Towards Putting Perspective into OWL (Extended Abstract)

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Introduction

Integrating knowledge from diverse, independently developed sources is a central problem in knowledge representation, particularly given the proliferation of available ontologies and other knowledge sources. Many of these ontologies – often expressed in W3C-standardized dialects of the Web Ontology Language (OWL) [1] – cover overlapping domains but embody varying conceptual frameworks and modelling choices. As an example scenario, imagine that some biomedical ontology ($\mathcal{O}_{\mathsf{Process}}$) might define $\mathtt{Tumour}$ as a dynamic biological process, whereas another ($\mathcal{O}_{\mathsf{Tissue}}$) might view it as a static abnormal tissue structure. While the description logics (DLs) [2, 3] underpinning OWL are well-suited to coherently model a domain, they lack refined mechanisms for managing heterogeneous or conflicting perspectives, leading to notorious challenges whenever such sources are to be integrated.

Standpoint logic (SL) [4] is a recently proposed modal logic framework for multi-perspective reasoning and ontology integration. Akin to epistemic logic, propositions with labelled modal operators $\square_{\mathsf{s}}\,\phi$ and $\lozenge_{\mathsf{s}}\,\phi$ express information relative to some *standpoint* s and read, respectively: "according to s, it is *unequivocal/conceivable* that ϕ". For instance, the formula $\square_{\mathsf{Process}}[\lozenge_{\mathsf{Tissue}}[\mathtt{Tumour}] \sqsubseteq {=}1\mathtt{TriggeredBy.Tumour}]$ expresses that, according to the $\mathsf{Process}$ standpoint, it is unequivocal that everything that is conceivably a $\mathtt{Tumour}$ from the Tissue standpoint has been triggered by exactly one $\mathtt{Tumour}$ (process). Similarly, $\square_{\mathsf{Tissue}}[\{\mathtt{patient1}\} \sqsubseteq \exists\mathtt{HasBodyPart.}(\mathtt{Tumour} \sqcap \{\mathtt{t1}\})]$ states that according to the Tissue standpoint, it is unequivocal that $\mathtt{patient1}$ has the $\mathtt{Tumour}$ $\mathtt{t1}$ as a body part. From both, we infer that according to the $\mathsf{Process}$ standpoint, $\mathtt{t1}$ was triggered by one $\mathtt{Tumour}$. Natural reasoning tasks over multi-standpoint specifications include gathering undisputed knowledge, determining knowledge that is relative to certain standpoints, and contrasting the knowledge from different standpoints.

The SL framework has promising applications in ontology integration, particularly in facilitating the interoperability of ontologies developed in isolation. For this reason, recent work has explored how it can be combined with logic-based formalisms underpinning the OWL family – most notably with the DLs $\mathcal{EL}$ [5], $\mathcal{EL}+$ [6] and $\mathcal{SHIQ}$ [7]. *Monodic* SL extensions of these languages (wherein modalities can occur both on the axiom level and the concept level) preserve the complexity of their standpoint-free counterpart, showing that in those cases, joint reasoning over the integrated combination of possibly many ontologies is not fundamentally harder than reasoning with the ontologies in separation.

Until recently, an open question has been whether the same holds for the very expressive side of modelling languages, in particular DLs that would fully cover high-end contemporary ontology languages such as OWL 2 DL, which – among other features – allow for nominal concepts. So far, for such languages, only *sentential* SL extensions [8] had been considered, which is a computationally easier but much more restricted case with no interplay between quantification and modal operators (e.g., in DLs, the modal operators can only occur on the axiom level). This extended abstract reports on our recent results [9] addressing this question by showing that the very expressive DLs $\mathcal{SHOIQB}_s$ and $\mathcal{SROIQB}_s$ which subsume the OWL 1 and OWL 2 ontology languages, also allow for monodic standpoint extensions without any increase of their standard reasoning complexity.

