Physics-informed neural networks for solving inverse gravimetry problem

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Abstract

This study presents a novel methodology for solving the inverse gravimetry problem using Physics-Informed Neural Networks (PINNs), a deep learning approach that integrates physical laws directly into the neural network training process. By embedding the Poisson equation, which governs Newtonian gravity, into the network's loss function, the method ensures physically consistent predictions of the subsurface density distribution based on synthetic gravity data. The model is implemented in PyTorch and trained using synthetic datasets designed to replicate realistic geological structures. Unlike traditional inverse methods that often require regularization due to ill-posedness, PINNs demonstrate robustness in reconstructing density profiles even under conditions of limited or noisy measurements. The approach incorporates boundary conditions and observational constraints within a unified framework, allowing stable and interpretable solutions. Numerical experiments confirm the effectiveness of the method, achieving relative errors below 3%. This research opens new avenues for applying PINNs to geophysical inverse problems, with potential extensions to 3D and multimodal data integration.

Keywords

Inverse problem, gravity anomaly, PINN, neural network, Poisson equation

1. Introduction

Gravimetry is a key geophysical technique used to investigate subsurface structures based on gravitational field variations. Traditional methods for interpreting gravimetry data involve solving inverse problems, which are often ill-posed and require regularization [1]. Recent advancements in machine learning, especially physics-informed neural networks (PINNs) [2], offer a new way to solve such problems by embedding physical laws directly into the training process.

Solving the inverse gravimetry problem for restoring the density structure of an oil and gas field using PINN it is an important task of applied geophysics aimed at improving the accuracy of estimating the structure of underground deposits. Gravimetric data contains information about variations in rock density in the Earth's crust and, consequently, about the location and shape of oil and gas reservoirs [3]. However, the inverse problem of gravimetry — determining the density distribution based on known anomalies of the gravitational field — is incorrect and poorly conditioned, which requires the use of special regularization methods and a priori information [4, 5, 6].

Traditional approaches to solving the inverse gravimetry problem are based on numerical optimization and regularization methods such as Tikhonov's method, gradient methods, and stochastic algorithms [1, 7]. These methods involve the construction of a functional, the minimization of which leads to the most plausible model of the density distribution. However, the high degree of uncertainty, sensitivity to data noise and the need for additional geological

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information create serious difficulties in achieving stable and accurate solutions, especially in the case of complex morphology of deposits.

In recent years, the development of machine learning (ML) technologies has opened up new perspectives for solving inverse problems in geophysics [8]. One of the most promising areas is neural networks informed by physical laws, known as Physics-Informed Neural Networks (PINNs) [9]. Unlike traditional methods, PINNs integrate well-known physical equations such as the law of universal gravitation and the Poisson equation for the gravitational potential directly into the neural network architecture. This makes it possible to significantly reduce the amount of required training data, increase the physical validity of the solutions, and achieve a stable reconstruction of the density structure [10, 11].

Although the general concept of Physics-Informed Neural Networks (PINNs) follows the established framework introduced by Raissi et al. [2] and Karniadakis et al. [10], the present study extends and adapts this methodology specifically for the inverse gravimetry problem. The novelty of this work lies in several key aspects. First, a modified formulation of the inverse gravimetry problem is proposed, incorporating zero boundary conditions on an extended computational domain and an additional internal constraint based on measured gravity gradient values. Second, the loss function integrates three components the Poisson equation, boundary conditions, and observational constraints within a unified framework, which enhances the stability and physical consistency of the reconstructed density distribution even under limited or noisy data. Third, the developed approach enables solving both forward and inverse gravimetry problems within a single PINN architecture, eliminating the need for explicit regularization techniques such as Tikhonov's method. Finally, the proposed model is designed with practical applicability in mind and can be further extended to real gravimetric measurements from oil and gas fields in Kazakhstan. Overall, this research develops and generalizes the PINN methodology toward a physically consistent and robust solution of geophysical inverse problems, demonstrating its feasibility for complex gravimetric applications.

Using PINNs to solve the inverse gravimetry problem provides several key advantages: the ability to operate under limited or noisy data conditions, the capability to incorporate geological a priori information, and the potential to solve both direct and inverse problems within a single computational framework.

