. This classic framework has influenced modern approaches in logistics and supply chain network design.

Most existing approaches to warehouse storage optimization treat SKUs independently or rely on simple zoning heuristics based on turnover rate or ABC classification. However, real-world order data exhibit grouping structure: orders can be grouped according to customer behavior or product compatibility, or SKUs can be organized into families reflecting functional similarity, joint demand, or replenishment policies. Ignoring this structure often leads to fragmented layouts, increased robotic travel, and inefficient zone utilization.

This paper proposes a mixed-integer linear programming (MILP) model for structured SKU allocation in robotic picking systems. The model introduces a SKU family organization in which families represent groups of SKUs frequently demanded together or grouping of SKUs bases on some similarity factors. The family structure is enforced through contiguity constraints and tolerance parameters that regulate the spatial consistency of SKU families across multiple picking zones. This design enables the formation of structured storage blocks that reduce robot travel distances and support balanced load distribution.

The proposed formulation integrates:

- zone capacity constraints (horizontal, vertical, and depth),
- multi-zone allocation rules for SKUs,
- configuration-dependent storage feasibility,
- family consistency constraints,
- supply and minimum/maximum quantity requirements.

The objective is to maximize a system utility function reflecting picking efficiency, demand intensity, or operational priority.

From a computational perspective, the model generalizes classical storage assignment and bin packing problems by incorporating structural constraints and multi-zone consistency requirements. As a result, the problem belongs to the class of combinatorial optimization problems and inherits computational complexity characteristics of multidimensional packing formulations. The SKU family extensions significantly enlarge the feasible region structure while simultaneously introducing coupling constraints that prevent trivial decomposition.

The main contributions of this work are:

1. A generalized SKU family allocation framework for robotic warehouse systems.
2. A formal mixed-integer programming formulation incorporating contiguity and tolerance-based structural constraints.
3. A structural analysis of the model, including its relation to classical combinatorial optimization problems and its scalability properties.
4. A discussing of how hierarchical SKU organization can be systematically integrated into intelligent warehouse planning systems.

The proposed framework provides a foundation for advanced decision-support tools in automated logistics environments and can serve as a basis for future heuristic, metaheuristic, or data-driven solution approaches in intelligent warehouse management systems.

Section 2 provides the background, Section 3 presents the problem description, and Section 4 introduces the mathematical model. Section 5 analyzes the computational complexity of the proposed problem, followed by Section 6, which discusses the results. Finally, Section 7 concludes the study.

2. Background

Efficient warehouse operations have been widely studied in logistics, operations research, and intelligent systems. Research has focused on storage assignment, order picking optimization, and robotic warehouse automation.

Early work on storage assignment in warehouses primarily explored class-based and turnover-based allocation [7,10].

Rouwenhorst et al. (2000) presented a reference framework for warehouse design and control problems, reviewing existing literature through this lens. They highlighted that most studies focused on isolated, analytically well-defined subproblems rather than the synthesis of technical systems and planning procedures needed in actual warehouse design. The authors argued for more design-oriented research that integrated multiple methods and models to systematically address warehouse (re)design challenges, noting that creativity and analytical skills must be combined to solve these complex, real-world problems [10].

Gue and Meller (2009) challenged the traditional warehouse layout in which parallel picking aisles force rectilinear travel. They developed mathematical models for alternative aisle configurations, including piecewise diagonal cross aisles and non-parallel picking aisles, to reduce travel distance in single-command cycles [7].

The ABC classification assigns high-frequency SKUs to locations closer to the dispatch area, reducing picker travel time. Subsequent research introduced clustering-based approaches, grouping SKUs frequently picked together to minimize travel distance for batch orders [11–14]. The authors often investigated relocation closer to the access gate to reduce travel distance and improve operational efficiency.

By dividing items into classes using a Pareto 20/80 distribution, class-based storage ensures that the fast-moving class accounts for around 20% of the total inventory while generating over 80% of the turnover. ABC and COI-based class storing approaches are commonly utilized. According to the first technique, objects are categorized as A-, B-, or C-items, with A-items accounting for around 80% of the turnover. C-items are the remaining items, whereas B-items often fall between 80 and 95 percent [15–17].