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We obtain these results through a corresponding result for $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$, the extension of $\mathcal{C}^2$ by monodic standpoints, where $\mathcal{C}^2$ is the counting two-variable fragment of first-order logic (a logic already well-established as an embedding target for very expressive DLs). Since the monadic standpoint extension of the DL $\mathcal{ALCOIQB}^{\mathsf{Self}}$ is in essence just a notational variant of $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$, the result carries over directly. Finally we sketch how this complexity-neutral monodic-standpoint-extension result can be lifted to monodic standpoint extensions of the very expressive DLs $\mathcal{SHOIQB}_s$ and $\mathcal{SROIQB}_s$, which subsume the OWL 1 and OWL 2 ontology languages. The full paper [9] also shows that NExpTime-hardness already occurs in much less expressive DLs as long as they feature both nominals and monodic standpoints. We also show that, with inverses, functionality, and nominals present, minimally lifting the monodicity restriction by allowing for one distinguished rigid binary predicate leads to undecidability.

Monodic Standpoint $\mathcal{ALCOIQB}^{\mathsf{Self}}$ Via Monodic Standpoint $\mathcal{C}^2$

We define $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$ as a fragment of first-order standpoint logic (FOSL) [10], which extends first-order logic with formulae $\square_{\mathsf{e}}\,\phi$ and $\lozenge_{\mathsf{e}}\,\phi$, where standpoint terms e are built from standpoint symbols s (including the universal standpoint $*$) under Boolean combinations $\mathsf{e}_1 \cap \mathsf{e}_2$, $\mathsf{e}_1 \cup \mathsf{e}_2$, and $\mathsf{e}_1 \setminus \mathsf{e}_2$. Then, $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$ is the fragment of this language with counting quantifiers explicitly present, but restricted to two variables, x and y, predicates of arity ≤ 2, and *monodic* formulae, i.e., subformulae preceded by a modal operator have at most one free variable. $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$ formulae are interpreted over *(first-order) standpoint structures*, which are tuples of the form $\langle \Delta, \Pi, \sigma, \gamma \rangle$ where Δ is the *domain*; Π is the set of *precisifications* or *worlds*; σ maps each standpoint symbol to a non-empty set of precisifications (i.e., a subset of Π), with $\sigma(*) = \Pi$ fixed; and γ maps each $\pi \in \Pi$ to an ordinary first-order structure $\mathcal{I}$ over the domain Δ. Satisfaction of a formula ϕ by a structure $\mathfrak{M}$ is inductively defined as usual, with $\mathfrak{M}, \pi \models \square_{\mathsf{e}}\,\phi$ iff $\mathfrak{M}, \pi' \models \phi$ for all $\pi' \in \sigma(\mathsf{e})$. Conveniently, $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$ formulae can be efficiently transformed into equisatisfiable nullary-predicate-free and constant-free $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$ formulae where the only standpoint symbol is $*$ [9]; so, in what follows, we will focus on formulae of this form, which we call *frugal*. Also, we will call formulae of the form $\square_{\mathsf{e}}\,\phi$ and $\lozenge_{\mathsf{e}}\,\phi$ *monodic* if they have one free variable and *sentential* if they have none.

Example 1 *Formula E in Fig. 1(1) expresses that there is one unequivocally good thing (E_0); that everything is either unequivocally good or conceivably the best (somewhere), with no two distinct things being the best simultaneously (E_1); and that it is conceivable that everything is good or the best (E_2).*

Notice that, to satisfy an $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}2}$ formula, a structure only needs at most one precisification to witness sentential subformulae (see E_2 witnessed by π_0 in $\mathfrak{M}_o$, Fig. 1(2)). However, to witness monodic subformulae, it may need one precisification per domain element, like in E_1 where each element must be the single `Best` *somewhere. Thus, in models of E with infinite domains, like $\mathfrak{M}_o$, Π must be infinite as well.* $\lozenge$

