

# Standpoint Logics with Defeasible Beliefs

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## Abstract

In this paper, we integrate the defeasible logic of Kraus, Lehmann and Magidor (KLM) with the standpoint logic framework of Gómez Álvarez and Rudolph. This is done with the goal of formally expressing knowledge taking into account multiple (possibly contradicting) viewpoints, which in turn may hold defeasible beliefs. In doing so, we utilise Defeasible Restricted Standpoint Logics (DRSL), introduced by Leisegang et al. Our work expands on previous work by providing a foundational representation result for DRSL semantics and systematically lifting several well-known entailment relations from the propositional case to the standpoint-enhanced setting. In particular, we characterise the semantics for DRSL through a set of KLM-style postulates adapted for the standpoints case. We furthermore provide a means to lift preferential entailment, and the class of entailment relations based on single ranking functions from the purely propositional to the standpoint-enhanced context, including rational and lexicographic closure. We show this can be done equivalently through semantic and algorithmic means. Furthermore, we show that, for each considered form of entailment, the complexity class of entailment checking does not change when moving from propositional KLM to DRSL.

## Keywords

Defeasible Reasoning, Standpoint Logics, Modal Logics

## 1. Introduction

*Standpoint logics* denote a modal logic framework introduced by Rudolph and Gómez Álvarez [1] with the purpose of expressing multiple standpoints or perspectives, which may hold conflicting beliefs about some topic, while still maintaining consistency in the logic. Much of the work in standpoint logics concerns the addition of *standpoint modalities* to base logics such as description logics [2, 3], while showing that the added expressivity does not increase the complexity of reasoning. Recent work has integrated standpoint modalities in non-monotonic formalisms, modelling situations where standpoints may encompass beliefs that resemble default rules or defeasible implications [4, 5]. As a motivating case, consider the following example.

**Example 1** ([4]). *In the 19th century, crops imported into the USA were divided into fruits and vegetables, where fruits were exempt from import tax. This led to a court case on whether a tomato should be legally classified as a fruit or a vegetable. From a botanical standpoint, tomatoes are fruits and all fruits are also vegetables. This can be expressed using standpoint logics with the formulas:  $\Box_B(\text{tomato} \rightarrow \text{fruit})$  and  $\Box_B(\text{fruit} \rightarrow \text{veg.})$ , where  $B$  represents the botanical standpoint. However, the court considered a different standpoint based on the culinary use of tomatoes, according to which vegetables are those crops which are savoury and fruits are those which are sweet, giving us the beliefs:  $\Box_C(\text{savoury} \leftrightarrow \text{veg.})$ ,  $\Box_C(\text{sweet} \leftrightarrow \text{fruit})$ ,  $\Box_C(\text{tomato} \vdash \text{savoury})$ , and  $\Box_C((\text{fruit} \vdash \neg \text{veg.}) \wedge (\text{veg.} \vdash \neg \text{fruit}))$ , where  $C$  represents the “culinary” standpoint. Here, the third statement tells us that tomatoes are usually used in savoury dishes, and the last proposition states that fruits and vegetables are usually considered distinct from each other. Ultimately, courts agreed with the culinary standpoint that tomatoes were usually considered vegetables. We can represent this by  $L \preceq C$ ,  $\Box_L(\text{veg.} \rightarrow \neg \text{fruit})$ , where  $L \preceq C$  tells us that the legal standpoint  $L$  holds true each conclusion of  $C$ ’s standpoint, and the second proposition states that fruits and vegetables are strictly distinct, legally speaking. This system allows for both internal exceptions, and strict disagreements between standpoints:  $B$  believes that every fruit is a vegetable, while  $L$  believes that no fruit*

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is a vegetable. We also see that it is possible that a certain kind of exceptional tomato is not savoury from a culinary standpoint. In fact, from a culinary perspective it is possible to consider a tomato both a fruit and vegetable, due to its exceptional nature (i.e.  $\Diamond_C(\text{tomato} \rightarrow (\text{fruit} \wedge \text{veg.}))$ ).

This shows us a scenario in which there are both different standpoints with conflicting points of view, as well as standpoints that hold beliefs which are defeasible in their nature. For example, the rule  $\text{fruit} \sim \neg \text{vegetable}$  allows for non-prototypical fruits to also be considered vegetables. The addition of standpoint modalities therefore adds a new layer of expressivity to the propositional defeasible reasoning of Kraus, Lehmann and Magidor (KLM) [6], in which multiple perspectives can be considered simultaneously. On the other hand, adding defeasibility to the beliefs held by standpoints allows for beliefs to be weakened, and therefore for more agreements in beliefs to be held across standpoints, ultimately increasing the inferences that can be drawn from the classical case. In our paper, we extend the work of Leisegang et al. [4], who introduce the language of Defeasible Restricted Standpoint Logic (DRSL), and the corresponding semantics of *preferential standpoint structures*. Our contribution characterises preferential standpoint structures by defining a set of KLM-style postulates, and providing a representation result that each set of DRSL statements closed under these postulates can be represented by a preferential standpoint structure. We then consider entailment for DRSL, where we first lift monotonic preferential entailment from the propositional case to the standpoints case, as well as providing a method for lifting the class of non-monotonic entailment relations which are based on single ranking functions to the standpoint logic case. We further show that entailment-checking for DRSL remains in the same complexity class as in the propositional case, in both the monotonic and non-monotonic case. Our paper is structured as follows: Section 2 introduces preliminary material from propositional KLM-style defeasible reasoning. Section 3 introduces the syntax and semantics for DRSL, and provides the representation result which shows soundness and completeness for our adapted set of postulates. Section 4 establishes results for both monotonic and non-monotonic entailments for DRSL. Section 5 considers related work and provides concluding remarks.

