# Reconciliation of average power calculations of pulse signals under dual definitions of electric current power in robotic control modules and joints*

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## Abstract

This work presents a new perspective on the duality of power definitions: calculating power either as the rate of energy use or as the product of voltage and current. The accuracy of power modeling in robotic control modules and joints, essential for energy efficiency and stability in future industrial systems, is examined. Discrepancies in average power calculations using different formulas are analyzed, and a review of the literature shows that this issue has not yet been systematically addressed, making the study scientifically innovative. Methodological imperfections arise from the dual definition of power in physics and insufficient attention to calculation formulas in engineering. The proposed “Energy Box” model and a formula for averaging the power of rectangular pulses in circuit sections provide a novel way to reconcile pulse-signal calculations with Ohm’s law, a detail often overlooked in standard practice. The next step is to establish principles for applying rectangular pulses to determine relationships between signals with different time profiles. These steps will create a rigorous mathematical basis for accurate modeling of average parameters of pulse signals with arbitrary profiles, according to Ohm’s law, thereby enhancing the efficiency of robotic systems.

## Keywords

rectangular pulses, triangular pulses, function of one variable, composite function, limiting behavior of the source, Ohm's law

## 1. Introduction

Robotic systems have ushered in a new era of human interplay with technical devices. These multi functional systems, which range from consumer technologies to military and aerospace applications, have been developed through synergies between achievements in various fields of science and engineering.

At the heart of this synergy lies power as a universal parameter reflecting various forms of energy conversion. This ranges from the detection and processing of low-power and time-varying signals by antennas and sensors, to the powerful and dynamic operation of drive motors in various environmental conditions and the forceful action of servomechanisms on target objects. All these processes require their power sources to operate stably and simultaneously. In control modules and joints of robotic systems, the predominant type of power source is direct current (DC), usually in the form of batteries or, for low-power devices, galvanic cells.

In this context, power is not merely a physical quantity that characterizes the processes of energy flow conversion and their technical application. It is also a criterion that enables quantitative comparison of the efficiency with which different types of energy are consumed by nodes in robotic systems. Refining the method of calculating power contributes to improving designs and technological solutions based on a universal criterion.

The need for refinement arises from the fact that many processes in robotic nodes occur in a pulsed mode, involving the short-term utilization of high-density energy. This enables the required

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power to be generated at a specific time and place in the system while reducing heat loss and other undesirable side effects effectively.

Historically, the earliest methods for analyzing electrical processes and systems – including the calculation of electric current power – were developed for galvanic direct current sources, the first of their kind, and were later adapted to harmonic currents.

Over time, these methods were refined for engineering practice, which mainly focused on periodic and random analogue processes. Discrete processes received far less attention than they do today. These processes have since formed the basis of technologies such as RADARS, LIDARS, SONARS, laser rangefinders and other innovative equipment. All of this stems from traditional pulse techniques and continually evolves.

Meanwhile, the existing method of calculating the parameters of pulsed signals is still based on provisions formed when the first mathematical apparatus for describing electric current sources was developed, before the concept of a signal itself had been introduced. As a result, this method remains not fully adapted to the properties of signals, especially pulsed ones.

This work presents a conceptual framework for optimizing average power calculations of rectangular pulses. Implementing this framework establishes a rigorous mathematical foundation for accurate modeling of time-varying signals with arbitrary profiles, in accordance with Ohm's law.

This reconciliation enhances the integration of pulsed electrical energy with other forms of energy, enabling precize of control modules and the efficiency of robotics joints and supporting the broad functionality of modern robotic systems.

## 2. The dilemma of the dual definition of electric current power, and an illustration of its relation to the calculation of average power, using rectangular pulses as an example

The discrepancies observed in calculating the average power of rectangular pulses using formulas derived from different definitions of electrical power highlight the limitations of traditional methods for determining average pulse-signals parameters. These limitations were first demonstrated in [1, pp. 56–66], and subsequent research – extending to software widely adopted in engineering practice and education – was presented in [2, pp. 24–26].

One of the reasons for the identified methodological limitations is the insufficient attention paid to the dualistic nature of the definition of power in physics, as well as to the engineering formula used for its calculation, expressed as the product of instantaneous voltage and current. As these and other associated provisions are widely recognized and extensively covered in educational and specialized literature, we will primarily reference publicly available sources that summarize only the principal aspects.