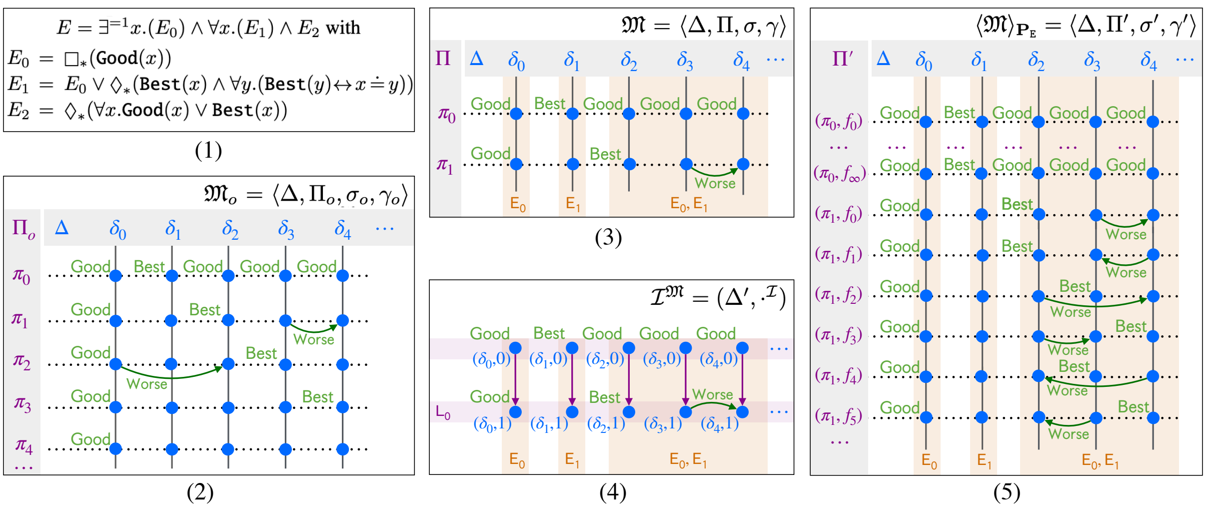

Figure 1: (1) The formula E from Example 1, and illustrations of (2) a model $\mathfrak{M}_o$ of E, (3) an interpretation $\mathfrak{M}$ with the additional predicates $\mathbf{P}_{\mathrm{E}}$, (4) the stacked interpretation $\mathcal{I}^{\mathfrak{M}}$ of $\mathfrak{M}$, and (5) $\langle \mathfrak{M} \rangle_{\mathbf{P}_{\mathrm{E}}}$, the $\mathbf{P}_{\mathrm{E}}$-stable permutational closure of $\mathfrak{M}$. A coloured area labelled with a unary predicate denotes its interpretation.