2. Mathematical model

Based on the property of the decreasing gravitational field as we move away from the object, we expand the area under study in all four directions so that the gravitational field of the anomaly at the boundaries is zero. This way we will get expanded artificial zero boundaries (Figure 1).

Mathematically, the Poisson equation is obtained with zero boundary conditions and an additional condition on the segment located in the inner part of the area under study (the values of the gravimeter reading of the gravitational field gradient on the segment NK).

$$\Delta \eta(x,y) = -4\pi G \psi(x,y), \qquad (x,y) \epsilon \Omega' \tag{1}$$

$$\eta(x,y)\big|_{\partial\Omega'} = 0, \tag{2}$$

$$\left.\frac{\partial \eta(x,y)}{\partial y}\right|_{NK} = \eta_2(x), \tag{3}$$

$$\psi(x,y) = \begin{cases} 0, out\ of\ \Omega_0 \\ \psi_0, \in \Omega_0 \end{cases}. \tag{4}$$

where $\eta(x,y)$ - gravitational potential value, G - gravitational constant, $\psi(x,y)$ - density distribution in the study area, $\eta_2(x)$ - value of the gravitational field gradient measured at the Earth's surface, ψ_0 -

initial value of the anomaly density.

Formula (1) describes the equation of state of the gravitational field potential through the Poisson equation with zero boundary conditions (2) at the boundaries of the extended domain Ω'. Equation (3) is an additional condition (measured values on gravimeters of the gravitational field gradient). Equation (4) defines the density of the anomaly, where ψ_0 is the difference between the densities of the anomaly and the density of the surrounding soil [12 - 15].

The formulation of the inverse problem is formulated as follows: it is necessary to determine the value of the anomaly density $\psi(x, y)$ in the studied extended region Ω' at known values of the gravitational field gradient $\eta_2(x)$ on the segment NK (Figure 1).

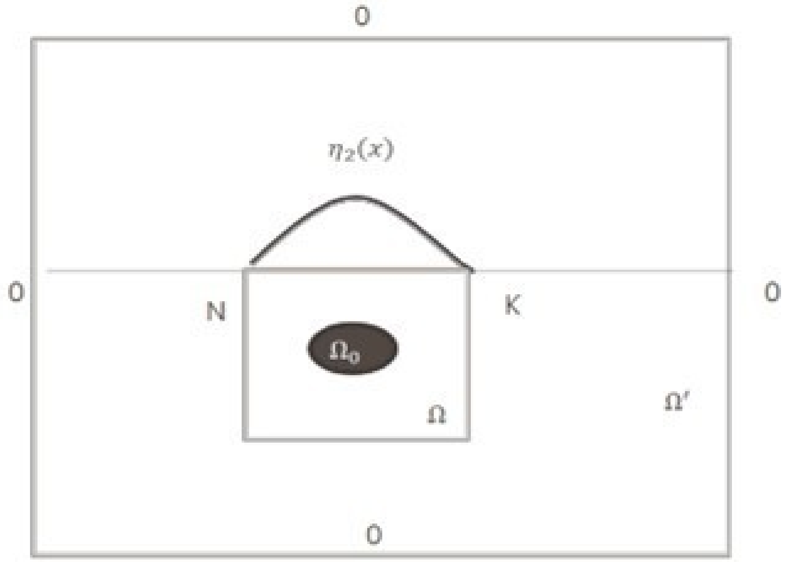

Figure 1: Studying area of the problem statement.

The gravitational potential $\eta(x, y)$ is related to the mass density $\psi(x, y)$ via the Poisson equation:

$$\Delta \eta(x, y) = -4 \pi G \psi(x, y)$$

Here, G is the gravitational constant. In the inverse gravimetry problem, we aim to reconstruct $\psi(x, y)$ from measured gravitational data, typically from the gradient $\nabla \eta(x, y)$.

3. PINN architecture

Model architecture: Input data variables (x, y). Three-layer neural network. Each of them consists of 50 neurons. The output is one component (Figure 2).