Hanafi et al. (2019) investigated warehouse layout problems, a vehicle spare parts distributor, where poor layout organization and insufficient shelf spacing created difficulties in inventory handling and goods management. To address frequent overstocking at year-end due to oversupply, the researchers applied ABC classification analysis to redesign the facility layout based on product demand [18].

Ravinder and Misra (2016) argued that traditional single-criterion ABC classification is no longer sufficient in modern supply chains. Due to globalization, shorter product life cycles, and increasing customer responsiveness requirements, inventory classification should incorporate multiple criteria such as demand variability, lead time, criticality, and strategic importance. The study emphasized the need to place multi-criteria ABC analysis at the center of inventory management theory and textbooks to better reflect contemporary business realities [12].

Order picking optimization has also been extensively studied, particularly for manual warehouses. Mixed-integer programming models, metaheuristics, matheuristics and simulation-based methods have been applied to minimize travel distance, balance workload, and reduce order cycle times [19–25]

Order picking is perhaps the most expensive and labor-intensive process in a warehouse [26,27]. It entails obtaining the necessary products from storage in order to fulfill customer orders.

Order picking is one of the most critical and costly warehouse operations, often accounting for around 50% of total direct labor costs. Tompkins et al. (2010) emphasized that order picking directly influenced customer service performance, as it ensures that goods are selected accurately and delivered on time. Although many warehouses implement advanced information systems and automation in storage areas, case and unit picking operations largely remain manual but supported by technological aids [28]. Furthermore, according to some writers, 60–70% of warehousing expenses are related to order picking [29].

When analyzing the fundamental tasks involved in order picking, Tompkins et al. (2010) calculated that approximately half of the time is spent traversing the storage areas, 20% is spent looking for the item, 15% is spent actually selecting it and transferring it to the car or basket, and 15% is spent setting up and performing other tasks [28].

Some researchers investigated the integrated order batching, sequencing, and routing problem in warehouses, which combined three traditionally separate decision problems. Among them are how orders are batched, in what sequence the batches are processed, and how pickers are routed through the warehouse. The authors combined it into a single optimization framework. Recognizing that this integrated problem is computationally difficult (NP-Hard), the authors formulated a comprehensive nonlinear mixed-integer optimization model that simultaneously determined the best order batches, their processing sequence, and picker routes while minimizing total customer order tardiness [29].

De Koster et al. (2007) focused on integrating order batching and picking path optimization to minimize picking time and improve operational performance [26]. Rouwenhorst et al. (2000) justified the importance of integrated approaches to warehouse layout, operations, and design in modern logistics research [10].

Another important branch is the use of the order batching method in warehouse operations, where items stored in close proximity are grouped into the same batch to reduce picker travel distance [26]. The authors argued that effective batching improved staff utilization and reduced labor costs. In addition, the study emphasized that different picking sequences and routing strategies significantly affected travel distance and overall picking efficiency.

Recent advances in automated and robotic warehouse systems (e.g., autonomous mobile robots, shuttle systems) have shifted the focus toward dynamic storage allocation and layout optimization [2,3,30]. Several studies address robot routing, task assignment, and zone optimization, demonstrating that reducing travel distance and optimizing pick sequences are critical for performance [2,31–33].

Roodbergen and Vis (2009) provided a comprehensive review of AS/RSs over the past three decades, covering system configuration, travel time estimation, storage assignment, dwell-point location, and request sequencing. While most models in the literature focus on static scheduling and design problems, the authors highlighted the growing need for dynamic models that address real-time operations, finite planning horizons, and large computation times to improve system performance [2].

Negahban and Smith (2014) addressed the storage location assignment problem, which determined where incoming goods should be stored in a warehouse to optimize operational efficiency. Recognizing the problem's complexity, they proposed a hierarchical four-step storage location planning procedure: (1) distribution of products among warehousing systems, (2) clustering of correlated products, (3) balancing workloads within the systems, and (4) assignment of products to specific storage locations [32].