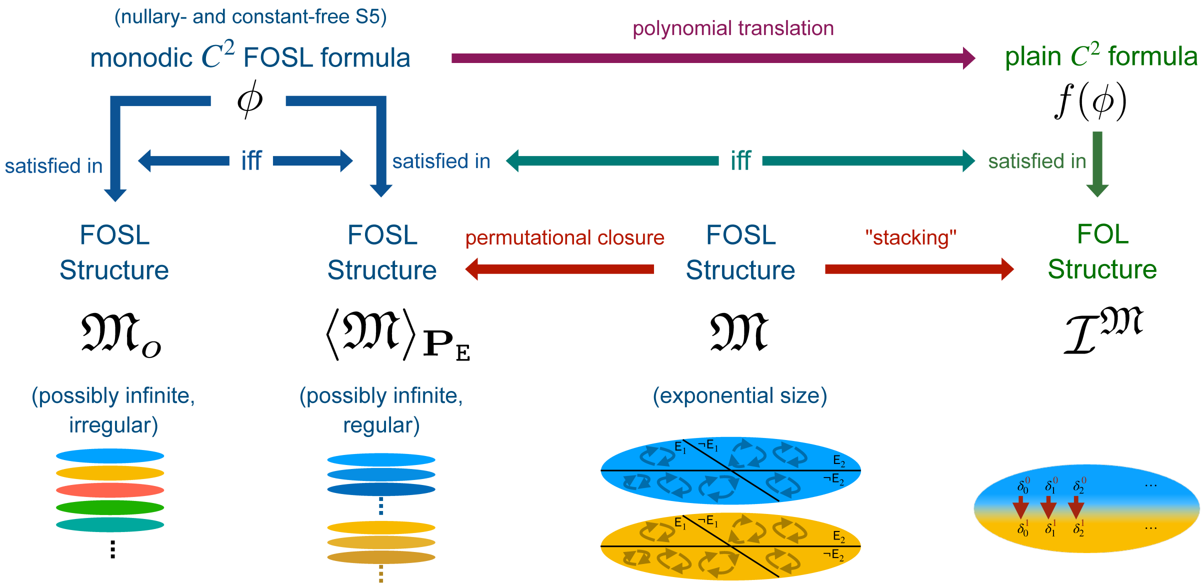

Figure 2: High-level overview of the reduction argument.

Argument Overview. The overall structure of the argument used to establish the complexity of the satisfiability problem for $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}^2}$ is illustrated in Fig. 2. The first part (cf. blue text in Fig. 2) consists in showing that, while we do not have a small (or even just finite) model property, the existence of a model $\mathfrak{M}_o$ of a frugal $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}^2}$ formula ϕ coincides with the existence of a structure $\mathfrak{M}$ whose number of precisifications is "only" exponential in ϕ and which – while not necessarily a model of ϕ itself – gives rise to a specific kind of (possibly infinite) ϕ-model $\langle\mathfrak{M}\rangle_{\mathbf{P}_\mathrm{E}}$, through a construction we call the $\mathbf{P}_\mathrm{E}$-*stable permutational closure*. In the second part (cf. green text in Fig. 2), we argue that such an $\mathfrak{M}$ can be represented by a so-called *stacked interpretation* $\mathcal{I}^{\mathfrak{M}}$, a plain first-order structure that closely mimics $\mathfrak{M}$. Stacked interpretations can be axiomatized in $\mathcal{C}^2$ and satisfaction of ϕ in $\langle\mathfrak{M}\rangle_{\mathbf{P}_\mathrm{E}}$ can be shown to coincide with satisfaction of $f(\phi)$ in $\mathcal{I}^{\mathfrak{M}}$, where f is a function mapping ϕ to a plain $\mathcal{C}^2$ sentence. This way, we establish an equisatisfiable polytime translation from $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}^2}$ to plain $\mathcal{C}^2$.

Permutational Representatives. To understand the structure of $\mathfrak{M}$, we first observe that some $\delta \in \Delta$ "fits" a monodic formula $\Diamond_* \psi(z)$ or $\Box_* \psi(z)$ if $\mathfrak{M}, \pi, \{z \mapsto \delta\} \models \psi$ for some/all $\pi \in \Pi$: thus by the semantics, the choice of π is irrelevant, hence monodic formulae are *rigid* – they hold synchronously across all precisifications. For instance, in Fig. 1(2), δ_0 is (rigidly) unequivocally Good since it is Good everywhere. Let us now consider the structure $\mathfrak{M}$, which contains exponentially many precisifications wrt. ϕ and, in addition to the predicates of the signature of ϕ, a set of special rigid unary predicates $\mathbf{P}_\mathrm{E}$. These are used to induce "E-types", corresponding to the subsets $T \subseteq \mathbf{P}_\mathrm{E}$, so a domain element is said to have the E-type T if it belongs to the interpretation of each E_i in T and to none outside it.

From $\mathfrak{M}$, we then produce $\langle\mathfrak{M}\rangle_{\mathbf{P}_\mathrm{E}}$, the $\mathbf{P}_\mathrm{E}$-stable permutational closure of $\mathfrak{M}$ (see Fig. 1(5)), a larger structure that contains, for each precisification in Π, the set of precisifications with all possible permutations between domain elements belonging to the same E-type. World-wise, all permuted versions of any $\pi \in \Pi$ in the closure are isomorphic to each other, thus they preserve the worlds' internal structure. At the global level, E-types of domain elements, which are preserved under the permutations (by construction), will be used to witness the elements' "membership" in monodic subformulae.