## 2. Preliminaries

In this section, we introduce some basic results and definitions from propositional defeasible reasoning. The framework for defeasible reasoning we use was proposed by Kraus, Lehmann and Magidor (KLM) [6], who introduced an additional defeasible implication “ $\sim$ ” to classical propositional logic, where  $\alpha \sim \beta$  reads as “ $\alpha$  typically implies  $\beta$ ”.

**Definition 1.** *The language of KLM propositional logic  $\mathcal{L}^\sim$  over a set of propositional atoms  $\mathcal{P}$  is given by*

$$\phi ::= \alpha \sim \beta \mid \phi \wedge \phi,$$

where  $\alpha$  and  $\beta$  are Boolean formulas with atoms in  $\mathcal{P}$ .

This definition is a slight extension of the original language of KLM, which does not allow conjunction in the language. This distinction is important in our case when we apply modal operators across conjunctions of KLM implications, rather than single implications. It is also worth noting that the  $\sim$  sign was originally introduced as a defeasible consequence operator on the meta level, whereas we use it as a form of defeasible implication within the language itself. Lastly, we recall that any Boolean formula  $\alpha$  is semantically equivalent to  $\neg \alpha \sim \perp$  [7], and so our language can express classical Boolean statements, as well as defeasible implications. Abusing notation, we use the Boolean formula  $\alpha$  as a shorthand for  $\neg \alpha \sim \perp$ . The semantics for  $\mathcal{L}^\sim$  is defined below.

**Definition 2.** [6] *A preferential interpretation over a set of propositional atoms  $\mathcal{P}$  is a triple  $\mathcal{I} = (W, l, <)$  where  $W$  is a (possibly empty) set of states,  $l : W \rightarrow 2^{\mathcal{P}}$  is a mapping from the set of states to the set of classical valuations on  $\mathcal{P}$ , and  $<$  is a strict partial order on  $W$  such that for every Boolean formula  $\alpha$ , the set  $\llbracket \alpha \rrbracket := \{w \in W \mid l(w) \models \alpha\}$  has a minimal element with respect to  $<$ .*

$$\begin{array}{lll}
(\text{Ref}) \frac{}{\alpha \sim \alpha} & (\text{LLE}) \frac{\alpha \equiv \beta, \alpha \sim \gamma}{\beta \sim \gamma} & (\text{RW}) \frac{\alpha \models \beta, \gamma \sim \alpha}{\gamma \sim \beta} \\
(\text{And}) \frac{\alpha \sim \beta, \alpha \sim \gamma}{\alpha \sim \beta \wedge \gamma} & (\text{Or}) \frac{\alpha \sim \gamma, \beta \sim \gamma}{\alpha \vee \beta \sim \gamma} & (\text{CM}) \frac{\alpha \sim \beta, \alpha \sim \gamma}{\alpha \wedge \beta \sim \gamma}
\end{array}$$

**Figure 1:** KLM Postulates

The satisfaction relation  $\models$  for a preferential interpretation is defined as follows for Boolean formulas  $\alpha$  and  $\beta$ :  $\mathcal{I} \models \alpha \sim \beta$  if  $l(w) \models \beta$  for all  $w \in \min_{<}[\llbracket \alpha \rrbracket]$ ; and  $\mathcal{I} \models \phi_1 \wedge \phi_2$  if  $\mathcal{I} \models \phi_1$  and  $\mathcal{I} \models \phi_2$ . A formula,  $\phi \in \mathcal{L}^\sim$  is satisfiable if there exists some preferential interpretation  $\mathcal{I}$  with  $\mathcal{I} \models \phi$ , and this is extended to sets of formulas in the usual way. However, it is trivially the case that every set  $A \subseteq \mathcal{L}^\sim$  is satisfiable, since the preferential model  $\mathcal{I}$  where the set of states is empty satisfies all formulas in  $\mathcal{L}^\sim$ . For our purposes, we want to consider a stronger notion of satisfiability, where we only consider non-empty preferential models. We say a set  $\mathcal{A} \subseteq \mathcal{L}^\sim$  is *non-trivially satisfiable* if there exists a preferential interpretation  $\mathcal{I} = (W, l, <)$  such that  $\mathcal{I} \models \phi$  for all  $\phi \in \mathcal{A}$  and  $W \neq \emptyset$ . A foundational result of preferential semantics is their relation to the *KLM postulates*. These are a set of properties, given in Figure 1 which describe the basic behaviour of the “ $\sim$ ” connective. The result below allows to establish that any set of KLM-style defeasible implications is closed under the KLM postulates iff it can be represented by a single preferential interpretation.

