# Preferential Temporal Description Logics with Typicality, Weighted KBs and Preference Combination

Mario Alviano[1,*], Laura Giordano[2,*] and Daniele Theseider Dupré[2,*]

[1]DEMACS, University of Calabria, Via Bucci 30/B, 87036 Rende (CS), Italy
[2]DISIT, University of Piemonte Orientale, Viale Michel 11, 1512 Alessandria, Italy

**Abstract**

In this paper we study a preferential temporal description logic with typicality, $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, which allows for defeasible reasoning in a temporal description logic. We consider an encoding of preferential temporal description logics with typicality into $LTL_{\mathcal{ALC}}$, a result which extends to the multi-preferential case. For the multi-preferential case, we revisit a previous semantics of weighted knowledge bases by considering preference combination.

**Keywords**

Typicality Logics, Description Logics, Temporal Logics

## 1. Introduction

Preferential and rational extensions of Description Logics (DLs) have been widely studied in the last decades, starting from [1, 2, 3]. They allow reasoning with exceptions through the identification of *prototypical properties* of individuals or classes of individuals. *Defeasible inclusions* are allowed in the knowledge base to model typical, but non-strict, properties of individuals, that is, defeasible properties which admit exceptions. Their semantics extends DL semantics with a preference relation among domain individuals, along the lines of the preferential semantics introduced by Kraus, Lehmann and Magidor [4, 5] (KLM for short). Preferential and rational extensions of the description logic $\mathcal{ALC}$ [6] have been studied [2, 3], and different closure constructions have been developed [7, 8, 9, 10, 11, 12], inspired by Lehmann and Magidor's rational closure [5] and Lehmann's lexicographic closure [13]. More recently, *multi-preferential* extensions of DLs have been developed, allowing multiple preference relations with respect to different concepts [14, 15, 16], as semantics for ranked and for weighted knowledge bases with typicality [14, 15, 16, 17], in the two-valued and in the many-valued case.

Temporal extensions of Description Logics are well-studied in the DL literature (see the survey in [18] on temporal DLs, their decidability and complexity). While preferential extensions of LTL with defeasible temporal operators have been recently studied [19, 20, 21] to enrich temporal formalisms with non-monotonic reasoning features, preferential extensions (*with typicality*) of temporal DLs have been considered in [22], building on $LTL_{\mathcal{ALC}}$ [18], a temporal extension of $\mathcal{ALC}$ based on Linear Time Temporal Logic (LTL).

Generalizing the approach in [3], the preferential temporal description logic with typicality, $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, allows for combining the typicality operator with the temporal modalities of LTL. The resulting temporal DL with typicality allows for representing temporal properties of concepts which admit exceptions, e.g., professors normally teach at least one course until the end of the semester, although exceptions are permitted.

DL 2026: 39th International Workshop on Description Logics, July 17–19, 2026, Lisbon, Portugal
*Corresponding author.
mario.alviano@unical.it (M. Alviano); laura.giordano@uniupo.it (L. Giordano); dtd@uniupo.it (D. Theseider Dupré)
https://alviano.net/ (M. Alviano); https://people.unipmn.it/laura.giordano/ (L. Giordano); https://people.unipmn.it/dtd/ (D. Theseider Dupré)
0000-0002-2052-2063 (M. Alviano); 0000-0001-9445-7770 (L. Giordano); 0000-0001-6798-4380 (D. Theseider Dupré)

The preferential temporal description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ can be polynomially encoded into $LTL_{\mathcal{ALC}}$, which allows borrowing decidability and complexity results from $LTL_{\mathcal{ALC}}$.

In [22], a multi-preferential extension of $LTL_{\mathcal{ALC}}$ with typicality is also considered, allowing a concept-wise preferential semantics where different preferences $<_{C_i}$ are associated to different distinguished concepts $C_i$. The encoding also applies to such a case, and allows for the definition of closure constructions based on weighted knowledge bases [15, 23] in the temporal case.

In this paper, we revisit the semantics of weighted knowledge bases proposed in [22]. To deal with general typicality concepts $\mathbf{T}(D)$, rather than introducing a *global* preference relation, we consider an approach for *combining preferences*, that was first proposed in [24] for weighted Conditional ASP programs. The idea is to define a preference relation $\leq_D$ with respect to a *boolean combination* $D$ of the distinguished concepts $C_i$.

## 2. A Temporal DL with Typicality

The concepts of the temporal description logic with typicality $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ can be formed from standard $\mathcal{ALC}$ constructors, using the temporal operators $\bigcirc$ (next), $\mathcal{U}$ (until), $\Diamond$ (eventually) and $\Box$ (always) of linear time temporal logic (LTL), as well as the *typicatily operator* $\mathbf{T}$.

Let $N_C$ be a set of concept names, $N_R$ a set of role names and $N_I$ a set of individual names. The set of $\mathcal{ALC}$ *extended concepts* can be defined inductively as follows:

$$C ::= A \mid \top \mid \bot \mid C \sqcap D \mid C \sqcup D \mid \neg C \mid \forall r.C \mid \exists r.C \mid \bigcirc C \mid C\mathcal{U}D \mid \Diamond C \mid \Box C \mid \mathbf{T}(C)$$

where $A \in N_C$, and $C$ and $D$ are extended concepts.

The instances of concept $\mathbf{T}(C)$ are intended to be the *typical instances* of concept $C$. When a concept $\mathbf{T}(C)$ occurs on the left hand side of a concept inclusion, it allows for defeasible properties of a concept $C$ to be expressed. A concept inclusion $\mathbf{T}(C) \sqsubseteq D$ means that the *typical* instances of concept $C$ are instances of concept $D$ (normally, $C$'s are $D$'s). One can therefore distinguish between *strict inclusions* ($C \sqsubseteq D$), describing properties that hold for all instances of $C$, and *defeasible inclusions* ($\mathbf{T}(C) \sqsubseteq D$) that only hold for the typical instances of $C$.

As usual, we assume that the typicality operator $\mathbf{T}$ cannot be nested. *Extended concepts* can be built by combining the concept constructors in $LTL_{\mathcal{ALC}}$ with the typicality operator. For instance, the defeasible inclusion $\mathbf{T}(\mathit{Professor}) \sqsubseteq (\exists \mathit{teaches}.\mathit{Course})\mathcal{U}\mathit{Semester\_End}$ means that normally professors teach at least one course until the end of the semester (but exceptions are allowed), while the inclusion $\exists \mathit{lives\_in}.\mathit{Town} \sqcap \mathit{Young} \sqsubseteq \mathbf{T}(\Diamond \exists \mathit{granted}.\mathit{Loan})$ means that persons living in town and being young are typical in the set of individuals eventually being granted a loan.

To define the semantics of $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ in terms of preferential models, we extend ordinary models of $LTL_{\mathcal{ALC}}$ [18] with a *preference relation* $<^n$ on the domain, for each time point $n$, whose intuitive meaning is to compare the "typicality" of domain elements at the different time points, where $x <^n y$ means that *domain element* $x$ *is more typical than* $y$ *at time point* $n$. The typical instances of an (extended) concept $C$ at time point $n$ (the instances of $\mathbf{T}(C)$ at time point $n$) are the instances $x$ of $C$ at time point $n$ that are minimal with respect to the preference relation $<^n$ (i.e., no other instances of $C$ are preferred to $x$ at time point $n$).

In the following, we introduce a relation $< \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and a collection of preference relations $<^n$, one for each time point $n$ (the projections of $<$ over the single time points).