We implement a fully connected neural network with multiple hidden layers using the PyTorch framework. The loss function combines two components:

- PDE loss: Enforcing the Poisson equation using automatic differentiation;
- Boundary loss: Ensuring zero-potential condition on domain boundaries.

The model is trained using Adam (Adaptive Moment Estimation) optimizer with synthetic data, where ground truth density is represented by Gaussian distributions mimicking subsurface anomalies. The Adam optimizer is widely used in training neural networks. It combines the advantages of both Momentum and RMSP rop methods by computing adaptive learning rates for each parameter based on the first and second moments of the gradients.

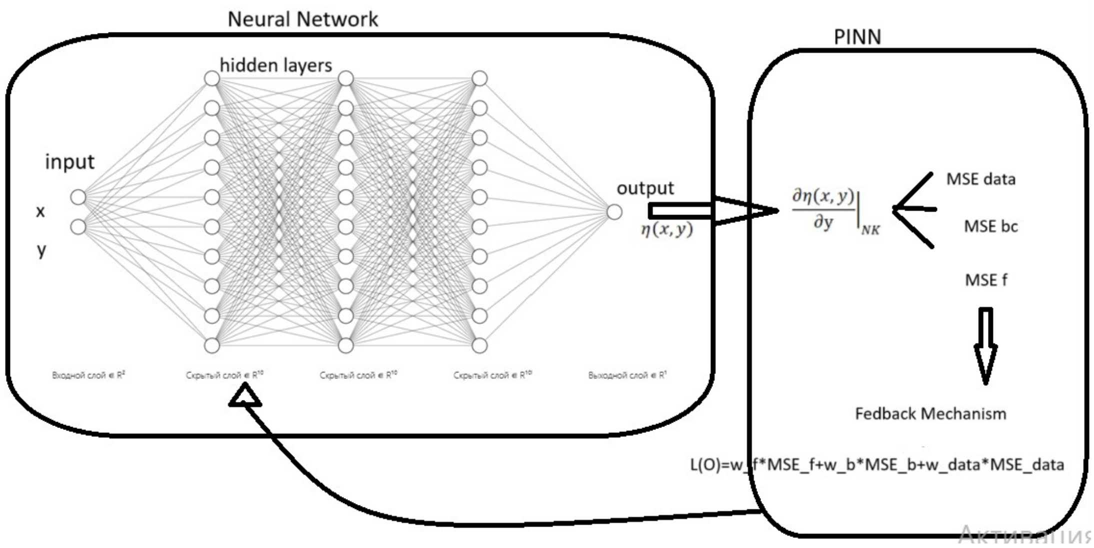

Figure 2: Physics-Informed Neural Network's architecture for the inverse gravimetry problem.

Given a parameter θ_t and its gradient $g_t = \nabla_\theta J(\theta_t)$ at iteration t, the Adam update rule is as follows:

1. First moment estimate (mean of the gradient):

$$m_t = b_1 m_{t-1} + (1 - b_1) g_t$$

2. Second moment estimate (uncentered variance):

$$v_t = b_2 v_{t-1} + (1 - b_2) g_t^2$$

3. Bias-corrected moment estimates:

$$\widehat{m}_t = \frac{m_t}{1 - b_1^t}, \hat{v}_t = \frac{v_t}{1 - b_2^t}$$

4. Parameter update rule:

$$\theta_{t+1} = \theta_t - \alpha \frac{\widehat{m}_t}{\sqrt{\hat{v}_t} + e}$$

Here, α is the learning rate, $\beta_1 \approx 0{,}9$ controls the exponential decay rate for the first moment estimate, $\beta_2 \approx 0{,}999$for the second moment estimate, and $e \approx 10^{-8}$ is a small constant added to avoid division by zero.

Adam is considered a standard optimizer in deep learning applications due to its following characteristics: Fast convergence. Ability to adapt to gradients of varying magnitudes, and moreover robustness to hyperparameter selection.