**Theorem 1.** [6] *If a set  $\mathcal{A} \subseteq \mathcal{L}^\sim$  is closed under the KLM postulates, then there is some preferential interpretation  $\mathcal{I}$  such that  $\phi \in \mathcal{A}$  iff  $\mathcal{I} \models \phi$ .*

We work under the assumption that defeasibility is added on top of classical propositional logic, and that any set of formulas which is considered closed under the KLM postulates is also closed under classical propositional deduction rules.

### 3. Defeasible Beliefs in Standpoint Logics

Standpoint logics are a family of modal logics used to describe scenarios involving different viewpoints which may hold conflicting beliefs. For each such standpoint  $s$ , one introduces modal operators  $\Box_s$  and  $\Diamond_s$ . Then,  $\Box_s \phi$  and  $\Diamond_s \phi$  read as “it is unequivocal to  $s$  that  $\phi$ ” and “it is possible to  $s$  that  $\phi$ ”, respectively. In this section, we consider an extension of propositional standpoint logic, as defined by Gómez Álvarez and Rudolph [1]. In particular, we apply standpoint modalities to KLM-style defeasible implications by introducing a logic with formulas of the form  $\Box_s(\alpha \sim \beta)$  or  $\Diamond_s(\alpha \sim \beta)$ . This represents the case where a standpoint holds a defeasible belief, and these sentences should be read as “it is unequivocal to  $s$  that  $\alpha$  usually implies  $\beta$ ” and “it is possible to  $s$  that  $\alpha$  usually implies  $\beta$ ”. In this section, we describe the syntax and semantics for DRSL, and show that this can be characterised through an extended system of postulates inspired by the original KLM postulates. We restrict the syntax to only allow for conjunction on the outermost level. This restriction is made with the spirit of propositional KLM in mind, in which disjunctions and negations of defeasible statements are not expressible. Such a restriction is also needed to facilitate the representation and complexity results found in the remainder of the paper.

**Definition 3.** [4] *A vocabulary is a pair  $\mathcal{V} = (\mathcal{P}, \mathcal{S})$  where  $\mathcal{P}$  is a finite set of propositional atoms and  $\mathcal{S}$  is a finite set of standpoint symbols. The language of Defeasible Restricted Standpoint Logic (DRSL)  $\mathcal{L}_\mathcal{S}^\sim$  is defined as follows,*

$$\psi ::= \phi \mid \Box_s \psi \mid \Diamond_s \psi \mid \psi \wedge \psi \text{ or } \psi ::= s \preceq t$$

where  $s, t \in \mathcal{S} \cup \{*\}$  and  $\phi \in \mathcal{L}^\sim$  with atoms in  $\mathcal{P}$ .

Statements of the form  $s \preceq t$  are referred to as *standpoint sharpening* statements, and intuitively, they say that  $s$  is a more specific version of  $t$ ’s viewpoint. The  $*$  symbol is referred to as the *universal*

standpoint and represents the beliefs which all standpoints agree upon. The semantics for this are given by *preferential standpoint structures*, which again are built on top of preferential interpretations given in propositional defeasible reasoning.

**Definition 4.** [4] A *preferential standpoint structure* is a triple  $M = (\Pi, \sigma, \tau)$ , where:

1.  $\Pi$  is a non-empty set of precisifications (possible worlds).
2.  $\sigma : \mathcal{S} \cup \{*\} \rightarrow 2^\Pi$  is a map which assigns a non-empty set of precisifications to each standpoint, such that  $\sigma(*) = \Pi$ .
3.  $\tau : \Pi \rightarrow \mathbb{I}$  is function where  $\mathbb{I}$  is the set of preferential interpretations over the set  $\mathcal{P}$ . That is,  $\tau$  is a map which assigns to each precisification a preferential interpretation.

**Definition 5.** [4] Given a *preferential standpoint structure*  $M$  and a precisification  $\pi \in \Pi$ , the *satisfaction relation*  $\Vdash$  is defined as followed (where  $\phi \in \mathcal{L}^\sim$ ,  $\psi \in \mathcal{L}_S^\sim$  and  $s, t \in \mathcal{S} \cup \{*\}$ ):

- $M, \pi \Vdash \phi$  iff  $\tau(\pi) \Vdash \phi$ ,
- $M, \pi \Vdash \Box_s \psi$  iff  $M, \pi' \Vdash \psi$  for all  $\pi' \in \sigma(s)$ ,
- $M, \pi \Vdash \Diamond_s \psi$  iff  $M, \pi' \Vdash \psi$  for some  $\pi' \in \sigma(s)$ ,
- $M, \pi \Vdash \psi_1 \wedge \psi_2$  iff  $M, \pi \Vdash \psi_1$  and  $M, \pi \Vdash \psi_2$ ,
- $M, \pi \Vdash s \preceq t$  iff  $\sigma(s) \subseteq \sigma(t)$ ,
- $M \Vdash \psi$  iff  $M, \pi \Vdash \psi$  for all  $\pi \in \Pi$ .

Intuitively,  $\sigma$  assigns to each standpoint  $s$  a set of reasonable ways to understand  $s$ 's set of beliefs, where each  $\pi \in \sigma(s)$  denotes one specific fixed way to understand  $s$ 's beliefs, via the preferential interpretation  $\tau(\pi)$ . These are closely derived from the semantics for classical propositional standpoint logic, where each  $\tau(\pi)$  is a classical valuation, rather than a preferential interpretation [1].