A *preferential temporal interpretation* for $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ is a triple $\mathcal{I} = (\Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}})$, where

- $\Delta^{\mathcal{I}}$ is a nonempty domain;
- the relation $< \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ associates to each time point $n$ a preference $<^n$ over the domain $\Delta^{\mathcal{I}}$ *such that*: for all $n \in \mathbb{N}$, $<^n = \{(c, d) \mid (n, c, d) \in <\}$ and $<^n$ is an *irreflexive*, *transitive* and *well-founded* relation over $\Delta^{\mathcal{I}}$;
- $\cdot^{\mathcal{I}}$ is an extension function that maps each concept name $C \in N_C$ to a set $C^{\mathcal{I}} \subseteq \mathbb{N} \times \Delta^{\mathcal{I}}$, each role name $r \in N_R$ to a relation $r^{\mathcal{I}} \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$, and each individual name $a \in N_I$ to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$.

Following [18] we have assumed individual names to be *rigid*, i.e., they have the same interpretation at any time point. For a pair $(n, d) \in \mathbb{N} \times \Delta^{\mathcal{I}}$, $n$ represents a time point and $d$ a domain element; $(n, d) \in C^{\mathcal{I}}$ means that $d$ is an instance of concept $C$ at time point $n$, and similarly for $(n, d_1, d_2) \in r^{\mathcal{I}}$. Function $\cdot^{\mathcal{I}}$ is extended to complex concepts as follows:

$$\top^{\mathcal{I}} = \mathbb{N} \times \Delta^{\mathcal{I}} \qquad \bot^{\mathcal{I}} = \emptyset \qquad (\neg C)^{\mathcal{I}} = (\mathbb{N} \times \Delta^{\mathcal{I}}) \backslash C^{\mathcal{I}}$$
$$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}} \qquad (C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$
$$(\exists r.C)^{\mathcal{I}} = \{(n, c) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \exists y.(n, c, d) \in r^{\mathcal{I}} \text{ and } (n, d) \in C^{\mathcal{I}}\}$$
$$(\forall r.C)^{\mathcal{I}} = \{(n, c) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \forall y.(n, c, d) \in r^{\mathcal{I}} \Rightarrow (n, d) \in C^{\mathcal{I}}\}$$
$$(\bigcirc C)^{\mathcal{I}} = \{(n, c) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid (n+1, c) \in C^{\mathcal{I}}\}$$
$$(\Diamond C)^{\mathcal{I}} = \{(n, c) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \exists m \geq n \text{ such that } (m, c) \in C^{\mathcal{I}}\}$$
$$(\Box C)^{\mathcal{I}} = \{(n, c) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \forall m \geq n, \ \ (m, c) \in C^{\mathcal{I}}\}$$
$$(C \mathcal{U} D)^{\mathcal{I}} = \{(n, c) \in \mathbb{N} \times \Delta^{\mathcal{I}} \mid \exists m \geq n \text{ s.t. } (m, c) \in D^{\mathcal{I}} \text{ and } (k, c) \in C^{\mathcal{I}}, \forall k \ (n \leq k < m)\}$$
$$(\mathbf{T}(C))^{\mathcal{I}} = \{(n, c) \mid c \in Min_{<^n}(C^{\mathcal{I}}_n), \text{ for } n \in \mathbb{N}\}$$

where $C^{\mathcal{I}}_n = \{d \mid (n, d) \in C^{\mathcal{I}}\}$ are the instances of $C$ at time point $n$, and $Min_{<^n}(S) = \{u : u \in S$ and $\nexists z \in S$ s.t. $z <^n u\}$.

Note that a preferential temporal interpretation, as defined above, is obtained by adding a preference relation $<$ to temporal interpretations for $LTL_{\mathcal{ALC}}$ in [18] and, additionally, by providing an interpretation $(\mathbf{T}(C))^{\mathcal{I}}$ to typicality concepts $\mathbf{T}(C)$. In the following we will refer to temporal interpretations for $LTL_{\mathcal{ALC}}$ as pairs $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, assuming the evaluation of $LTL_{\mathcal{ALC}}$ constructs reported above.

For each timepoint $n$, relation $<^n$ has the properties of a preference relation in KLM preferential interpretations [4, 5]. When *modularity* also holds for $<^n$, the relation $<^n$ has the properties of a preference relation in rational (or ranked) KLM interpretations [5]. The fact that each irreflexive and transitive relation $<^n$ on $\Delta$ is well-founded guarantees that there are no infinite descending chains $x_0, x_1, x_2, \ldots$ of elements of $\Delta^{\mathcal{I}}$, such that $x_1 <^n x_0, x_2 <^n x_1, \ldots$.

Although we have considered a constant domain $\Delta^{\mathcal{I}}$, as for $LTL_{\mathcal{ALC}}$ [18], expanding domains can be considered, by letting a domain $\Delta^{\mathcal{I}}_n$, for each time point $n$, with $\Delta^{\mathcal{I}}_n \subseteq \Delta^{\mathcal{I}}_{n+1}$, for all $n$.

An $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ *knowledge base* $K = (\mathcal{T}, \mathcal{A})$, where $\mathcal{T}$ is a set of concept inclusions and $\mathcal{A}$ is a set of assertions of the form $C(a)$ or $r(a, b)$, where $C$ is an extended concept, $r$ is a role name in $N_R$, and $a$ and $b$ are individual names in $N_I$.

The notions of satisfiability and model of a knowledge base can be extended to $LTL^{\mathbf{T}}_{\mathcal{ALC}}$: the assertions in $\mathcal{A}$ are evaluated at time point 0; the inclusions $C \sqsubseteq D$ in $\mathcal{T}$ have to be satisfied at all time points[1].

**Definition 1** (Satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$). *Given an $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$: $\mathcal{I}$ satisfies a concept inclusion $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$; $\mathcal{I}$ satisfies an assertion $C(a)$ (resp., $r(a, b)$) iff $(0, a^{\mathcal{I}}) \in C^{\mathcal{I}}$ (resp., $(0, a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$). $\mathcal{I}$ satisfies a TBox $\mathcal{T}$ if $\mathcal{I}$ satisfies all the concept inclusions $C \sqsubseteq D$ in $\mathcal{T}$. $\mathcal{I}$ satisfies an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies all the assertions in $\mathcal{A}$.*
*A concept $C$ is satisfiable w.r.t. TBox $\mathcal{T}$, if there is an interpretation $\mathcal{I}$ such that $C^{\mathcal{I}} \neq \emptyset$, and $\mathcal{I}$ satisfies all the concept inclusions in $\mathcal{T}$.*
*Given an $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ knowledge base $K = (\mathcal{T}, \mathcal{A})$, the interpretation $\mathcal{I}$ is a* model *of $K$ if $\mathcal{I}$ satisfies all concept inclusions in $\mathcal{T}$ and all assertions in $\mathcal{A}$.*

In [25] a modal interpretation of the typicality operator $\mathbf{T}$ in terms of a Gödel-Löb modality $\Box_<$ has been used to define an encoding of a preferential extension of $\mathcal{SHIQ}$ with typicality into $\mathcal{SHIQ}$, by introducing a new role $P_<$ in the DL language to represent the preference relation.

In the next section, we will extend this encoding to the temporal case.

[1] Note that, among the linear time temporal description logics studied in [18], in this paper we are building on $LTL_{\mathcal{ALC}}$ with *non-temporal TBoxes*, where the TBox $\mathcal{T}$ is a set of concept inclusions $C \sqsubseteq D$, where $C, D$ contain temporal operators, but no temporal operator is applied to concept inclusions themselves.