In classical physics, the power (P) of a direct current (DC) is defined as the rate at which electrical energy (W) is consumed in order to perform useful work or convert energy into other forms over a given time period (T) [3]:

$$P=\frac{W}{T}\Rightarrow W=P\cdot T. \tag{1}$$

In technical applications, power refers to the ability of a power source to supply power to loads, as well as to the energy conversion within the loads themselves. This is formally described as a function of two basic physical quantities – voltage (U) and current (I) – with power being defined as their product [4]:

$$P=U\cdot I. \tag{2}$$

The parameter that reflects the relationship between voltage and current is the resistance (R) of a circuit section [5]:

$$\frac{U}{I}=R. \tag{3}$$

Alongside Ohm's law, the definition in (2) is fundamental to analyzing electrical processes and systems. For example, applying Ohm's law (3) together with (2) yields a formula for Joule heat power [6] that can be used as a reliable criterion for validating power calculation methods:

$$Q=I^2 \cdot R. \tag{4}$$

For harmonic signals, formula (2) is expressed in terms of the instantaneous power p(t), voltage u(t) and current i(t) values as follows [7]:

$$p(t)=u(t) \cdot i(t), \tag{5}$$

where $u(t) = U_m \cdot \sin(\omega \cdot t + \varphi_u)$, $i(t) = I_m \cdot \sin(\omega \cdot t + \varphi_i)$. Here, $U_m$ and $I_m$ are the voltage and current amplitudes, respectively; ω is the angular frequency; t is time; and $\varphi_u$ and $\varphi_i$ are the initial phases of voltage and current, respectively.

In its general form, equation (5) is used to calculate the instantaneous power of continuous and discrete deterministic signals described by arbitrary time functions [5; 7; 11; 12].

In the sense defined in (1), the energy of continuous and discrete deterministic signals is a physical quantity that can be objectively determined by integrating the measured instantaneous power values over a period of time T. According to the definition in (1), power and energy are related by the integral of instantaneous power, as shown below [11-13]:

$$P=\frac{1}{T}\int_0^T p(t)dt \Rightarrow W=\int_0^T p(t)dt \; . \tag{6}$$

The pulse power ($P_i$) and average power ($P_{avg}$) are derived from the instantaneous power p(t) [8-15]. For example, the formulas for the power and energy of rectangular pulses can be considered:

$$P_i=\frac{1}{t_i}\int_0^{t_i} p(t)dt \Rightarrow W=\int_0^{t_i} p(t)dt = P_i \cdot t_i, \tag{7}$$

where $t_i$ is the duration of the active phase of pulses. The average power is given by:

$$P_{avg}=\frac{1}{T}\int_0^{t_i} p(t)dt = P_i \cdot \frac{t_i}{T}. \tag{8}$$

Together with formulas (7) and (8), the literature invariably employs a simplified graphical representation of pulsed and average power, the essence of which is demonstrated in Figure 1.

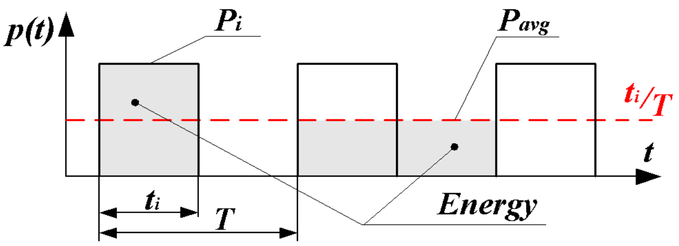

**Figure 1:** A generalized graphical representation of pulse power $P_i$ and average power $P_{avg}$ for a rectangular pulse as a function of one variable, in the form of instantaneous power p(t), where the pulse energy remains constant and equal to a fixed area.

This approach of presenting power as a function of a single variable relies on a non-alternative model, which serves to justify the method for calculating the average voltage and current values of pulses. The ratio $t_i/T$, highlighted in Figure 1 and represented in equation (8), is usually denoted by the special symbol D in scientific literature and is known as the Duty Cycle [14; 15]. However, in this paper, we intentionally consider it only in the form of a ratio. This approach provides a clearer understanding of the issues related to the representation in Figure 1 which are discussed here.