**Definition 6.** A set of DRSL formulas  $\mathcal{A} \subseteq \mathcal{L}_S^\sim$  is *satisfiable* if there exists some *preferential standpoint structure*  $M = (\Pi, \sigma, \tau)$  in which, for all  $\pi \in \Pi$  the set of states in  $\tau(\pi)$  is non-empty, and such that  $M \Vdash \phi$  for all  $\phi \in \mathcal{A}$ ,

This is stronger than satisfiability in the propositional KLM setting, and is closer to non-trivial satisfiability for propositional KLM. In particular, every set of defeasible implications in  $\mathcal{L}^\sim$  is satisfiable, while not every set in  $\mathcal{L}_S^\sim$  is satisfiable.

**Example 2.** Consider the “tomato” knowledge base  $\mathcal{K}_T$  from Example 1, with the propositional atoms shortened:

$$\mathcal{K}_T = \{\Box_B(t \rightarrow v), \Box_B(f \rightarrow v), \Box_C(v \leftrightarrow s_a), \Box_C(f \leftrightarrow s_w), \\ \Box_C(t \vdash s_a), \Box_C((f \vdash \neg v) \wedge (v \vdash \neg f)), L \preceq C, \Box_L(v \rightarrow \neg f)\}$$

We exhibit a *preferential standpoint structure*  $M = (\Pi, \sigma, \tau)$  which satisfies  $\mathcal{K}_T$  as follows: Let  $\Pi = \{\pi_1, \pi_2, \pi_3\}$  where  $\tau(\pi_1)$  and  $\tau(\pi_2)$  are preferential interpretations each with a single state which have underlying valuations of  $\{t, f, v\}$  and  $\{t, v, s_a\}$  respectively. Let  $\tau(\pi_3) = (W, l, <)$  where  $W$  has 3 states  $s_1, s_2$  and  $s_3$ ;  $< = \{(s_1, s_3), (s_2, s_3)\}$ ; and  $l(s_1) = \{f, s_w\}$ ,  $l(s_2) = \{v, s_a\}$ ,  $l(s_3) = \{t, f, s_w, v, s_a\}$ . Then defining  $\sigma$  by  $\sigma(B) = \{\pi_1\}$ ,  $\sigma(L) = \{\pi_2\}$  and  $\sigma(C) = \{\pi_2, \pi_3\}$  we see that  $M$  satisfies  $\mathcal{K}_T$ .

We now describe a set of proof-theoretic postulates which characterise preferential semantics for DRSL. As with KLM and propositional standpoint logic, we operate under the assumption that our logic is built on top of Boolean propositional logic. We therefore accept that any axioms or rules of inference which hold in Boolean propositional logic hold in our logic. Besides this, the postulates we give here can be broadly split into two groups. The first of these is an adapted set of the modal axioms which characterise standpoint modalities. In the classical case standpoint logic acts as a multimodal variant of **KD45** with additional axioms tailored for standpoint sharpenings.  $\Box_*$  acts as an **S5** modality, and therefore has additional properties [1]. In our setting, we include an adapted version of the original

$$\begin{array}{c}
(\mathbf{RN}) \frac{\phi}{\Box_s \phi} \quad (\mathbf{K.a.}) \frac{\Box_s(\phi \rightarrow \psi), \Box_s \phi}{\Box_s \psi} \quad (\mathbf{K.b.}) \frac{\Box_s \phi, \Diamond_s \psi}{\Diamond_s(\phi \wedge \psi)} \quad (\mathbf{K.c.}) \frac{\Diamond_s((\phi \rightarrow \psi) \wedge \phi \wedge \Gamma)}{\Diamond_s(\psi \wedge (\phi \rightarrow \psi) \wedge \phi \wedge \Gamma)} \\
(5') \frac{\Diamond_t \Box_s \phi}{\Box_s \phi} \quad (\mathbf{T}^*) \frac{\Box_s \phi}{\phi} \quad (\mathbf{P.a.}) \frac{s \preceq t, \Box_t \phi}{\Box_s \phi} \quad (\mathbf{P.b.}) \frac{s \preceq t, \Diamond_s \phi}{\Diamond_t \phi} \quad (\mathbf{D.}) \frac{\Box_s \phi}{\Diamond_s \phi} \quad (4') \frac{\Diamond_t \Diamond_s \phi}{\Diamond_s \phi} \\
(\Box\text{-Dist.a.}) \frac{\Box_s(\phi \wedge \psi)}{\Box_s \phi \wedge \Box_s \psi} \quad (\Box\text{-Dist.b.}) \frac{\Box_s \phi \wedge \Box_s \psi}{\Box_s(\phi \wedge \psi)} \quad (\Diamond\text{-Dist.}) \frac{\Diamond_s(\phi \wedge \psi)}{\Diamond_s \phi \wedge \Diamond_s \psi} \\
(\preceq\text{-Refl.}) \frac{}{s \preceq s} \quad (*\text{-Top}) \frac{}{s \preceq *} \quad (\preceq\text{-Trans.}) \frac{s \preceq t, t \preceq u}{s \preceq u}
\end{array}$$