## 3. Encoding $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ in $LTL_{\mathcal{ALC}}$

For the (non-temporal) case of $\mathcal{ALC}$ with typicality [3], it was observed that the meaning of $\mathbf{T}$ can be split into two parts: for any element $x \in \Delta^{\mathcal{I}}$, $x \in (\mathbf{T}(C))^{\mathcal{I}}$ when (i) $x \in C^{\mathcal{I}}$, and (ii) there is no $y \in C^{\mathcal{I}}$ such that $y < x$. In order to isolate the second part of the meaning of $\mathbf{T}$, a Gödel-Löb modality $\Box_<$ was introduced (not to be confused with the temporal modality $\Box$), and the preference relation $<$ was interpreted as the inverse of the accessibility relation of this modality. Well-foundedness of $<$ was needed to ensure that typical elements of $C^{\mathcal{I}}$ exist whenever $C^{\mathcal{I}} \neq \emptyset$, by avoiding infinitely descending chains of elements. For $\mathcal{ALC}$ with typicality, it was proven that $x$ is a typical instance of $C$ if and only if it is an instance of $C$ and $\Box_< \neg C$, that is: given an interpretation $\mathcal{I}$, a concept $C$ and an element $x \in \Delta$, $x \in (\mathbf{T}(C))^{\mathcal{I}}$ iff $x \in (C \sqcap \Box_< \neg C)^{\mathcal{I}}$ [3].

In the temporal case, the interpretation of $\mathbf{T}(C)$ at a time point $n$ depends on the preference relation $<$ at time point $n$, i.e., on $<^n$. An instance of $\mathbf{T}(C)$ (at time point $n$) is an instance of $C$ such that all elements preferred to it (at time point $n$) are not instances of $C$ (at time point $n$). We represent the preference relation $<$ in a preferential temporal interpretation $I$, by introducing a new role name $P_<$ in the language. Also, we represent a concept $\mathbf{T}(C)$ with the concept $C \sqcap \Box_{\neg C}$, where $\Box_{\neg C}$ is a *new concept name* which is intended to capture the idea that no preferred domain elements is an instance of concept $C$. Finally, we introduce additional concept inclusion axioms to capture the interplay between role name $P_<$ and the new concepts $\Box_{\neg C}$, as well as to enforce the properties of the preference relations $<^n$.

For a TBox $\mathcal{T}$ in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, we define an encoding $\mathcal{T}'$ in $LTL_{\mathcal{ALC}}$ as follows. For each $\mathbf{T}(E)$ occurring in $K$ (where $E$ is any extended concept not containing the typicality operator), we introduce a new atomic concept $\Box_{\neg E}$ and, for each inclusion $C \sqsubseteq D \in \mathcal{T}$, we introduce in $\mathcal{T}'$ the inclusion $C' \sqsubseteq D'$, where $C'$ and $D'$ are obtained from $C$ and $D$, respectively, by replacing the occurrence of any concept $\mathbf{T}(E)$ with the concept $E \sqcap \Box_{\neg E}$. Note that concept $\Box_{\neg E}$ (as any other concept) may have a different interpretation at different time points. The following concept inclusion axioms are introduced in $\mathcal{T}'$, for all concepts $E$ such that $\mathbf{T}(E)$ occurs in $\mathcal{T}$:

$$\Box_{\neg E} \sqsubseteq \forall P_<.(\neg E \sqcap \Box_{\neg E}) \tag{1}$$

$$\neg \Box_{\neg E} \sqsubseteq \exists P_<.(E \sqcap \Box_{\neg E}) \tag{2}$$

The first inclusion is exploited to account for the transitivity of the preference relations $<^n$. The second inclusion accounts for the smoothness (see [5]) of the preference relations $<^n$, i.e., the fact that if an element is not a typical $E$ element at a time point $n$, then there must be a typical $E$ element preferred to it according to $<^n$. The property holds for a well-founded relation $<^n$.

We also define the ABox $\mathcal{A}'$ by replacing each occurrence of the concept $\mathbf{T}(E)$ in any individual assertions $C(d)$ in $\mathcal{A}$, with the concept $E \sqcap \Box_{\neg E}$, and by including in $\mathcal{A}'$ all the resulting assertions. All the assertions of the form $R(a, b) \in \mathcal{A}$ are included unaltered in $\mathcal{A}'$. The following result holds.

**Proposition 1.** *For a temporal knowledge base $K = (\mathcal{T}, \mathcal{A})$ in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, let $K' = (\mathcal{T}', \mathcal{A}')$ be the encoding of $K$ in $LTL_{\mathcal{ALC}}$. It holds that (i) $K$ is satisfiable in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ iff $K'$ is satisfiable in $LTL_{\mathcal{ALC}}$. Also, (ii) $C$ is satisfiable wrt $\mathcal{T}$ in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ iff $C'$ is satisfiable wrt $\mathcal{T}'$ in $LTL_{\mathcal{ALC}}$.*

*Proof.* We provide a detailed sketch of the proof of part *(i)* (the proof of part *(ii)* is similar).

Let us consider the two directions for statement *(i)*.

*(If)* Assume there is a preferential model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$ of $K$ in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$. We build an $LTL_{\mathcal{ALC}}$ model $\mathcal{I}' = \langle \Delta^{\mathcal{I}'}, \cdot^{\mathcal{I}'} \rangle$ satisfying $K'$ as follows:

- $\Delta^{\mathcal{I}'} = \Delta^{\mathcal{I}}$;
- for all concept names $B$ in $N_C$, $B^{\mathcal{I}'} = B^{\mathcal{I}}$;
- for all role names $R$ in $N_R$, $R^{\mathcal{I}'} = R^{\mathcal{I}}$;
- for all individual names $a$ in $N_I$, $a^{\mathcal{I}'} = a^{\mathcal{I}}$;
- $(n, y, x) \in P_<^{\mathcal{I}'}$ iff $(n, x, y) \in <$ in the model $\mathcal{I}$;

- $\Box_{\neg E}^{\mathcal{I}'} = \{(n, x) :$ for all $y \in \Delta^{\mathcal{I}}$, if $y <^n x$ in $\mathcal{I}$, then $(n, y) \in (\neg E)^{\mathcal{I}}\}$.

We have to prove that $\mathcal{I}'$ is a model of $K'$. First, observe that, as $<$ is transitive and antisymmetric, by construction the relation $P_<^{\mathcal{I}'}$ is transitive and antisymmetric as well.

From the construction, it can be proven by induction on the structure of concepts that, for all concepts $C$ occurring in the KB:

$$(n, y) \in C^{\mathcal{I}} \text{ if and only if } (n, y) \in (C')^{\mathcal{I}'}. \tag{3}$$

For the case $C = \mathbf{T}(E)$, we can prove that:

$$(\mathbf{T}(E))^{\mathcal{I}} = (E \sqcap \Box_{\neg E})^{\mathcal{I}'} \tag{4}$$

If $(n, x) \in (\mathbf{T}(E))^{\mathcal{I}}$, for some $n \in \mathbb{N}$, then $x \in Min_{<^n}(E_n^{\mathcal{I}})$. Hence, $(n, x) \in E^{\mathcal{I}}$ and for all $y <^n x$, $y \notin E_n^{\mathcal{I}}$, i.e., $(n, y) \notin E^{\mathcal{I}}$. From (3), $(n, x) \in E^{\mathcal{I}'}$ and by construction it follows that $(n, x) \in \Box_{\neg E}^{\mathcal{I}'}$. Therefore, $(n, x) \in (E \sqcap \Box_{\neg E})^{\mathcal{I}'}$. The vice-versa holds as well.