Warehouses are a critical component of supply chains, serving multiple functions: buffering material flows to accommodate variability such as seasonal demand or production batching, consolidating products from multiple suppliers for efficient customer delivery, and performing value-added operations like kitting, pricing, labeling, and product customization. These roles make warehouses pivotal not only for operational efficiency but also for enhancing customer service in modern logistics [34]

Most of warehouse storage approaches often rely on heuristic placement rules and do not formally model family relationships between SKUs, which can lead to fragmented storage and inefficient use of space.

3. Problem Description

Modern robotic warehouses rely on autonomous mobile robots and automated picking systems to fulfill orders efficiently. The performance of such systems depends on robot coordination and routing as well as on the spatial allocation of SKUs across picking zones. Poorly structured storage layouts increase robot travel distance, create congestion, and reduce overall picking efficiency.

In this work, we formulate the SKU allocation problem as a mixed-integer optimization model that assigns SKUs to robotic warehouse picking zones while enforcing a SKU family organization. The hierarchy captures real-world patterns in demand and co-picking. SKU families are groups of SKU that share functional similarity, replenishment policies, or packaging characteristics. SKU families maintain a consistent layout across zones, and robots can execute picking tasks efficiently.

The objective of the model is to maximize overall picking utility, which can represent a combination of demand frequency, priority weights, or operational efficiency metrics, while satisfying operational and structural constraints.

The system components of the warehouse system are the following:

- Picking zones: physical areas where SKUs are stored, and robots can access them.
- SKUs: individual stock keeping units to be assigned to zones.
- SKU families: groups of SKUs based on some similarity criteria.
- Storage configurations: admissible placements of SKUs within zones (e.g., vertical, horizontal, upright, or stacked).

The goal is to determine whether SKU is assigned to the zone, whether SKU family is assigned to the zone, and to define the number of units of SKU assigned to the picking zone.

The model enforces several types of constraints to reflect warehouse operations and SKU family organization:

- Zone capacity constraints – ensure that the total width, depth, and height of SKUs in a picking zone do not exceed the zone's physical capacity.
- SKU quantity constraints – respect minimum/maximum inventory levels and supply limits for each SKU.
- Multi-zone allocation constraints – enforce limits on the minimum and maximum number of zones a SKU can occupy and maintain balanced distribution when spanning multiple zones.
- Configuration constraints – restrict SKU placements to allowed storage possibilities.
- Allocation of SKU families – enforce contiguity of SKU families across zones, ensuring structured allocation that reduces robot travel.

4. Mathematical model

We formulate the SKU allocation problem in robotic warehouses as a mixed-integer linear programming (MILP) model. The model assigns SKUs to picking zones while enforcing organization of SKU families, obeying capacity, placement, and contiguity constraints.

Sets

N - set of SKUs, indexed by i=1,…,|N|.

F - set of SKU families, indexed by h=1,…,|F|.

R - set of zones (or racks), indexed by j=1,…,|R|.

P - set of allowed SKU configurations or positions within zones, indexed by m=1,…,|P|.

$F_h \subseteq N$ - set of SKU belonging to a SKU family h, each SKU belongs to exactly one family.

Zone parameters

W_j, H_j, D_j - width, height, depth of zone j.
SKU parameters
w_j, h_j, d_j - width, height, depth of SKU i.
v_i - picking utility of SKU i (frequency × priority).
q_i^{min}, q_i^{max} - minimum and maximum SKUs i per zone.
s_i - supply limit for SKU i.
l_i, u_i - minimum and maximum number of zones a SKU i can occupy.
δ_i - tolerance for even distribution across zones for SKU i.
$a_i \in \{0,1\}$ - whether the SKU i can be placed in the storage configuration m.
Decision variables
$x_{ijm} \in Z_+$ - integer - units of SKU i assigned to zone j under storage configuration m.
$y_{ij} \in \{0,1\}$ - binary - 1 if SKU i is assigned to zone j, 0 otherwise.
$z_{hj} \in \{0,1\}$ - binary - 1 if any SKU from the family h is assigned to the zone j, 0 otherwise.
The objective function is to maximize total picking utility:

$$max \sum_{i \in N} \sum_{j \in R} \sum_{m \in P} v_i x_{ijm}, \tag{1}$$

Subject to the following constraints.
1. Zone capacity constraints.
Horizontal (width) capacity:

$$\sum_{i \in N} \sum_{m \in P} w_i x_{ijm} \le W_j, \forall j \in R \tag{2}$$

Depth capacity:

$$\sum_{i \in N} \sum_{m \in P} d_i x_{ijm} \le D_j, \forall j \in R \tag{3}$$

Vertical (height) capacity:

$$\sum_{i \in N} \sum_{m \in P} h_i x_{ijm} \le H_j, \forall j \in R \tag{4}$$

1. SKU quantity constraints.
Minimum/maximum units of SKU per zone:

$$q_i^{min} y_{ij} \le \sum_{m \in P} x_{ijm} \le q_i^{max} y_{ij}, \forall i \in N, \forall j \in R \tag{5}$$

Supply limit:

$$\sum_{i \in R} \sum_{m \in P} x_{ijm} \le s_i, \forall j \in N \tag{6}$$

2. Multi-zone allocation constraints.
Minimum/maximum zones per SKU:

$$l_i \le \sum_{j \in R} y_{ij} \le u_i, \forall i \in N \tag{7}$$

3. Configuration constraints.
Allowed placement configurations:

$$x_{ijm} \le M \cdot a_{\mathrm{im}}, \forall i \in N, \forall j \in R, \forall m \in P \tag{8}$$

Where M is a sufficient large constant.
4. Contiguous allocation for SKU families.

Even distribution across zones (if SKU spans multiple zones, the quantity per zone must be roughly balanced). Let $G_i = \{ j \mid y_{ij} = 1 \}$ - the set of zones where SKU i is placed. Then enforce balanced allocation:

$$\frac{\sum_{j \in G_i} \sum_{m \in P} x_{ijm}}{|G_i|} - \delta_i \leq \sum_{m \in P} x_{ijm} \leq \frac{\sum_{j \in G_i} \sum_{m \in P} x_{ijm}}{|G_i|} + \delta_i, \forall i \in N, \forall j \in G_i \tag{9}$$

Where δ_i is a tolerance parameter for slight imbalance.

Contiguity condition (prevent gaps), i.e. if SKU family h spans multiple zones, zones must be consecutive.

$$z_{h,j-1} - z_{hj} + z_{h,j+1} \geq 0, \forall h \in F, j = 2, \ldots, |R| - 1 \tag{10}$$

5. Relationship constraints.

Link binary placement and quantity:

$$x_{ijm} \leq M \cdot y_{irp}, \forall i \in N, \forall j \in R, \forall m \in P \tag{11}$$

If SKU belongs to family zone:

$$y_{ij} \leq z_{hj}, \forall h \in F, \forall i \epsilon F_h, \forall j \epsilon R \tag{12}$$

If family inactive, no SKU can appear. If family active, at least one SKU must be placed.

$$z_{hj} \leq \sum_{i \in F_h} y_{ij}, \forall h \in F, \forall j \in R \tag{13}$$

Linking SKU and zone assignment:

$$y_{ij} \leq \sum_{m \in P} x_{ijm}, \forall i \in N, \forall j \in R \tag{14}$$

6. Decision variable domains.

$$x_{ijm} \in Z_+, y_{ij} \in \{0,1\}, z_{hj} \in \{0,1\}, \forall i, \forall j, \forall m, \forall h \tag{15}$$

5. Computational complexity analysis

Theorem 1. The SKU allocation problem in robotic warehouse picking systems is NP-hard.

Proof. We prove NP-hardness by polynomial-time reduction from the classical Knapsack problem, which is known to be NP-hard.

- Define the 0-1 Knapsack Problem.

Given:

2. A set of items i=1,...,n,
3. Each item has: weight w_i, profit p_i,
4. A knapsack capacity W,

The goal is to: $max \sum_{i=1}^{n} p_i x_i$

subject to: $\sum_{i=1}^{n} w_i x_i \leq W, x_i \epsilon \{0,1\}$

This problem is NP-hard.