**Figure 2:** Modality Postulates for DRSL

axioms for standpoint modalities, which are sufficient to cover the restrictions in our syntax. These axioms rephrased in terms of Gentzen-style rules in order to be expressible in our restricted version of standpoint logic, which does not allow for unrestricted material implication or disjunction. Furthermore, the restriction of negation means  $\Box_s$  and  $\Diamond_s$  must be treated separately, and not simply as duals of each other; although semantically they behave as duals. The derived modality postulate for DRSL are given in Figure 2. They are named directly after the original classical standpoint logic axioms from which they are derived [1], with some of the original modal axioms requiring two or more rules in our setting of restricted expressivity (for example, **P.a** and **P.b**). Additional postulates describe how modalities distribute over conjuncts, and govern the behaviour of standpoint sharpenings. Besides these modal rules which govern the external structure between precisifications in the semantics, we also require rules which describe how the preferential interpretations underlying the precisifications behave. In order to describe this, we adapt the original KLM postulates, as given by Kraus et al. [6], into the standpoint logic modal case. These are given in Figure 3. From each original KLM postulate, we derive a pair of postulates which correspond to preferential reasoning which is both global or local to a given standpoint. Each “a.” postulate describes how the KLM postulates apply when defeasible beliefs hold throughout an entire standpoint, while each “b.” postulate describes conclusions reached in a specific possible precisification for a standpoint. That is, in each rule we cumulatively add a new conjunct to the premise, ultimately obtaining a large  $\Diamond$ -bound conjunction of defeasible implications. This is done in order to collect every conclusion valid at a possible precisification, and make sure that we do not lose any conclusions which may be derived relative to the same possible world.

**Definition 7.** A set  $\mathcal{A} \subseteq \mathcal{L}_S^{\sim}$  is preferentially closed if it is closed under classical propositional logic and the rules in Figures 2 and 3. That is, if the premises of any of the rules occur in  $\mathcal{A}$ , then the consequences of the rule are in  $\mathcal{A}$ .

We show that these postulates accurately characterise the semantics of DRSL. We do this by showing our semantics are sound and complete with respect to the rules in Figures 2 and 3. Moreover, we show that any set of DRSL formulas closed under preferential reasoning can be represented by a single preferential standpoint structure. We begin with soundness:

**Lemma 1.** Any preferential standpoint structure satisfies the rules given in Figures 2 and 3, and satisfies classical propositional logic.

We now show the completeness result that says any set of DRSL sentences which is preferentially closed can be characterised by some preferential standpoint structure. In order to show this theorem holds, we utilise the original representation result given in Theorem 1 for preferential consequence relations in the propositional case. We introduce a lemma which allows us to only consider DRSL formulas in some normal form. That is, we show that any DRSL formula (which is not a standpoint sharpening) can be reduced to an equivalent formula in this normal form, and the reduction of a formula to normal form can be done through applying the postulates and through semantic means.

$$\begin{array}{ll}
(\text{Ref.}) \frac{}{\alpha \sim \alpha} & (\text{LLE.a.}) \frac{\Box_s(\alpha \leftrightarrow \beta), \Box_s(\alpha \sim \gamma)}{\Box_s(\beta \sim \gamma)} \quad (\text{LLE.b.}) \frac{\Diamond_s((\alpha \leftrightarrow \beta) \wedge (\alpha \sim \gamma) \wedge \Gamma)}{\Diamond_s((\beta \sim \gamma) \wedge \Gamma')} \\
(\text{RW.a.}) \frac{\Box_s(\alpha \rightarrow \beta), \Box_s(\gamma \sim \alpha)}{\Box_s(\gamma \sim \beta)} & (\text{RW.b.}) \frac{\Diamond_s((\alpha \rightarrow \beta) \wedge (\gamma \sim \alpha) \wedge \Gamma)}{\Diamond_s((\gamma \sim \beta) \wedge \Gamma')} \\
(\text{And.a.}) \frac{\Box_s(\alpha \sim \beta), \Box_s(\alpha \sim \gamma)}{\Box_s(\alpha \sim (\beta \wedge \gamma))} & (\text{And.b.}) \frac{\Diamond_s((\alpha \sim \beta) \wedge (\alpha \sim \gamma) \wedge \Gamma)}{\Diamond_s((\alpha \sim (\beta \wedge \gamma)) \wedge \Gamma')} \\
(\text{Or.a.}) \frac{\Box_s(\alpha \sim \gamma), \Box_s(\beta \sim \gamma)}{\Box_s((\alpha \vee \beta) \sim \gamma)} & (\text{Or.b.}) \frac{\Diamond_s((\alpha \sim \gamma) \wedge (\beta \sim \gamma) \wedge \Gamma)}{\Diamond_s(((\alpha \vee \beta) \sim \gamma) \wedge \Gamma')} \\
(\text{CM.a.}) \frac{\Box_s(\alpha \sim \beta), \Box_s(\alpha \sim \gamma)}{\Box_s((\alpha \wedge \beta) \sim \gamma)} & (\text{CM.b.}) \frac{\Diamond_s((\alpha \sim \beta) \wedge (\alpha \sim \gamma) \wedge \Gamma)}{\Diamond_s(((\alpha \wedge \beta) \sim \gamma) \wedge \Gamma')}
\end{array}$$

**Figure 3:** KLM Postulates for DRSL. Here,  $\Gamma'$  is used as shorthand to denote the original diamond-bound sentence occurring in the premise of each rule.