A careful examination of equations (1) to (8) reveals that the average power is not derived equally from definitions (1) and (2). According to definition (2), we have:

$$P_{avg}=U_{avg}\cdot I_{avg}. \qquad (9)$$

A series of equivalent transformations of expression (9), based on Ohm's law and including the formula for Joule heating (4), leads to the final formula which does not coincide with (8):

$$P_{avg}=U_{avg}\cdot I_{avg}=(U_m\cdot\frac{t_i}{T})\cdot(I_m\cdot\frac{t_i}{T})=U_m\cdot I_m\cdot(\frac{t_i}{T})^2\Rightarrow$$

$$\frac{(U_m^2)}{R}\cdot(\frac{t_i}{T})^2=R\cdot I_m^2\cdot(\frac{t_i}{T})^2=P_i\cdot(\frac{t_i}{T})^2, \qquad (10)$$

where $U_m$ and $I_m$ are the amplitude of the voltage and current of the rectangular pulse.

It is important that formulas (10) and (2) demonstrate the same functional relationship between power, voltage and current, since this indicates their compliance with Ohm's law. According to Ohm's law, for example, a twofold change in the voltage across the active resistance of a circuit section results in an equal change in the current flowing through it. Therefore, a twofold change in both voltage and current in formula (2) leads to a fourfold change in power. Analogously, a twofold change in the $t_i/T$ ratio in formula (10) results in a fourfold change in the average power.

The discrepancy between the well-known formula (8) and formula (10), first proposed in [1], stems from a methodological dilemma concerning the dualistic definition of electric current power in formulas (1) and (2). A systematic review of available sources shows that this duality has so far gone unnoticed, and its practical significance remains undetermined.

In general, the dilemma of the duality of definitions according to (1) and (2) can be summarized as outlined below. Formula (1) provides a comprehensive definition of power and is therefore ideal for calculating the total energy consumed by a source, particularly when a highly stable power supply is required. Formula (2), meanwhile, expresses instantaneous power and is therefore better suited to local analysis of the section of the circuit where time-varying signal power is dissipated. This is due to the nature of signals being dynamic, whereas stability is required for sources.

In practice, ensuring consistent use of formulas (8) and (10) alongside the dual definitions in (1) and (2) is extremely important. This is because improving the stability of the power supply for robotics nodes where signal processing occurs reduces internal distortions and increases their resistance to external interference. Therefore, the reconciliation of the concept of average power in pulsed signals with the definitions in formulas (1) and (2), as well as with Ohm's law, eliminates the contradictions that were previously identified. This represents a scientifically innovative approach that expands our knowledge base.

## 3. Discussion of a graphical-analytical model for reconciling the concept of average power with the dual definition of electric current power

Today, in all known educational and specialized literature, the graphical representation of power is simplified to a 'flat' function of time, as shown in Figure 1. This is analogous to voltage or current wave-forms, which are functions of a single variable.

This not only contradicts the fundamental definition of power as a composite function of the product of instantaneous current and voltage values, but also hinders accurate modeling of energy formation as the integral sum of instantaneous power values over a given time period.

A more accurate methodological presentation of electrical power and energy is provided by 3D graphical-analytical visualization, which includes the fundamental physical quantities of current and voltage, as well as all the derived parameters necessary for practical calculations. It accurately reflects the relationship between energy parameters, thereby avoiding the methodological errors that are typical of simple models.

Figure 2 shows a detailed model of the 'Energy Box' applied to a uni-polar rectangular pulse and a DC voltage and current, as well as to the results of averaging according to formulas (8) and (10).