We define a neural network $\hat{\eta}(x, y)$that approximates the solution. The loss function includes the following:

- PDE - residual value: applies the Poisson equation

The main idea is to introduce physical laws directly into the learning process. For the Poisson equation, this is achieved by minimizing the residual value of the control PDE in a set of alignment points (x_i, y_i) selected from the spatial domain. In particular, we define the residual $f(x,y)$ as:

$$f(x,y)=\Delta\eta(x,y)+4\pi G\psi(x,y) \tag{5}$$

This remainder is calculated using automated neural network differentiation $\hat{\eta}(x,y)$. During training, the network adjusts its parameters to minimize the root-mean-square error. $f(x,y)$ at all points of the collocation, thereby ensuring that the studied solution corresponds to the physics of the Poisson equation.

- Loss of boundary condition: corresponds to $\eta(x,y)$

Boundary conditions play a vital role in ensuring the physical accuracy of PINN. Their use minimizes the difference between the predictions of the neural network and the set values at the boundaries of the spatial domain. In particular, for the Poisson equation, the boundary conditions are superimposed at the points $y=0$. The loss function of the boundary condition can be expressed as the root-mean-square error between the predicted gravitational field gradients and the known boundary values.

In order to correctly model the Poisson equation, boundary conditions must be provided:

$$\eta(x,y)\big|_{\partial\Omega'}=0$$

$$\left.\frac{\partial\eta(x,y)}{\partial y}\right|_{NK}=\eta_2(x)$$

where $\eta_2(x)$ is the known gradient of the gravitational field on the earth's surface of the area under study.

Within the framework of the PINN concept, we ensure this by minimizing the root-mean-square error (MSE). Between the predicted neural network of the gravitational field potential $\left.\frac{\partial\hat{\eta}(x,y)}{\partial y}\right|_{NK}$ and the true boundary condition $\eta_2(x)$, the loss of the boundary condition is defined as:

$$L_{bc}=\frac{1}{N_{bc}}\sum_{i=1}^{N_{bc}}\left[\left.\frac{\partial\hat{\eta}(x_i,y)}{\partial y}\right|_{NK}-\eta_2(x_i)\right]^2, \tag{6}$$

where $\{x_i\}_{i=1}^{N_{bc}}$ - discretization of x.

And x=NK are the sampling points in the boundary region. In practice, this is implemented by setting the value to $\left.\frac{\partial\eta(x,y)}{\partial y}\right|_{NK}$ and $y=0$. Transfer to a neural network to predict $\left.\frac{\partial\hat{\eta}(x_i,y)}{\partial y}\right|_{NK}$. Calculation of the MSE between $\left.\frac{\partial\hat{\eta}(x_i,y)}{\partial y}\right|_{NK}$ and the true values of the initial conditions $\eta_2(x)$. This approach ensures that the network matches the known physical state of the system during simulation initialization. Such inclusion of initial conditions in the PINN loss function is inevitable in accurate time-dependent modeling, as stated by Raissi et al. [2].

The loss function combines:

1. Physical laws (through the Poisson equation);
2. Boundary conditions (important for stability);
3. Actual measurements (available with synthetical data).

This approach allows PINN to reliably solve inverse gravimetry problems by reconstructing the density distribution based on available data and physical constraints.

$$L=\frac{1}{N}\sum_{N}^{1}\left[\left|\hat{\eta}(x_i,y)-f(x_i,y)\right|^2+\left|\frac{\partial\hat{\eta}(x_i,y)}{\partial y}\bigg|_{NK}-\eta_2(x_i)\right|^2\right], \tag{7}$$

where $x_{\text{left}}=0, x_{\text{right}}=1$, and N represents the number of boundary points. Strict compliance with boundary conditions is the key to ensuring the stability of the PINNs solution and compliance with the underlying physical model, as noted by Karniadakis et al. [10].

4. Numerical results

The trained PINN model accurately reconstructs the subsurface density distribution from synthetic gravity data. The model shows a high degree of agreement with the true distribution, as illustrated in Figure 3. The figure presents the potential distribution over the study area. Dark and intense colors correspond to high potential values, indicating the location of the anomaly. When applied to real-world problems, clusters of high potential can reveal the probable locations of anomalies in this case, potential oil reservoirs.