**Definition 8.** [4] A DRSL formula  $\phi$  is in normal form if it is in the form

$$\phi = \bigwedge_{i=1}^n \phi_i$$

where for each  $i \in \{1, \dots, n\}$  either (a)  $\phi_i \in \mathcal{L}^\sim$ , (b)  $\phi_i = \Box_s \psi$ , or (c)  $\phi_i = \Diamond_s \psi$  for some  $\psi \in \mathcal{L}^\sim$  and  $s \in \mathcal{S}$ . That is,  $\phi$  is a conjunction of propositional KLM formulas bound by at most one standpoint modality.

Leisegang et al. [4] show that each DRSL formula has a semantically equivalent formula in normal form.

**Lemma 2.** For any  $\phi \in \mathcal{L}_S^\sim$  which is not a standpoint sharpening, there exists a formula  $\phi' \in \mathcal{L}_S^\sim$  in normal form such that, for any preferential standpoint structure  $M$ , we have  $M \models \phi$  iff  $M \models \phi'$ .

The following result shows that a formula in normal form can be generated via the rules in Figures 2 and 3. Furthermore, since we have shown soundness for each of our postulates, it follows that the formula in normal form obtained through applying the postulates is a formula which is equivalent in the semantics.

**Lemma 3.** For any  $\phi \in \mathcal{L}_S^\sim$  such that  $\phi$  is not a standpoint sharpening, there exists some  $\phi' \in \mathcal{L}_S^\sim$  in normal form such that for any preferentially closed set  $\mathcal{A} \subseteq \mathcal{L}_S^\sim$ ,  $\phi \in \mathcal{A}$  iff  $\phi' \in \mathcal{A}$ .

In our following results, we therefore assume without losing generality that any DRSL formula considered is in normal form. We utilise this in the following results leading up to our main representation theorem. In our representation result, we aim to take a preferentially closed set of DRSL statements  $\mathcal{A}$  and show that there is a preferential standpoint structure  $M$  which satisfies formulas iff they are in  $\mathcal{A}$ . In order to do this, we induce sets of formulas in  $\mathcal{L}^\sim$  based on the formulas in  $\mathcal{A}$  which are closed under propositional preferential reasoning.

**Definition 9.** Given a set  $\mathcal{A} \subseteq \mathcal{L}_S^\sim$  and a standpoint symbol  $s \in \mathcal{S}$ , the derived set  $\mathcal{A}_s \subseteq \mathcal{L}^\sim$  is given by

$$\mathcal{A}_s := \{\phi \in \mathcal{L}^\sim \mid \Box_s \phi \in \mathcal{A}\}$$

This intuitively defines the set defeasible beliefs which necessarily hold for the standpoint  $s$ . We additionally need to consider sets in  $\mathcal{L}^\sim$  which consider additional information which is possible, but not necessary for a standpoint. We start with a preliminary definition.

**Definition 10.** For a set  $\mathcal{A} \subseteq \mathcal{L}_S^\sim$  and a formula of the form  $\Diamond_s \psi \in \mathcal{A}$  where  $\psi \in \mathcal{L}^\sim$  we define a set of  $(\psi, s)$ -conjuncts in  $\mathcal{A}$  as a finite set  $C \subseteq \mathcal{L}^\sim$  such that  $\psi \in C$  and  $\Diamond_s(\bigwedge C) \in \mathcal{A}$ . We denote the set of all  $(\psi, s)$ -conjuncts in  $\mathcal{A}$  by  $\text{Conj}_{\mathcal{A}}(\psi, s)$ .

This allows us to induce the sets derived from  $\mathcal{A}$  which collect the local consequences relative to a formula of the form  $\Diamond_s \phi$ .

**Definition 11.** Suppose  $\mathcal{A} \subseteq \mathcal{L}_S^\sim$  and  $\Diamond_s \psi \in \mathcal{A}$  where  $\psi \in \mathcal{L}^\sim$ . For each sequence of conjuncts  $\omega = (C_i)_{i \in \mathbb{N}}$  in  $\mathbf{Conj}_{\mathcal{A}}(\psi, s)$  such that  $C_i \subset C_{i+1}$  for all  $i \in \mathbb{N}$ , we define  $\mathcal{A}_{s,\omega}^\psi$  as the maximal element of this chain. That is,

$$\mathcal{A}_{s,\omega}^\psi := \bigcup_{i=1}^{\infty} C_i$$

for a fixed  $\Diamond_s \psi \in \mathcal{A}$ . We denote the set of all such union as  $\mathbf{LimConj}_{\mathcal{A}}(\psi, s)$ . We further denote the subset-maximal elements of  $\mathbf{LimConj}_{\mathcal{A}}(\psi, s)$  as  $\mathbf{MaxConj}_{\mathcal{A}}(\psi, s)$ .