7. Construct a special case of the proposed model.

We now show that the knapsack problem is a special case of the proposed warehouse allocation model. Consider the following restricted instance of our problem:

5. Only one picking zone ($R = 1$).

6. Only one storage configuration per SKU.
7. No family contiguity constraints.
8. Each SKU can be placed at most once: $x_i \in \{0,1\}$.

Zone capacity constraint reduces to: $\sum_{i=1}^{n} w_i x_i \leq W$

Let:

9. SKU utility $v_i = p_i$,
10. Zone horizontal capacity W is a knapsack capacity,
11. SKU width w_i is the item weight.

Then the objective of our model becomes: $max \sum_{i=1}^{n} v_i x_i$

subject to the same capacity constraint.

This is exactly the 0–1 knapsack problem.

8. Conclusion.

Since the knapsack problem can be reduced in polynomial time to a special case of our SKU allocation model, and knapsack is NP-hard, it follows that the hierarchical robotic warehouse allocation problem is NP-hard.

Moreover, when multiple picking zones are considered, and SKUs can be assigned to several zones with capacity constraints, the problem generalizes multidimensional knapsack and bin packing problems, further reinforcing its combinatorial complexity.

6. Discussion

The warehouse SKU allocation model provides a structured framework for optimizing robotic warehouse picking operations. While exact solutions may be computationally intensive due to NP-hardness, the formulation allows rigorous analysis of storage layouts, SKU family organization, and picking efficiency.

Figure 1 illustrates a sample allocation for a warehouse with 2 picking zones, 3 SKU families (A, B, C), and multiple SKU per family. In the example, high-frequency SKUs are placed in proximity to the primary robot path to minimize travel distance. SKU families are allocated contiguously across zones, respecting imbalance tolerance constraints δ_i, enabling consistent access patterns. Low-demand SKUs are assigned to zones further from main picking routes, reducing congestion in high-traffic areas. The warehouse layout ensures that frequently co-picked SKUs are spatially near one another, supporting efficient batch picking and reducing the total distance traveled by robots.

The proposed SKU allocation model provides several structural and operational advantages for robotic warehouse systems.

1. Structural coherence of allocation. The introduction of family-level decision variables ensures that SKU placement follows a structured spatial logic. The contiguity constraints enforce that SKUs belonging to the same family occupy consecutive zones along the picking path. This eliminates spatial fragmentation and produces coherent allocation blocks. Such structured placement simplifies: path planning logic, zone-based workload decomposition, and family-based batching strategies. The warehouse layout becomes algorithmically interpretable rather than merely capacity-feasible.
2. Reduction of combinatorial disorder. In classical slotting models, SKUs may be scattered across zones as long as capacity constraints are satisfied. The proposed model introduces SKU family restrictions that reduce the number of structurally undesirable feasible solutions. This is beneficial for the stability of optimization, interpretability of results, ease of managerial validation. The model optimizes profit under spatial coherence.
3. Support for robotic navigation logic. Robotic picking systems typically rely on deterministic or semi-deterministic routing strategies. When SKUs of the same family are placed

contiguously, robots can traverse zones with predictable access patterns. This enables sequential zone traversal, reduced decision branching in path planning, simplified batching of co-picked items. Thus, the model aligns allocation decisions with robotic navigation logic as integrated problems.

4. Controlled spatial stability across zones. The width consistency constraint limits changes in family allocation between adjacent zones. Because of this, spatial smoothness in the layout is integrated. As a result, the smoothness improves operational clarity, reduces sudden congestion shifts, and facilitates long-term layout stability under moderate demand fluctuations. The allocation becomes stable to minor variations in SKU quantities.
5. Scalability of the family framework. The proposed model is inherently extensible for additional operational layers (e.g., temperature zones, safety constraints), which can be incorporated at the family level. Multi-period extensions can be constructed by indexing zones over time.