To prove that $\mathcal{I}'$ is a model of $K'$, first we prove that $\mathcal{I}'$ satisfies $\mathcal{T}'$, that is, for each concept inclusion $C \sqsubseteq D$ in $\mathcal{T}$, $\mathcal{I}'$ satisfies the corresponding concept inclusion $C' \sqsubseteq D'$ in $\mathcal{T}'$. From $(n, y) \in (C')^{\mathcal{I}'}$, it follows that $(n, y) \in C^{\mathcal{I}}$ by (3). As $\mathcal{M}$ is a model of $K$, $(n, y) \in D^{\mathcal{I}}$ holds. Hence, by (3), $(n, y) \in (D')^{\mathcal{I}'}$ also holds.

It is easy to see that the new concept inclusion axioms (1) and (2) added in $\mathcal{T}'$ are also satisfied in $\mathcal{I}'$. For axiom (1), observe that:

$\Box_{\neg A}^{\mathcal{I}'} = \{(n, x) :$ for all $y \in \Delta^{\mathcal{I}}$, if $y <^n x$ in $\mathcal{I}$, then $(n, y) \in (\neg A)^{\mathcal{I}}\}$.

and, by definition of $P_<^{\mathcal{I}'}$ and (3):

$\Box_{\neg A}^{\mathcal{I}'} = \{(n, x) :$ for all $y \in \Delta'^{\mathcal{I}}$, if $(n, x, y) \in P_<^{\mathcal{I}'}$, then $(n, y) \in (\neg A)^{\mathcal{I}'}\}$.

To prove that axiom (1) is satisfied in $\mathcal{I}'$, we have to prove that for all $x \in \Delta^{\mathcal{I}}$, if $(n, x) \in (\Box_{\neg A})^{\mathcal{I}'}$, then $(n, x) \in (\forall P_<.(\neg A \sqcap \Box_{\neg A}))^{\mathcal{I}'}$.

Assume $(n, x) \in (\Box_{\neg A})^{\mathcal{I}'}$. For all $y \in \Delta^{\mathcal{I}}$ such that $(n, x, y) \in P_<^{\mathcal{I}'}$, we have already shown that $(n, y) \in (\neg A)^{\mathcal{I}'}$. We still have to prove that $(n, y) \in (\Box_{\neg A})^{\mathcal{I}'}$.

By contradiction, if $(n, y) \notin (\Box_{\neg A})^{\mathcal{I}'}$, from the construction, there is a $z$ such that $z <^n y$ and $(n, z) \notin (\neg A)^I$. As $y <^n x$, by the transitivity of $<^n$, it follows that $z <^n x$, which contradicts the hypothesis that $(n, x) \in (\Box_{\neg A})^{I'}$. Therefore, $(n, y) \in (\Box_{\neg A})^{\mathcal{I}'}$ and, hence, $(n, x) \in (\forall P_<.(\neg A \sqcap \Box_{\neg A}))^{I'}$. Thus, inclusion axiom (1) is satisfied in $\mathcal{I}'$.

Let us prove that axiom (2) is satisfied in $\mathcal{I}'$.

Assume $(n, x) \in (\neg\Box_{\neg A})^{\mathcal{I}'}$. Then there is a $z \in \Delta^{\mathcal{I}}$ such that $z <^n x$, and $(n, z) \in A^{\mathcal{I}}$. As the relation $<^n$ is well-founded, there must be a $<^n$-minimal element $y <^n x$, such that $(n, y) \in A^{\mathcal{I}}$, and such that, for all $w <^n y$, $(n, w) \notin A^{\mathcal{I}}$.

Hence, by construction and by (3), there is an element $y \in \Delta^{\mathcal{I}'}$, such that $(n, x, y) \in P_<^{\mathcal{I}'}$, and $(n, y) \in A^{\mathcal{I}'}$. Furthermore, by construction, $(n, y) \in \Box_{\neg A}^{\mathcal{I}'}$.

Therefore, $(n, x) \in (\exists P_<.(A \sqcap \Box_{\neg A}))^{\mathcal{I}'}$, and we can conclude that all the inclusion axioms in the TBox $\mathcal{T}'$ are satisfied in $\mathcal{I}'$.

Let us prove that all assertions in $\mathcal{A}'$ are satisfied in $\mathcal{I}'$. For all the individual assertions $C'(b) \in \mathcal{A}'$, $C(b)$ is in $\mathcal{A}$. As $\mathcal{I}$ satisfies $\mathcal{A}$, $(0, b^I) \in C^{\mathcal{I}}$. As $b^{\mathcal{I}'} = b^{\mathcal{I}}$, by property (3), we have that $(0, b^{\mathcal{I}'}) \in (C')^{\mathcal{I}'}$. Assertions of the form $r(a, b)$ in $\mathcal{A}'$ are as well satisfied in $\mathcal{I}'$, as they also occur in $\mathcal{A}$ and the interpretation of roles and individual names is the same in $\mathcal{I}'$ and in $\mathcal{I}$. This concludes the proof that $\mathcal{I}'$ is a model of $K'$.

*(Only if)* Assume there is an $LTL_{\mathcal{ALC}}$ model $\mathcal{I}' = \langle \Delta^{\mathcal{I}'}, \cdot^{\mathcal{I}'} \rangle$ satisfying $K'$. We build a preferential model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$ of $K$ in $LTL_{\mathcal{ALC}}^{\mathbf{T}}$ as follows:

- $\Delta^{\mathcal{I}} = \Delta^{\mathcal{I}'}$;
- for all concept names $B$ in $N_C$, $B^{\mathcal{I}} = B^{\mathcal{I}'}$;
- for all role names $R$ in $N_R$, $R^{\mathcal{I}} = R^{\mathcal{I}'}$;
- for all individual names $a$ in $N_I$, $a^{\mathcal{I}} = a^{\mathcal{I}'}$.

It would seem natural to define each preference relations $<^n$ in $\mathcal{I}$ as the transitive closure of the set of pairs: $P^{\mathcal{I}'}_{<,n} = \{(x,y) : \ (n,x,y) \in P^{\mathcal{I}'}_{<}\}$. However, this would not guarantee that the resulting relation $<^n$ is well-founded. To define the relation $<$ in $\mathcal{I}$ so that it is transitive, well-founded and irreflexive we introduce some more notation. For all $y \in \Delta^{\mathcal{I}'}$, we let $\mathcal{S}_{n,y}$ be the set of all concepts of the form $\Box_{\neg A}$ such that $(n,y) \in (\Box_{\neg A})^{\mathcal{I}'}$ i.e.,

$$S_{n,y} = \{\Box_{\neg A} \ | \ \Box_{\neg A} \text{ occurs in KB and } \ (n,y) \in (\Box_{\neg A})^{\mathcal{I}'}\}.$$

Observe that, given a sequence of elements $x_0, x_1, \dots, x_i, \dots$ such that $(n, x_i, x_{i+1}) \in P_<{}^{\mathcal{I}'}$, for all $i = 0, 1, \dots$, the sets $S_{n,x_i}$ are non-decreasing: it holds that $S_{n,x_i} \subseteq S_{n,x_{i+1}}$.

In the model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, <, \cdot^{\mathcal{I}} \rangle$ we define the preference relation $<$ by letting:

$$(n,y,x) \in < \text{ iff } (x,y) \in (P^{\mathcal{I}'}_{<,n})^+ \text{ and } S_{n,x} \subset S_{n,y}.$$

That is,

$$y <^n x \text{ iff } (x,y) \in (P^{\mathcal{I}'}_{<,n})^+ \text{ and } S_{n,x} \subset S_{n,y}.$$

In essence, the pairs of elements $x$ and $y$ such that $(x,y) \in (P^{\mathcal{I}'}_{<,n})^+$ but $x$ and $y$ are instances of exactly the same boxed concepts ($S_{n,x} = S_{n,y}$) do not give rise to a triple $(n,y,x)$ in $<$. In this way, for the elements $x_i$ in a descending chain $x_0 >^n \dots >^n x_i >^n x_{i+1} >^n \dots$ the sets $S_{n,x_i}$ are strictly increasing, i.e., it holds that $S_{n,x_i} \subset S_{n,x_{i+1}}$.