That is, if  $X \in \mathbf{MaxConj}_{\mathcal{A}}(\psi, s)$ , then there is no  $X' \in \mathbf{LimConj}_{\mathcal{A}}(\psi, s)$  such that  $X \subset X'$ . Note that, in general we are guaranteed the existence a non-trivial infinite chain of conjuncts for each  $\Diamond_s \psi \in \mathcal{A}$ . This follows from the fact that there are infinitely many syntactically distinct Boolean formulas over a finite set of atoms, and so infinitely many formulas of the form  $\alpha \sim \alpha$  will occur in  $\mathcal{A}_{s,\omega}^\psi$ , due to an application of the rules **Ref**, **RN** and **K.b**. We derive a set of formula in  $\mathcal{L}^\sim$  for *each chain* in order to maintain satisfiability. If instead we define the set  $\mathcal{A}_s^\psi$  as the union of all conjuncts in  $\mathbf{Conj}_{\mathcal{A}}(\psi, s)$ , we may obtain contradictory information in  $\mathcal{A}_s^\psi$  that does not occur in  $\mathcal{A}$ . For example, we may have that  $\Diamond_s(\psi \wedge p), \Diamond_s(\psi \wedge \neg p) \in \mathcal{A}$  and so the union of  $\mathbf{Conj}_{\mathcal{A}}(\psi, s)$  would contain  $p$  and  $\neg p$  and thus be unsatisfiable in non-trivial cases. However,  $p$  and  $\neg p$  would not occur in the same chain of conjuncts unless  $\Diamond_s(\psi \wedge p \wedge \neg p) \in \mathcal{A}$ , which would make  $\mathcal{A}$  unsatisfiable. The following result shows that sets we derive from  $\mathcal{A}$  preserve closure under the KLM postulates.

**Lemma 4.** If  $\mathcal{A}$  is preferentially closed, then each derived set of the form  $\mathcal{A}_s$  is closed under the KLM postulates and classical propositional logic. Furthermore, for all  $s \in \mathcal{S}$  and all  $\Diamond \psi \in \mathcal{A}$  where  $\psi \in \mathcal{L}^\sim$ , we have that each set in  $\mathbf{MaxConj}_{\mathcal{A}}(\psi, s)$  is closed under the KLM postulates and classical propositional logic.

We also show that our derived sets preserve satisfiability.

**Lemma 5.** If  $\mathcal{A}$  is satisfiable, then  $\mathcal{A}_s$  and  $\mathcal{A}_{s,\omega}^\psi \in \mathbf{MaxConj}_{\mathcal{A}}(\psi, s)$  are non-trivially satisfiable.

It follows that, when  $\mathcal{A}$  is satisfiable and preferentially closed, for each derived set of the form  $\mathcal{A}_s$  there exists some preferential interpretation  $\mathcal{I}_s$  such that  $\mathcal{I}_s \models \phi$  iff  $\phi \in \mathcal{A}_s$ . Similarly, for each derived set of the form  $\mathcal{A}_{s,\omega}^\psi$ , there exists some preferential interpretation  $\mathcal{I}_{s,\omega}^\psi$  such that  $\mathcal{I}_{s,\omega}^\psi \models \phi$  iff  $\phi \in \mathcal{A}_{s,\omega}^\psi$ . We use these derived preferential interpretations to define the preferential standpoint structure which characterises  $\mathcal{A}$ .

**Definition 12.** Given a preferentially closed set  $\mathcal{A}$  of DRSL formulas, we define the preferential standpoint structure  $M_{\mathcal{A}} = (\Pi, \sigma, \tau)$  as follows:

1.  $\Pi = \{\pi_s \mid s \in \mathcal{S} \cup \{*\}\} \cup \{\pi_{s,\omega}^\psi \mid \Diamond_s \psi \in \mathcal{A}, \psi \in \mathcal{L}^\sim \text{ and } \mathcal{A}_{s,\omega}^\psi \in \mathbf{MaxConj}_{\mathcal{A}}(s, \psi)\}$ .
2.  $\sigma(s) = \{\pi_t \mid t \preceq s \in \mathcal{A}\} \cup \{\pi_{t,\omega}^\psi \mid t \preceq s \in \mathcal{A}\}$ .
3.  $\tau(\pi_s) = \mathcal{I}_s$  and  $\tau(\pi_{s,\omega}^\psi) = \mathcal{I}_{s,\omega}^\psi$ .

As a result of Lemma 4,  $\tau(\pi)$  is a well-defined preferential interpretation for each  $\pi \in \Pi$ . Moreover,  $\tau(s)$  is non-empty for all  $s \in \mathcal{S}$  and  $\tau(*) = \Pi$  since  $s \preceq * \in \mathcal{A}$  for all  $s \in \mathcal{S}$ . That is,  $M_{\mathcal{A}}$  is a well-defined preferential standpoint structure. The following lemma shows us that  $M_{\mathcal{A}}$  is in fact the standpoint structure we use to represent the set  $\mathcal{A}$ .

**Lemma 6.** For any satisfiable, preferentially closed set  $\mathcal{A}$ , we have that  $\phi \in \mathcal{A}$  iff  $M_{\mathcal{A}} \models \phi$ .

As a result of this, we obtain the following representation theorem, which is the main contribution of this section.

**Theorem 2.** A set  $\mathcal{A} \subseteq \mathcal{L}_S^\sim$  of DRSL formulas is satisfiable and preferentially closed iff there exists some preferential standpoint structure  $M$  such that  $\phi \in \mathcal{A}$  iff  $M \models \phi$ .