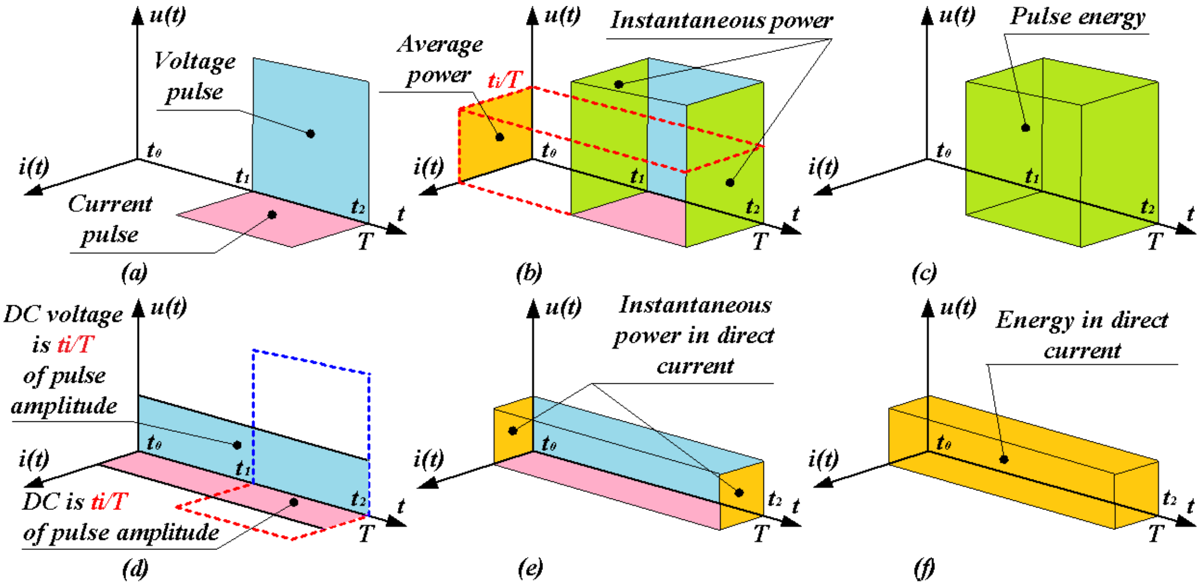

**Figure 2:** Graphical-analytical model of the 'Energy Box' based on conjugated models of voltage, current and power, presented as areas, and energy presented as volume.

Figure 2(a) shows the pulse repetition period starting at time $t_0$ and ending at time $t_2$, with a duration of $T = t_2 - t_0$, and a rectangular pulse with a duration of $t_i = t_2 - t_1$.

The instantaneous power model is the area formed by the product of the instantaneous voltage and current, for example, at times $t_1$ and $t_2$. The model of pulse energy in Figure 2(c) is the volume formed by integrating the instantaneous power values over the pulse duration $t_i$. This corresponds to formulas (1) and (6), on the condition that the voltage and current remain constant for the full duration of the rectangular pulse.

The red dashed line in Figure 2(b) illustrates the average power, obtained from the pulse power using the ratio $t_i/T$ according to formula (8). As is evident from Figure 2(c), the actual energy value of the pulses remains fixed for the time period $t_i = t_2 - t_1$. This is more clear than the simplified graphical representation shown in Figure 1.

Figure 2(d) shows a DC voltage and current that matches the average voltage and current of the pulse in Figure 2(a). The instantaneous powers at times $t_0$ and $t_2$ are equivalent to the average power given by formula (10) for the pulse in Figure 2(a), as shown in Figure 2(e). This is $T/t_i$ times smaller than the average power in Figure 2(b).

This example illustrates the methodological dilemma that arises when power electric current is considered simultaneously as a function of a single variable, according to definition in (1), and as a composite function, under definition in (2), without critically examining the practical contexts in which these definitions are employed.

As evident from Figures 2(a)–(c), definition in (1) refers to the power of sources that must provide stable voltage or currents. *This reflects the limiting behavior of the source, which does not* *obey Ohm's law if the load resistance (R) changes.* So, if the internal resistance of the source r ⟶ 0, then it behaves as an ideal voltage source for which U = constant as R ⟶ 0 and I ⟶ ∞. If r ⟶ ∞, it behaves as an ideal current source for which I = constant as R ⟶ 0 and U ⟶ 0 or R ⟶ ∞ and U ⟶ ∞.

For these cases, formula (8) is correct since averaging is only performed on one variable (voltage or current) and the other quantity remains constant. This reduces the power to a function of one variable, thereby simplifying the analysis.