As the number of formulas $\Box_{\neg A}$ in $K$ is finite, it is not possible that there is an infinite descending chains of elements $x_0 >^n \dots >^n x_i >^n x_{i+1} >^n \dots$, for some $n$. It is easy to see that, for any $n$, $<^n$ is a transitive, irreflexive and well-founded relation.

As done in the *(If)* direction, it can proven that that $C^{\mathcal{I}} = (C')^{\mathcal{I}'}$, for all concepts $C$ occurring in the KB, and that $\mathcal{I}$ is a model of $K$. □

The encoding above is polynomial in the size of the knowledge base $K$ and, more precisely, if $|K|$ is the size of $K$, the size of $K'$ is $O(|K|)$. While we have considered a constant domain in the proof above, the same encoding can be adopted also for the case of expanding domains.

It was proven [26, 18] that concept satisfiability w.r.t. TBoxes for $LTL_{\mathcal{ALC}}$ is ExpTime-complete, both with expanding domains and with constant domains. This complexity result, as a consequence of Proposition 1 above, extends to the preferential temporal description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$.

**Corollary 1.** *Concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ w.r.t. TBoxes is ExpTime-complete.*

In the next sections, we move to consider a *multi-preferential semantics* for temporal $\mathcal{ALC}$ with typicality, as well as an approach for combining preferences for weighted knowledge bases.

## 4. A Multi-preferential Temporal Extension of $\mathcal{ALC}$

In [22] we considered a *multi-preferential* temporal extension of $\mathcal{ALC}$ with typicality, called $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$. In $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ semantics, a preference relation $<_{C_i}$ is associated with each concept $C_i$ in a set of *distinguished concepts* $\mathcal{C} = \{C_1, \dots, C_k\}$, where the distinguished concepts $C_i$ represent the aspects with respect to which domain individuals are compared. For instance, Tom may be more typical than Bob as a student ($tom <_S bob$), but less typical as an employee ($bob <_E tom$).

In the temporal case, at each time point $n$, there are different preference relations $<^n_{C_1}, \dots, <^n_{C_k}$ one for each $C_i \in \mathcal{C}$. As the typicality operator only applies to distinguished concepts $C_i$, a notion of multi-preferential temporal interpretation $\mathcal{M} = (\Delta^{\mathcal{I}}, <_{C_1}, \dots, <_{C_k}, \cdot^{\mathcal{I}})$ can be defined for $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$, where the preference relations $<_{C_i}$ provide a semantic interpretation of typicality concepts $\mathbf{T}(C_i)$ occurring in an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ knowledge base.

**Definition 2.** *An $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$* multi-preferential temporal interpretation *over $\mathcal{C}$ is a tuple $\mathcal{I} = (\Delta^{\mathcal{I}}, <_{C_1}, \dots, <_{C_k}, \cdot^{\mathcal{I}})$ where:*

- $\Delta^{\mathcal{I}}$ *is a nonempty domain;*
- *for each* $C_i \in \mathcal{C}$, *the relation* $<_{C_i} \subseteq \mathbb{N} \times \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ *associates to each time point* $n$ *a preference* $<^n_{C_i}$ *over the domain* $\Delta^{\mathcal{I}}$ *such that* $<^n_{C_i} = \{(a,b) \mid (n,a,b) \in <_{C_i}\}$ *is an irreflexive, transitive and well-founded relation over* $\Delta^{\mathcal{I}}$*;*
- *The interpretation function* $\cdot^{\mathcal{I}}$, *introduced in Section 2 for temporally extended goals, is extended to typicality concepts* $\mathbf{T}(C_i)$ *as follows:*

$$(\mathbf{T}(C_i))^{\mathcal{I}} = \{(n,d) \mid d \in Min_{<^n_{C_i}}(C^{\mathcal{I}}_{i,n}),\ \textit{for}\ n \in \mathbb{N}\}$$

*where* $C^{\mathcal{I}}_{i,n} = \{d \mid (n,d) \in C^{\mathcal{I}}_i\}$ *are the instances of* $C_i$ *at time point* $n$.

For the moment, let us define an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ knowledge base $K = \langle \mathcal{T}, \mathcal{A} \rangle$ as an $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ knowledge base in which only typicality concepts of the form $\mathbf{T}(C_i)$ may occur. Later, in section 6 we will lift this restriction.

The notions of satisfiability and model of a knowledge base, defined in Section 2 for $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, naturally extend to $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$. In particular, given an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, <_{C_1}, \ldots, <_{C_k}, \cdot^{\mathcal{I}})$: $\mathcal{I}$ satisfies a concept inclusion $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$; $\mathcal{I}$ satisfies an assertion $C(a)$ (resp., $r(a,b)$) iff $(0, a^{\mathcal{I}}) \in C^{\mathcal{I}}$ (resp., $(0, a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$).

As for $LTL^{\mathbf{T}}_{\mathcal{ALC}}$, in the multi-preferential case, an encoding of an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ knowledge base $K$ into an $LTL_{\mathcal{ALC}}$ knowledge base $K'$ can be defined and it can be proven that satisfiability of an $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ knowledge base can be polynomially reduced to satisfiability of an $LTL_{\mathcal{ALC}}$ knowledge base.

To deal with multiple preferences, a new role $P_{<_{C_i}}$ has to be introduced in the language for each distinguished concept $C_i \in \mathcal{C}$, and new concept names $\Box_{\neg C_i}$ are needed to encode each typicality concept $\mathbf{T}(C_i)$ occurring in the TBox with a concept $C_i \sqcap \Box_{\neg C_i}$ in $LTL_{\mathcal{ALC}}$. Axioms corresponding to axioms (1) and (2) in Section 3 need as well to be introduced for all $C_i \in \mathcal{C}$.

The result that concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ w.r.t. TBoxes is ExpTime-complete also extends to the multi-preferential temporal $\mathcal{ALC}$ under the $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ semantics.

## 5. Temporal Weighted KBs and weakly faithful interpretations

Many closure constructions have been proposed for preferential description logics [7, 10, 11, 14]. In this section, we revisit the notion of from *temporal weighted* $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ *knowledge base* from [22]. Weighted knowledge bases have been previously considered in [15, 23] for non-temporal DLs with typicality, and more recently in [24] in a conditional answer set programming framework (CondASP). The idea is to allow for the definition of prototypical properties of a class, with (positive or negative) weights representing the degree of plausibility/implausibility of each property, including the temporal dimension.

A *temporal weighted* $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ *knowledge base* $K$ *over* $\mathcal{C}$ is a tuple $\langle \mathcal{T}, \mathcal{T}_{C_1}, \ldots, \mathcal{T}_{C_k}, \mathcal{A} \rangle$, where $\mathcal{T}$ is a TBox and $\mathcal{A}$ an ABox as in $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$, and, for each $C_i \in \mathcal{C}$, $\mathcal{T}_{C_i}$ is a set of *weighted defeasible inclusions*, of the form $\mathbf{T}(C_i) \sqsubseteq D_h$, having a weight $w^i_h$, a real number. Here, both $C_i$ and $D_h$ are $LTL_{\mathcal{ALC}}$ concepts, and do not contain occurrences of the typicality operator. Let us report the following example from [22].