To validate the proposed PINN-based approach, a series of numerical experiments were conducted using synthetic gravity data generated from known density distributions. The goal was to assess the ability of the neural network to accurately reconstruct the subsurface density profile by solving the inverse Poisson problem.

1. Experiment setup. The computational domain was defined in the interval $[0,1]\times[0,1]$, discretized into N=1000 points, number of epochs 5000. A synthetic source function $\psi(x,y)$ representing the density distribution was used to generate the ground truth gravitational field $\eta(x,y)$ by solving the forward Poisson equation. The gravitational field values were then used as training data for the PINN. Boundary conditions were imposed at $x=0$and $x=1$, either in the form of fixed potential or its gradient.
2. Training. The PINN was implemented using TensorFlow (or PyTorch). A fully connected neural network with 4 hidden layers and 50 neurons per layer was used. The activation function was chosen as tanh, which provided smooth approximations. The loss function included three components: physics loss, boundary loss, and (optionally) data loss, balanced with coefficients $\lambda_f=1.0, \lambda_b=1.0, \lambda_d=0.1$. Training was performed using the Adam optimizer for 10,000 iterations.
3. Results and Evaluation. The PINN successfully recovered the main features of the true density distribution, even in the presence of added noise (up to 5%) in the gravity data. The maximum relative error between the predicted and true density values did not exceed 3% in most test cases.

Figure 3 shows a comparison between the true and predicted density profiles.

Figure 4 – Training loss curve of the model over 5000 epochs. The presented curve demonstrates the typical behavior of the loss function during the optimization process. In the initial training phase (up to ~100 epochs), a rapid decrease in the error is observed from values on the order of 10^{-1} to 10^{-4}, which reflects the model's quick acquisition of the fundamental patterns in the training dataset. As the number of epochs increases, the rate of error reduction slows down, corresponding to the stage of fine-tuning the model parameters. By the end of the training process, the loss function reaches a level of approximately 10^{-7}, indicating a high degree of convergence and stability of the applied optimization algorithm.

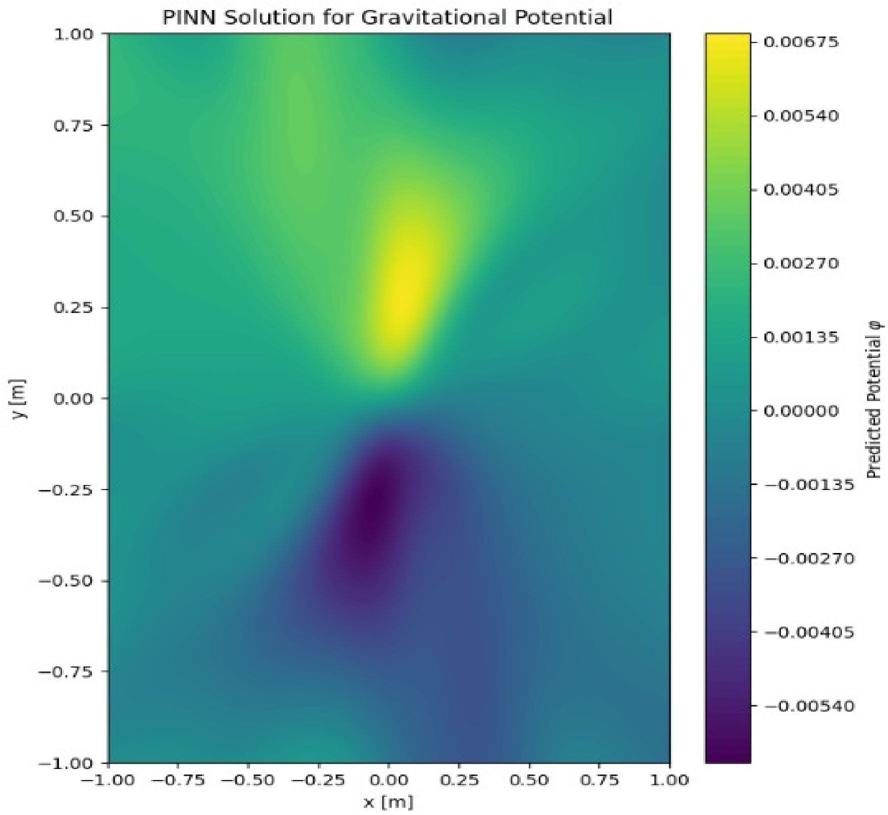

Figure 3: PINN Solution for Gravitational Potential.