The example in Fig. 2(b) demonstrates consistency with formula (6) for the average power of an ideal current source. Here, the average power is derived from an unchanging current and a voltage that is $T/t_i$ times smaller than the amplitude of the voltage pulse. Therefore, the average power in Figure 2(b) is $T/t_i$ times greater than that in Figure 2(e), as discussed in [1; 2].

Rather, Figures 2(d)–(f) show that definition in (2) is valid for time-varying signal power in a fixed, linear, resistivity load. This is evident from the equivalence of the DC voltage and current to the average voltage and current of the pulsed signal, which is expressed as the ratio $t_i/T$ in terms of amplitude.

In this case, neither the current nor the voltage is constant, and the power of the signal is a composite function of these two variables. In such cases, we propose formula (8), which involves averaging two variables: voltage and current. This is consistent with formula (3) and Ohm's law for instantaneous values, which states that u(t)/i(t) = R, when u(t) and i(t) not constant.

## 4. An illustration of the principle of using rectangular pulses as the basis for determining the relations between parameters of pulses with different time profiles

Studies [1; 2] have currently been extended to include pulses with linear time profiles, particularly triangular ones, as indicated in [2]. These studies focus on formulating principles for using rectangular pulses to analyses the relationships between the parameters of pulses with different time profiles.

Figure 3 shows the traditional 'flat' graphical representation of the average current and voltage values as a function of a single variable for a series of continuous uni-polar triangular pulses with different time profiles.

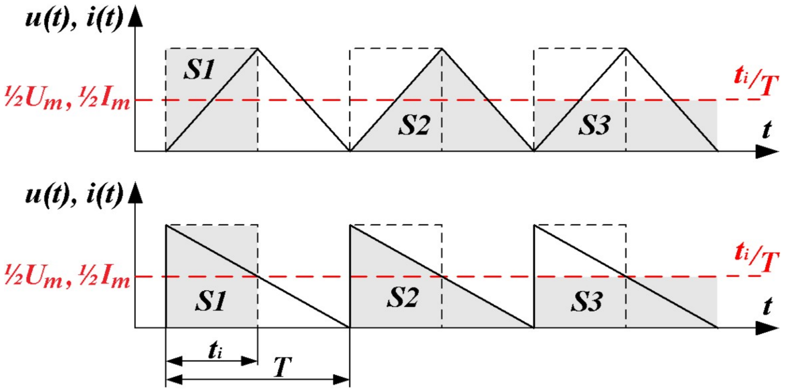

**Figure 3:** Identical average voltage and current values for rectangular and triangular pulses, derived from their amplitude based on the equivalence of areas S1, S2 and S3.

For comparison, rectangular pulses with the same period (T) and a ratio of $t_i/T = ½$ are also shown. The average values in Figure 3 correspond to the height of the rectangle with area S3, which is equal to the area of the triangle S2, as well as the area of the rectangle S1. Therefore, since the ratio $t_i/T$ is ½, the height of a rectangle with an area of S3 is equal to ½ of the amplitude of both rectangular and triangular pulses. Evidently, the average power of the rectangular and triangular pulses in Figure 3 should be equal since it is defined as the product of identical average voltage and current values, each of which is ½ the amplitude value.

Moreover, the same rectangular pulses can be used to determine the average values of the parameters of pulses with arbitrary time profiles. However, this requires unambiguous mathematical functions to be established to describe their relationship.

Figure 4 illustrates the construction of a model based on the 'Energy Box' concept for the triangular pulse shown in Figure 3. The dotted line indicates the base rectangular pulse. The peak power of the triangular pulse is equal to the instantaneous power of the rectangular pulse at time $t_1$. The pulse's energy is represented by the volume of an irregular quadrangular bi-pyramid.