**Example 1.** *Let* $K = \langle \mathcal{T}, \mathcal{T}_{Emp}, \mathcal{T}_{Student}, \mathcal{A} \rangle$ *be a weighted knowledge base over the set of distinguished concepts* $\mathcal{C} = \{Emp, Student, Professor\}$, *with empty ABox, where* $\mathcal{T}$ *containing the inclusions:*

$Emp \sqsubseteq Adult$ $\qquad$ $Adult \sqsubseteq \exists has\_SSN.\top$ $\qquad$ $PhdStudent \sqsubseteq Student$

$\mathbf{T}(Professor) \sqsubseteq (\exists Teaches.Course) \mathcal{U} Semester\_End$

*where, for instance,* $\mathcal{T}_{Emp}$ *contains the weighted defeasible inclusions:*

$(d_1)$ $\mathbf{T}(Emp) \sqsubseteq Young$, *- 50*

$(d_2)$ $\mathbf{T}(Emp) \sqsubseteq \exists has\_boss.Emp$, *100*

$(d_3)$ $\mathbf{T}(Emp) \sqsubseteq \exists has\_classes.\top$, *-70;*

*and $\mathcal{T}_{Student}$ contains the defeasible inclusions:*

$(d_4)$ $\mathbf{T}(Student) \sqsubseteq Young,$ *90*

$(d_5)$ $\mathbf{T}(Student) \sqsubseteq \exists has\_classes.\top,$ *80*

$(d_6)$ $\mathbf{T}(Student) \sqsubseteq \exists hasScholarship.\top,$ *-30*

$(d_7)$ $\mathbf{T}(Student) \sqsubseteq \Diamond(Promoted \sqcup Rejected),$ *100*

*The meaning is that, while an employee normally has a boss, he is not likely to be young or to have classes. Furthermore, between the two defeasible inclusions $(d_1)$ and $(d_3)$, the second one is considered to be less plausible than the first one. Negative weights represent penalties. Given two employees Tom and Bob such that, at time point $n$, Tom is not young, has no boss and has classes, while Bob is not young, has a boss and has no classes, considering the weights above, we will regard Bob as being more typical than Tom as an employee at time point $n$.*

In the definition of the semantic of the language we depart from [22]. Given a *temporal multi-preferential interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, <_{C_1}, \ldots, <_{C_k}, \cdot^{\mathcal{I}})$, a concept $C_i \in \mathcal{C}$ and a domain element $x \in \Delta^{\mathcal{I}}$, the *weight* $W^{\mathcal{I}}_{C_i,n}(x)$ *of* $x$ *w.r.t.* $C_i$ *in* $\mathcal{I}$ *at time point* $n$, is defined considering the inclusions $(\mathbf{T}(C_i) \sqsubseteq D_h\ , w^i_h) \in \mathcal{T}_{C_i}$, as follows:

$$W^{\mathcal{I}}_{C_i,n}(x) = \sum_{h:(n,x)\in D^{\mathcal{I}}_h} w^i_h$$

Informally, the weight $W^{\mathcal{I}}_{C_i,n}(x)$ of $x$ wrt $C_i$ at time point $n$ is the sum of the weights of all defeasible inclusions $\mathbf{T}(C_i) \sqsubseteq D_h$, for $C_i$ such that $x$ is an instance of $D_h$ in $\mathcal{I}$. The more plausible are such inclusions, the higher is the weight of $x$ wrt. $C_i$; the less plausible they are, the lower is the weight. If no defeasible inclusion $(\mathbf{T}(C_i) \sqsubseteq D_h\ , w_{i,h})$ is in $K$ for a distinguished concept $C_i$, then $W_{C_i,n}(x) = 0$ holds for all domain elements $x$.

Given a set $\mathcal{T}_{C_i}$ of defeasible inclusions for concept $C_i$, the weights $W^{\mathcal{I}}_{C_i,n}(x)$, for the different domain elements $x$, are intended to constrain the preference relation $<^n_{C_i}$. At time point $n$, $x$ is preferred to $y$ wrt $C_i$ (i.e., $x <^n_{C_i} y$) if the weight $W^{\mathcal{I}}_{C_i,n}(x)$ is higher than the weight $W^{\mathcal{I}}_{C_i,n}(y)$. That is, for $x, y \in \Delta^{\mathcal{I}}$,

$$x <^n_{C_i} y \quad \text{iff} \quad W^{\mathcal{I}}_{C_i,n}(x) > W^{\mathcal{I}}_{C_i,n}(y) \tag{5}$$

The higher is the weight of an element $x$ wrt. $C_i$ at time point $n$, the more preferred is the element wrt. $C_i$ at time point $n$. In the example above, $W^{\mathcal{I}}_{C_i,n}(bob) = 100 > W^{\mathcal{I}}_{C_i,n}(tom) = -70$ (for $C_i = Emp$) and, hence, $bob <_{Emp} tom$, i.e., Bob is more typical than Tom as an employee.

Let us define a *concept-wise multi-preferential temporal semantics* (cw$^m$ temporal semantics) for a weighted knowledge base.

**Definition 3.** *Given a weighted $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$ knowledge base $K = \langle \mathcal{T}, \mathcal{T}_{C_1}, \ldots, \mathcal{T}_{C_k}, \mathcal{A} \rangle$ over $\mathcal{C}$, a* concept-wise multi-preferential temporal model *(cw$^m$-model) of $K$ is a multi-preferential temporal interpretation $\mathcal{I} = \langle \Delta^{\mathcal{I}}, <_{C_1}, \ldots, <_{C_k}, \cdot^{\mathcal{I}} \rangle$, such that: $\mathcal{I}$ satisfies all the inclusions in $\mathcal{T}$ and assertions in $\mathcal{A}$ and, for all $j = 1, \ldots, k$,*

$$<_{C_j} = \{(n, x, y) : n \in \mathbb{N} \text{ and } x <^n_{C_j} y\},$$

*where each $<^n_{C_j}$ is defined according to condition (5).*

As in the non-temporal case, one can strengthen the semantics by enforcing an *agreement* between the interpretation of concepts $C_i$ at some time point $n$ in $\mathcal{I}$, and the preference relations $<^n_{A_i}$ among domain elements at the same time point. In particular, we may require that the interpretation $\mathcal{I}$ is *weakly faithful wrt. $C_i$ at time point $n$*, by requiring that, for all $x, y \in \Delta^{\mathcal{I}}$,

$$x <^n_{C_i} y \text{ and } (n, y) \in C^{\mathcal{I}}_i \text{ then } (n, x) \in C^{\mathcal{I}}_i.$$

This condition prevents that, when $x$ is preferred to $y$ wrt. $C_i$ at time point $n$, $y$ is an instance of $C_i$ at timepoint $n$, while $x$ is not.

Note that here we are departing from the definition of the *concept-wise multi-preferential temporal semantics* in [22], as we do not introduce a *global* preference relation $<$ in the semantics, and we have also exploited a slightly different notion of weight of a domain element $W^{\mathcal{I}}_{C_i,n}(x)$. This is motivated by the approach we adopt for combining preferences, which is described in the next section.

## 6. Combining preferences from weighted knowledge bases

In example 1, one may be willing to consider typicality concepts such as $\mathbf{T}(Emp \sqcap Student)$ or to verify the satisfiability of an inclusion such as $\mathbf{T}(Emp \sqcap Student) \sqsubseteq has\_Classes$. But how can we define the interpretation of a concept $\mathbf{T}(C)$, when $C$ is a boolean combination of distinguished concepts $C_i \in \mathcal{C}$? In previous approaches to multi-preferential semantics in the two-valued case, e.g., [12, 27, 14, 22], a global preference relation $<$ was defined from the preferences $<_{C_i}$, and the semantics of typicality concepts $\mathbf{T}(D)$ is defined by identifying the typical $D$ elements as the $<$-preferred $D$ elements.