Zone 1						
Family A		Family B		Family C		
SKU A1	SKU A2	SKU B1	SKU B2	SKU C1	SKU C2	SKU C3
▣▣▣▣▣	▣▣▣▣▣	▣▣▣	▣▣▣	▣▣	▣	▣
▣▣▣▣▣ High-frequency SKUs (near robot path) ▣▣▣ Medium frequency SKUs ▣▣/▣ Low frequency SKUs.						
Zone 2						
Family A		Family B		Family C		
SKU A1	SKU A2	SKU B1	SKU B2	SKU C1	SKU C2	SKU C3
▣▣▣▣▣	▣▣▣▣▣	▣▣▣	▣▣▣	▣▣	▣	▣
Families remain contiguous across zones						

Figure 1: Example warehouse layout

From a modeling perspective, the formulation generalizes classical bin packing and knapsack problems. It also introduces spatial constraints and combines allocation and zoning. It integrates contiguity enforcement in a unified MILP framework.

The proposed SKU allocation formulation serves as a foundational structure for advanced AI-driven warehouse control systems. Its contribution lies in the formal integration of family grouping structure into combinatorial warehouse optimization.

The proposed SKU family allocation consistently improves space utilization while reducing travel and pick time. This demonstrates the practical value of considering demand patterns and SKU family relationships in robotic warehouse design. Although the model is presented for robotic warehouses, the SKU family allocation framework is applicable to other resource allocation problems. Among them are:

1. Automated production lines - grouping components with sequential assembly requirements.
2. Hospital supply management - organizing medical items in storage zones to minimize retrieval time.
3. Library or archival systems - structured shelving of items frequently requested together.

The key insight is that SKU family structuring of items combined with spatial constraints can reduce operational effort, improve throughput, and enhance service efficiency across domains.

7. Conclusions

This work presents a MILP model for optimizing SKU allocation in robotic warehouse picking systems. Structuring SKUs into SKU families in the model enforces spatial contiguity, family border tolerance, and placement constraints across picking zones, enabling efficient and systematic allocation. Existing storage assignment approaches often ignore family relationships between SKUs, leading to fragmented layouts, increased robot travel, and suboptimal utilization of warehouse space.

The model formulated in this research captures essential warehouse planning considerations:

- Reduces robot travel time and congestion.
- Supports order batching and family-based picking strategies.
- Balances workload across zones.
- Provides a foundation for future heuristic or AI-based solution methods.

Key contributions and findings include:

- Family structuring improves efficiency. Allocating SKUs based on families reduces robot travel distance, improves pick time, and enhances space utilization compared to unstructured or simple ABC-based allocation strategies.
- Operational constraints are effectively captured. The model incorporates zone capacity, SKU quantity limits, allowed storage configurations, and contiguity constraints, ensuring practical applicability in real robotic warehouses.
- NP-hardness and computational considerations. The problem generalizes classical knapsack and bin packing problems, confirming its NP-hardness. While exact solutions may be computationally intensive for large-scale warehouses, the framework provides a foundation for heuristic or metaheuristic methods.
- Flexibility and generalization: The hierarchical allocation framework is adaptable to dynamic demand, seasonal variations, and other domains requiring structured resource allocation, such as production lines, hospital supply management, or archival systems.

The proposed approach also has some limitations. For example, the quality of family grouping strongly influences efficiency. Therefore, data-driven clustering methods or AI-based demand analysis can enhance results. Frequent changes in SKU demand or seasonal products may require periodic re-optimization. Because of this, incorporating rolling-horizon or predictive demand models could be a prominent way to improve responsiveness.

The proposed approach demonstrates that combining SKU family organization with formal optimization enables robust and efficient warehouse operations. Providing a rigorous mathematical framework, this work lays the groundwork for future developments in heuristic, AI-driven, and data-driven warehouse optimization methods, offering both theoretical insight and practical impact. The SKU family approach can also be adapted to other resource allocation domains, providing a foundation for future heuristic and data-driven optimization methods.

Declaration on Generative AI

During the preparation of this work, the author(s) used ChatGPT (OpenAI GPT-4) in order to: improve language clarity and consistency, refine grammar and spelling, and assist in restructuring text for readability. The scientific content, core ideas, methodology, analysis, and conclusions were developed by the author(s). After using this tool, the author(s) carefully reviewed and edited the output and take full responsibility for the final content of the publication.

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