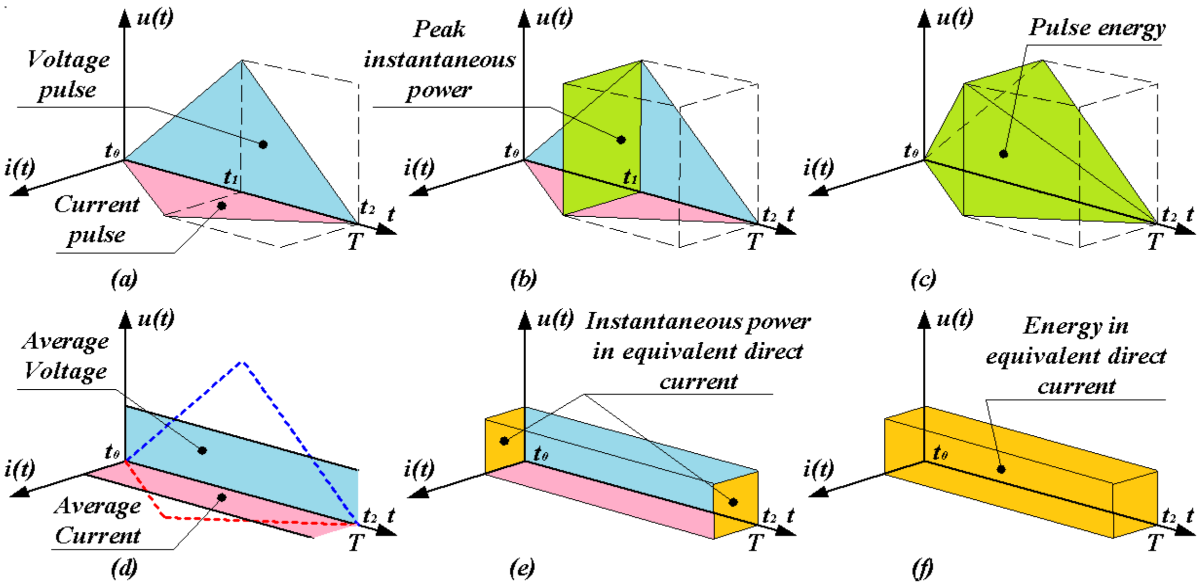

**Figure 4:** The 'Energy Box' model for a triangular pulse has a duration ranging from $t_0$ to $t_2$ when constructed over the period T of a basic rectangular pulse with a duration ranging from $t_1$ to $t_2$ and a ratio of $t_i/T$ of ½.

Further research should focus on pulses with non-linear time profiles to establish a rigorous and reliable mathematical foundation for accurately modelling the average parameters of pulse signals with arbitrary time profiles.

## 5. Conclusions

The average power of pulsed signals cannot be reliably determined from classical definitions without accounting for their duality within the framework of Ohm's law – a property inextricably embedded in formulations that implement the engineering and the physical approaches. This issue has not yet been studied by anyone, and its decision not only contributes to the further development of existing theoretical knowledge but also helps solve practical engineering tasks.

Due to the diversity of practical needs and the lack of a clear differentiation between the areas of application of the dual definitions of electric current power established in classical physics and engineering, harmonization of methods for calculating statistical parameters of pulsed signals and modeling the behavior of electrical circuits is particularly important. The following is proposed as a first step towards reconciling the application of the concept of average power in accordance with Ohm's law:

1. A formula for averaging the time-varying power of rectangular pulses in a active resistance of circuit section.

2. The concept of applying rectangular pulses as a basis for establishing the relationships between the parameters of pulses with different time profiles.

3. A detailed 3D graphical-analytical model showing how the derived quantities of power and energy are formed from the basic physical quantities of voltage and current.

The next step should be to develop a method of using basic rectangular pulses to establish relationships between pulses with different time profiles, as well as quantitative criteria for consistently assessing their parameters.

Together, these steps will provide a robust mathematical foundation for accurately modeling the average parameters of pulse signals with arbitrary time profiles in accordance with Ohm's law.

The coordinated application of these approaches will contribute to the practical realization of the hidden potential of the dual definitions of electric current power, while simultaneously expanding our knowledge base.

This will ultimately contribute to the development of precise of control modules and efficient of robotics joints for modern automation that work closely with other functional units, such as mechanical drives, hydraulic and pneumatic systems, sensor subsystems and energy units, for ground, air, surface and underwater vehicles, in accordance with optimal energy consumption and interaction criteria.

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## Declaration on Generative AI

During the preparation of this work, the authors used ChatGPT-5 to check the grammar and spelling of the text and to translate it. Having used this tool, the authors reviewed and edited the content as necessary, bearing full responsibility for the publication's content.

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