However, choosing a single global preference relation $<$ may be restrictive, as one may want to exploit different minimization criteria and allow for different user-defined preference relations. Following [24], rather than defining a single global preference relation $<$, we allow for the definition of different preference relations, associated with complex concepts obtained by combining the distinguished concepts $C_1, \ldots, C_n$. The idea is related to Brewka's framework for preference combination [28], which introduces a logical preference description language for combining basic preference descriptions $d_1$ and $d_2$ (defining preorders on models) into complex preference descriptions $d_1 \wedge d_2$, $d_1 \vee d_2$, $\neg d_1$ and $d_1 > d_2$. However, as observed by Brewka [28], the preference description $\neg(d_1 \vee d_2)$ is different from $\neg d_1 \wedge \neg d_2$, which would lead to break some desired KLM properties of a nonmonotonic consequence relation [4].

In this section, following the approach proposed for Conditional ASP with weighted KBs [24], we develop an alternative approach for preference combination, which exploits the weights $W^{\mathcal{I}}_{C_i,n}(x)$, introduced above to compute the preference relation $<^n_D$, for any concept $D$ which is a boolean combination of the distinguished concepts.

Let $D$ be a *boolean combination of the distinguished concepts* (i.e., $D$ is obtained by combining the distinguished concepts $C_i$ using $\sqcap$, $\sqcup$ and $\neg$). We define the interpretation of a typicality concept $\mathbf{T}(D)$, as:

$$(\mathbf{T}(D))^{\mathcal{I}} = \{(n, d) \mid d \in Min_{<^n_D}(D^{\mathcal{I}}_n), \text{ for } n \in \mathbb{N}\}$$

where $D^{\mathcal{I}}_n = \{d \mid (n, d) \in D^{\mathcal{I}}\}$ are the instances of $D$ at time point $n$, and $<^n_D$ is the preference relation associated to $D$.

To define the preference relation $<^n_D$ associated with a concept $D$ (a boolean combination of the $C_i$'s) we inductively extend the notion of the weight of a domain element with respect to a distinguished concept $C_i$ to any boolean concepts. Given given a knowledge base $K$ and an interpretation $\mathcal{I}$, let $Max$ and $Min$ be, resp., the maximum and the minimum value of the weight $W^{\mathcal{I}}_{C_i,n}(x)$, for any distinguished concept $C_i$ in $K$ and domain element $x$ in $\mathcal{I}$. We define $W^{\mathcal{I}}_{D,n}(x)$ inductively on the structure of $D$:

$$W^{\mathcal{I}}_{D_1 \sqcap D_2,n}(x) = min(W^{\mathcal{I}}_{D_1,n}(x), W^{\mathcal{I}}_{D_2,n}(x))$$
$$W^{\mathcal{I}}_{D_1 \sqcup D_2,n}(x) = max(W^{\mathcal{I}}_{D_1,n}(x), W^{\mathcal{I}}_{D_2,n}(x))$$
$$W^{\mathcal{I}}_{\neg D_i,n}(x) = Max - W^{\mathcal{I}}_{D_i,n}(x) + Min$$

**Proposition 2.** *The following properties hold:*

$$W^{\mathcal{I}}_{\neg\neg D,n}(S) = W^{\mathcal{I}}_{D,n}(S)$$
$$W^{\mathcal{I}}_{\neg(D_1 \sqcap D_2),n}(x) = W^{\mathcal{I}}_{\neg D_1 \sqcup \neg D_2,n}(x) \qquad W^{\mathcal{I}}_{\neg(D_1 \sqcup D_2)}(S) = W^{\mathcal{I}}_{\neg D_1 \sqcap \neg D_2}(S)$$

$$W^{\mathcal{I}}_{D_1 \sqcup D_2,n}(x) = W^{\mathcal{I}}_{D_1 \sqcup D_2,n}(x) \qquad W^{\mathcal{I}}_{D_1 \sqcap D_2,n}(x) = W^{\mathcal{I}}_{D_2 \sqcap D_1,n}(x)$$
$$W^{\mathcal{I}}_{D \sqcup D,n}(x) = W^{\mathcal{I}}_{D,n}(x) \qquad W^{\mathcal{I}}_{D \sqcap D,n}(x) = W^{\mathcal{I}}_{D,n}(x)$$
$$W^{\mathcal{I}}_{(D_1 \sqcup D_2) \sqcup D_3,n}(x) = W^{\mathcal{I}}_{D_1 \sqcup (D_2 \sqcup D_3),n}(x) \qquad W^{\mathcal{I}}_{(D_1 \sqcap D_2) \sqcap D_3,n}(x) = W^{\mathcal{I}}_{D_1 \sqcap (D_2 \sqcap D_3),n}(x)$$

Assuming that $C_1, \ldots, C_n$ are named concepts, for two equivalent boolean concepts $D_1$ and $D_2$, the weight of a domain element $x$ wrt. $D_1$ and wrt. $D_2$ at a time point $n$ have to be the same.

**Lemma 1.** *Let $D_1$ and $D_2$ be boolean combinations of the distinguished concepts $C_1, \ldots, C_n$. Assume the distinguished concepts $C_1, \ldots, C_n$ are named concepts in $N_C$.*

*If $D_1$ and $D_2$ are equivalent concepts (in $\mathcal{ALC}$), then $W^{\mathcal{I}}_{D_1,n}(x) = W^{\mathcal{I}}_{D_2,n}(x)$, for all domain elements $x$.*

The preference relation associated with a complex concept $D$ can be defined by generalizing condition (5) to any formula $D$ which is a boolean combination of distinguished propositions. For all $x, y \in \Delta^{\mathcal{I}}$, we let:

$$x <^n_D y \quad \text{iff} \quad W^{\mathcal{I}}_{D,n}(x) > W^{\mathcal{I}}_{D,n}(y)$$

The associated preorder relation $\leq_D$ is a total preorder. As a direct consequence of Lemma 1, the following proposition holds:

**Proposition 3.** *Let $D_1$ and $D_2$ be boolean combinations of the distinguished concepts $C_1, \ldots, C_n$. Assume the distinguished concepts $C_1, \ldots, C_n$ are all named concepts in $N_C$.*

*If $D_1$ and $D_2$ are equivalent in $\mathcal{ALC}$, then, for all domain elements $x, y \in \Delta^{\mathcal{I}}$, $x \leq_{D_1} y$ iff $x \leq_{D_2} y$.*

This result shows that the semantics is well behaved and we conjecture that suitably reformulated KLM properties may hold at each time point $n$. We will leave the study of the KLM properties of the logic for future work.

## 7. Conclusions

In this paper we have developed an encodings of the preferential temporal description logics with typicality $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ in the temporal logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$. The formalism has also been extended to define a multi-preferential semantics for weighted knowledge bases, by introducing multiple preferences with respect to distinguished concepts. The paper introduces a concept-wise multi-preferential semantics for weighted temporal knowledge bases and develops an approach for preference combination in the temporal case, based on weights, by generalizing the approach proposed for a conditional answer set programming [24].

On a different route, a preferential LTL with defeasible temporal operators has been studied in [20, 21]. The decidability of meaningful fragments of the logic has been proven, and tableaux based proof methods for such fragments have been developed [19, 21]. Instead, our approach does not consider defeasible temporal operators (nor preferences over time points), but it combines standard LTL operators with the typicality operator in a temporal $\mathcal{ALC}$ (where preferences are over the domain elements).