Note that the set of E-types must be at least as large as the set of types induced by the monodic formulae ($\Diamond$-types) for $\langle\mathfrak{M}\rangle_{\mathbf{P}_\mathrm{E}}$ to witness them, and the two sets will be aligned: any E-type determines a $\Diamond$-type. Moreover, for each formula in a realised $\Diamond$-type, $\mathfrak{M}$ must include one world in which some element of that type satisfies ψ. The construction of $\langle\mathfrak{M}\rangle_{\mathbf{P}_\mathrm{E}}$ then ensures that every other element of the same E-type also satisfies ψ in some "permutational copy" of that world. By Theorem 11 in [9], a frugal $\mathbb{S}^{\mathrm{mon}}_{\mathcal{C}^2}$ formula ϕ is satisfiable iff it is satisfied in $\langle\mathfrak{M}\rangle_{\mathbf{P}_\mathrm{E}}$ of an exponential structure $\mathfrak{M}$.

Stacked Interpretations. We then define $\mathcal{I}^{\mathfrak{M}}$, the *stacked interpretation* of $\mathfrak{M}$, a specific $\mathcal{C}^2$-structure obtained from a standpoint structure $\mathfrak{M}$ with 2^m precisifications, closely mirroring its shape.

Our approach constructs a stacked domain by creating one copy of the original domain Δ for each precisification in $\mathfrak{M}$, so that each new element (δ, π) mimics δ at precisification π. A set of new unary predicates $\mathtt{L}_0, \ldots, \mathtt{L}_{m-1}$ bit-encodes the index of the associated precisification (each $< 2^m$). Also, a new binary predicate $\mathtt{F}$ links each element (δ, π_i) to its "next-world-twin" (δ, π_{i+1}). Thus, for every original element δ, the stacked interpretation forms an $\mathtt{F}$-chain tracking δ across all precisifications in $\mathfrak{M}$. In Fig. 1, (4) depicts the stacked interpretation of (3); The $\mathtt{F}$-chains (purple arrows) link all the copies of each domain element in Δ, and a single predicate $\mathtt{L}_0$ is enough to number the two precisifications, hence the copy of π_0 is labelled with $\{\}$ and the copy of π_1 is labelled with $\{\mathtt{L}_0\}$ (highlighted in purple).

Stacked Formulae and Translation. Finally, we can establish a translation f from a $\mathbb{S}^{\text{mon}}_{\mathcal{C}^2}$ formula ϕ to a plain $\mathcal{C}^2$ formula $f(\phi) = \phi^m_{\text{stack}} \wedge \phi^\ell_{rig\mathtt{E}} \wedge \mathsf{Trans}_m(\phi)$. The first component, ϕ^m_{stack}, is the stacked formula of size m, and is used to enforce that all models of $f(\phi)$ are stacked interpretations of some structure $\mathfrak{M}$ (Theorem 16 in [9]). The second component, $\phi^\ell_{rig\mathtt{E}}$, ensures that all predicates from $\mathbf{P}_{\mathtt{E}}$ are interpreted "rigidly". Finally, $\mathsf{Trans}_m(\phi)$ translates ϕ into plain $\mathcal{C}^2$, with its key components being the handling of counting quantification and modal operators. Intuitively, the translation of a counting existential quantification ranges only over elements belonging to the current layer (each layer being determined by a subset of the counting predicates $\mathtt{L}_i$) of the stacked interpretation – namely, those whose counterparts correspond to the domain elements at the current precisification. In contrast, the translation of modal subformulae of the form $\Diamond_* \psi$ ranges over the elements belonging to the current $\mathtt{E}$-type. Recall that $\mathcal{I}^{\mathfrak{M}}$ mirrors $\mathfrak{M}$, from which in turn we obtain the model $\langle\mathfrak{M}\rangle_{\mathbf{P}_{\mathtt{E}}}$. Consequently, if for some δ, we find any element of the same $\mathtt{E}$-type that satisfies ψ, then there is some permutation within $\langle\mathfrak{M}\rangle_{\mathbf{P}_{\mathtt{E}}}$ in which ψ holds for δ, and thus the formula $\Diamond_* \psi$ is satisfied. When $\Diamond_* \psi$ is sentential, the variable assignment is irrelevant. By Theorem 22 in [9], ϕ is satisfiable iff $f(\phi)$ is satisfiable.