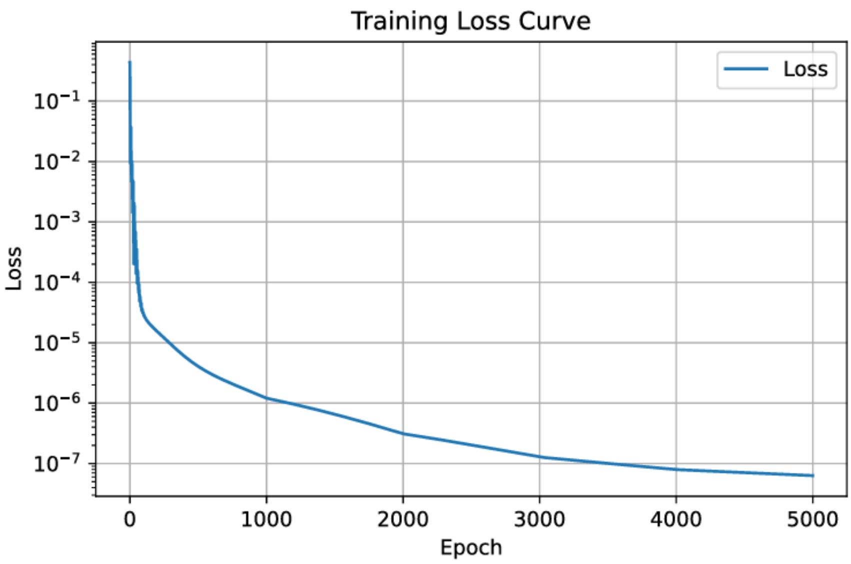

Figure 4: The dynamics of the loss function during the PINN training process.

The absence of pronounced oscillations or regular spikes in the curve suggests that the chosen configuration of hyperparameters (Adam optimizer with a learning rate of lr = 1e-3 and the applied network architecture) ensures a stable training process without signs of overfitting or numerical instability.

Thus, the presented graph confirms the correctness of the chosen hyperparameters and architectural decisions, and demonstrates the effectiveness of the applied training methodology for solving the given task.

The results confirm the capability of the PINN approach to handle the inverse gravimetry problem with high accuracy and robustness, especially under limited or noisy observations.

5. Conclusion

This work has demonstrated the applicability and potential of Physics-Informed Neural Networks (PINNs) for solving the inverse gravimetry problem. By directly incorporating physical laws into the neural network training process, PINNs provide not only high accuracy but also interpretability of the obtained solutions—an aspect particularly valuable in geophysical applications where measurements are often sparse or noisy.

The proposed methodology is based on the integration of the Poisson equation, boundary conditions, and experimental measurements into a unified loss function. This multi-component formulation ensures that the solutions are physically consistent, stable, and robust even under conditions of limited or imperfect data. The careful design of the loss function allows the network to go beyond simple interpolation of observations and instead learn in accordance with fundamental physical principles.

Numerical experiments performed on synthetic datasets confirmed the effectiveness of the approach: high accuracy in density reconstruction was achieved, and the potential of PINNs for addressing gravimetric problems was clearly demonstrated. These results suggest that PINNs can serve as a reliable tool for modeling and analyzing complex geophysical processes.

Future research will focus on extending the proposed methodology to multidimensional formulations and more complex three-dimensional geological structures. To enhance the practical significance of the study, future work will also include testing on real gravimetric observations and comparisons with classical inversion approaches, such as Tikhonov regularization. Additionally, integrating complementary geophysical datasets such as seismic and magnetic measurements may further improve the reliability and resolution of interpretations. The inclusion of a real case study or field measurements will contribute to demonstrating the applicability of the proposed framework and support more informed decision making in geology and natural resource management.

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Declaration on Generative AI

The authors have not employed any Generative AI tools.

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