The encoding of $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ into $LTL_{\mathcal{ALC}}$ provides decidability and complexity results for concept satisfiability wrt. Tboxes, for the monotonic logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$. For the multi-preferential case, proof methods for defeasible temporal reasoning with weighted knowledge bases have to be investigated, possibly for fragments of $LTL^{\mathbf{T},m}_{\mathcal{ALC}}$, by exploiting preference combination as done in [24] for Conditional ASP, based on Answer Set Programming techniques. This will be subject of future work.


## Acknowledgments

We thank the anonymous referees for their helpful comments and suggestions. This work was partially supported by GNCS-INdAM. Mario Alviano was partially supported by the Italian Ministry of

Health (MSAL) under POS projects CAL.HUB.RIA (CUP H53C22000800006) and RADIOAMICA (CUP H53C22000650006); by the Italian Ministry of Enterprises and Made in Italy under project STROKE 5.0 (CUP B29J23000430005); under PN RIC project ASVIN "Assistente Virtuale Intelligente di Negozio" (CUP B29J24000200005); by Regione Calabria under project NextGenGuides "Innovazione Tecnologica per le Guide Turistiche del Futuro tramite l'ausilio di tecnologie innovative AI e sistemi di geolocalizzazione avanzata" (CUP J39I24002040005); under project MOZART "Modelli e tecniche di mOdernizzazione di applicaZioni e data silos a supporto di clienti esterni, field force e impiegati di backoffice basati su AI GeneRativa e apprendimento auTomatico" (CUP J49I24001740005); by the LAIA lab (part of the SILA labs).

## Declaration on Generative AI

The authors have not employed any Generative AI tools.

## References

[1] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Preferential Description Logics, in: LPAR 2007, volume 4790 of *LNAI*, Springer, Yerevan, Armenia, 2007, pp. 257–272.
[2] K. Britz, J. Heidema, T. Meyer, Semantic preferential subsumption, in: G. Brewka, J. Lang (Eds.), KR 2008, AAAI Press, Sidney, Australia, 2008, pp. 476–484.
[3] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, ALC+T: a preferential extension of Description Logics, Fundamenta Informaticae 96 (2009) 1–32.
[4] S. Kraus, D. Lehmann, M. Magidor, Nonmonotonic reasoning, preferential models and cumulative logics, Artificial Intelligence 44 (1990) 167–207.
[5] D. Lehmann, M. Magidor, What does a conditional knowledge base entail?, Artificial Intelligence 55 (1992) 1–60.
[6] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, P. Patel-Schneider, The Description Logic Handbook - Theory, Implementation, and Applications, Cambridge, 2007.
[7] G. Casini, U. Straccia, Rational Closure for Defeasible Description Logics, in: T. Janhunen, I. Niemelä (Eds.), JELIA 2010, volume 6341 of *LNCS*, Springer, Helsinki, 2010, pp. 77–90.
[8] G. Casini, U. Straccia, Defeasible inheritance-based description logics, Journal of Artificial Intelligence Research (JAIR) 48 (2013) 415–473.
[9] G. Casini, T. Meyer, I. J. Varzinczak, , K. Moodley, Nonmonotonic Reasoning in Description Logics: Rational Closure for the ABox, in: 26th International Workshop on Description Logics (DL 2013), volume 1014 of *CEUR Workshop Proceedings*, 2013, pp. 600–615.
[10] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Semantic characterization of rational closure: From propositional logic to description logics, Art. Int. 226 (2015) 1–33.
[11] K. Britz, G. Casini, T. Meyer, K. Moodley, U. Sattler, I. Varzinczak, Principles of KLM-style defeasible description logics, ACM Trans. Comput. Log. 22 (2021) 1:1–1:46.
[12] L. Giordano, V. Gliozzi, A reconstruction of multipreference closure, Artif. Intell. 290 (2021).
[13] D. J. Lehmann, Another perspective on default reasoning, Ann. Math. Artif. Intell. 15 (1995) 61–82.
[14] L. Giordano, D. Theseider Dupré, An ASP approach for reasoning in a concept-aware multipreferential lightweight DL, TPLP 10(5) (2020) 751–766.
[15] L. Giordano, D. Theseider Dupré, Weighted defeasible knowledge bases and a multipreference semantics for a deep neural network model, in: Proc. JELIA 2021, May 17-20, volume 12678 of *Lecture Notes in Computer Science*, Springer, 2021, pp. 225–242.
[16] L. Giordano, D. Theseider Dupré, An ASP approach for reasoning on neural networks under a finitely many-valued semantics for weighted conditional knowledge bases, Theory Pract. Log. Program. 22 (2022) 589–605.
[17] M. Alviano, F. Bartoli, M. Botta, R. Esposito, L. Giordano, D. Theseider Dupré, A preferential

interpretation of multilayer perceptrons in a conditional logic with typicality, Int. Journal of Approximate Reasoning 164 (2024). URL: https://doi.org/10.1016/j.ijar.2023.109065.

[18] C. Lutz, F. Wolter, M. Zakharyaschev, Temporal description logics: A survey, in: TIME, 2008, pp. 3–14.

[19] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, A one-pass tree-shaped tableau for defeasible LTL, in: TIME 2021, September 27-29, 2021, Klagenfurt, Austria, volume 206 of *LIPIcs*, 2021.

[20] A. Chafik, F. C. Alili, J. Condotta, I. Varzinczak, On the decidability of a fragment of preferential LTL, in: TIME 2020, September 23-25, 2020, Bozen-Bolzano, Italy, volume 178 of *LIPIcs*, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2020.

[21] A. Chafik, Defeasible temporal logics for the specification and verification of exception-tolerant systems, PhD Thesis, Artois University, 2022.

[22] M. Alviano, L. Giordano, D. Theseider Dupré, Preferential temporal description logics with typicality and weighted knowledge bases, in: Proc. 38th Italian Conf. on Computational Logic (CILC 2023), volume 3428 of *CEUR Workshop Proceedings*, CEUR-WS.org, 2023.

[23] L. Giordano, D. Theseider Dupré, Weighted conditional EL$^{\perp}$ knowledge bases with integer weights: an ASP approach, in: Proc. 37th Int. Conf. on Logic Programming, ICLP 2021 (Technical Communications), Porto, Sept. 20-27, 2021, volume 345 of *EPTCS*, 2021, pp. 70–76. Https://doi.org/10.4204/EPTCS.345.19.

[24] M. Alviano, L. Giordano, D. Theseider Dupré, A framework for conditional reasoning in answer set programming, in: Proceedings 41st International Conference on Logic Programming, ICLP 2025, Rende, Italy, 12-19th September 2025, EPTCS, 2025, pp. 188–201. URL: https://doi.org/10.4204/EPTCS.439.13.

[25] L. Giordano, V. Gliozzi, Encoding a preferential extension of the description logic *SROIQ* into *SROIQ*, in: Proc. ISMIS 2015, volume 9384 of *LNCS*, Springer, 2015, pp. 248–258.

[26] K. Schild, Combining terminological logics with tense logic, in: EPIA, 1993, pp. 105–120.

[27] L. Giordano, D. Theseider Dupré, A framework for a modular multi-concept lexicographic closure semantics, in: Proc. 18th Int. Workshop on Non-Monotonic Reasoning, NMR2020, September 12th - 14th, 2020.

[28] G. Brewka, A rank based description language for qualitative preferences, in: 6th Europ. Conf. on Artificial Intelligence, ECAI'2004, Valencia, Spain, August 22-27, 2004, 2004, pp. 303–307.