Together with the translation from $\mathbb{S}^{\text{mon}}_{\mathcal{C}^2}$ to frugal $\mathbb{S}^{\text{mon}}_{\mathcal{C}^2}$, this entails that there is a polytime equisatisfiable translation from $\mathbb{S}^{\text{mon}}_{\mathcal{C}^2}$ to plain $\mathcal{C}^2$. On the other hand, every plain $\mathcal{C}^2$ formula is $\mathbb{S}^{\text{mon}}_{\mathcal{C}^2}$, thus in view of the known NExpTime completeness of plain $\mathcal{C}^2$ [11], we can establish that satisfiability in monodic standpoint $\mathcal{C}^2$ is NExpTime-complete. We conclude this section by mentioning that monodic standpoint $\mathcal{ALCOIQB}^{\mathsf{Self}}$ admits a rather straightforward polytime translation into $\mathbb{S}^{\text{mon}}_{\mathcal{C}^2}$. Since satisfiability is already NExpTime-hard for the sublogic $\mathcal{ALCOIF}$ [12], we obtain that the satisfiability of monodic standpoint $\mathcal{ALCOIQB}^{\mathsf{Self}}$ is NExpTime-complete (cf. Theorem 26 in [9]).

Tackling $\mathcal{SHOIQB}_s$ and $\mathcal{SROIQB}_s$: Handling Role Chain Axioms

In order to cover the Web ontology languages OWL 1 and OWL 2 DL, we need to extend our formalism by so-called *role chain axioms*, arriving at the description logics $\mathcal{SHOIQB}_s$ (when allowing just role chain axioms expressing transitivity such as $\mathtt{FriendOf} \circ \mathtt{FriendOf} \sqsubseteq \mathtt{FriendOf}$) or $\mathcal{SROIQB}_s$ (when admitting more complex forms like $\mathtt{FriendOf} \circ \mathtt{EnemyOf} \sqsubseteq \mathtt{EnemyOf}$), respectively.[1] As these types of axioms do not contain any concept expressions, we only need to care about the case of modal operators occurring in front of full axioms. Luckily, by combining known standpoint encoding tricks [8] and well-established removal techniques for role-chain axioms [13, 14] with some novel ideas, it is possible to translate monodic standpoint $\mathcal{SHOIQB}_s$ and $\mathcal{SROIQB}_s$ sentences back into monodic standpoint $\mathcal{ALCOIQB}^{\mathsf{Self}}$. The translation is polynomial for $\mathcal{SHOIQB}_s$ and exponential for $\mathcal{SROIQB}_s$.

Thus, by Theorem 26 in [9], checking satisfiability of monodic standpoint $\mathcal{SHOIQB}_s$ sentences is NExpTime-complete, and by Theorem 27 in [9], checking satisfiability of monodic standpoint $\mathcal{SROIQB}_s$ sentences is N2ExpTime-complete. Therein, the hardness part for monodic standpoint $\mathcal{SROIQB}_s$ follows from the known N2ExpTime hardness of its fragment $\mathcal{SROIQ}$ [13]. Together with the previous section's result, this concludes our argument that adding monodic standpoints to OWL 1 and OWL 2 does not increase complexity of standard reasoning tasks.

[1] Recall that the $\mathcal{B}_s$ at the end of the DL names stands for full Boolean expressions over simple roles.

Declaration on Generative AI

During the preparation of this work, the authors used Claude in order to: Improve writing style. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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