## 4. Entailment

In this section, we turn our attention to defining and computing entailment from DRSL knowledge bases. In the propositional setting, KLM-style defeasible reasoning has several non-equivalent notions of entailment defined for a given knowledge base [7]. In this paper, we do not focus on a single form of defeasible entailment, but rather we attempt to show principled means to lift classes of entailments from the propositional setting to the standpoints setting. The next sections characterise the lifting of several well-known defeasible entailment operators from the propositional case. We assume that each knowledge base is given in *conjunction free normal form*. That is, for each formula in  $\mathcal{K}$  in normal form that has the shape of a conjunction  $\bigwedge_{i=1}^n \phi_i$ , we replace it by its set of conjuncts  $\{\phi_i \mid 1 \leq i \leq n\}$ . It is clear to see that this is logically equivalent, and this assumption allows us simpler definitions for characterizing derived propositional knowledge bases within the following sections.

### 4.1. Preferential Entailment

The first form of defeasible entailment we consider is preferential entailment, which is the monotonic core of KLM-style reasoning. This is defined via the usual Tarskian methods. We provide both the propositional and DRSL definition here.

**Definition 13.** Given a (finite) knowledge base  $\mathcal{K} \subseteq \mathcal{L}^\sim$ , we say that  $\phi \in \mathcal{L}^\sim$  is *preferentially entailed* by  $\mathcal{K}$ , denoted  $\mathcal{K} \models_{P,prop} \phi$  if for any preferential interpretation  $\mathcal{I}$ , we have that  $\mathcal{I} \models \mathcal{K}$  implies  $\mathcal{I} \models \phi$ .

**Definition 14.** Given a (finite) knowledge base  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$ , we say that  $\phi \in \mathcal{L}_S^\sim$  is *preferentially entailed* by  $\mathcal{K}$ , denoted  $\mathcal{K} \models_P \phi$  if for any preferential standpoint structure  $M$ , we have that  $M \models \mathcal{K}$  implies  $M \models \phi$ .

From Theorem 2, we obtain the following corollary.

**Corollary 1.**  $\phi \in \mathcal{L}_S^\sim$  is preferentially entailed by  $\mathcal{K}$  iff  $\phi \in C(\mathcal{K})$ , where  $C(\mathcal{K})$  is the set obtained by exhaustively applying the rules in Figures 2 and 3, as well as the rules of classical propositional logic to  $\mathcal{K}$ .

The above corollary shows that we are able to compute preferential entailment through applications of the rules given in Figures 2 and 3, in order to determine whether  $\mathcal{K} \models_P \phi$  for some  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$  and  $\phi \in \mathcal{L}_S^\sim$ . Next we show that we can reduce entailment-checking in the standpoint case to entailment checking in the propositional case. We first note following preliminary lemma.

**Lemma 7.** For any  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$ , we have that a standpoint sharpening statement  $s \preceq t \in C(\mathcal{K})$  iff  $t = *$  or  $t = s$  or  $s \preceq t$  is in the transitive closure of  $\preceq_{\mathcal{K}} = \{(s_1, s_2) \mid s_1 \preceq s_2 \in \mathcal{K}\}$ .

The above holds since it is clear from the rules in Figure 2 that the only way to obtain standpoint sharpenings as conclusions are through one of the cases above. We are therefore able to compute the set of standpoint sharpening statements in PTIME, since this amounts to computing transitive closure. With this in mind, we derive a set of KLM propositional statements from  $\mathcal{K}$  for each standpoint. Intuitively, this set represents the “base case” of beliefs for this standpoint.

**Definition 15.** For a DRSL knowledge base  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$ , and a standpoint  $s \in \mathcal{S}$  we define the derived set  $\mathcal{K}_s$  as:

$$\mathcal{K}_s := \{\phi \in \mathcal{L}^\sim \mid \phi \in \mathcal{K} \text{ or } \Box_t \phi \in \mathcal{K}, s \preceq t \in C(\mathcal{K})\}.$$

We then define another set of propositional KLM statements for each diamond-bound statement occurring in  $\mathcal{K}$ . Intuitively, this represents constructing the precise “version” of a standpoint  $s$ ’s beliefs once we take into account an extra possibility belief made explicit in the knowledge base.

**Definition 16.** For a DRSL knowledge base  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$ , a standpoint  $s \in \mathcal{S}$  and a statement  $\Diamond_s \phi \in \mathcal{K}$ , we define the set  $\mathcal{K}_s^\phi$  as:

$$\mathcal{K}_s^\phi := \mathcal{K}_s \cup \{\phi\}.$$

Moreover, from this we can find a collection of sets in  $\mathcal{L}^\sim$  which are all those that ought to be associated with a specific standpoint, representing each distinguished possibility for a standpoint which is found in the knowledge base.

**Definition 17.** For a knowledge base  $\mathcal{K}$  and a standpoint  $s \in \mathcal{S}$ , we define the set of  $s$ -associated propositional knowledge bases, or  $\mathbf{PropKB}_{\mathcal{K}}(s)$  as

$$\mathbf{PropKB}_{\mathcal{K}}(s) := \{\mathcal{K}_t \mid t \preceq s \in C(\mathcal{K})\} \cup \{\mathcal{K}_t^\phi \mid t \preceq s \in C(\mathcal{K}), \Diamond_s \phi \in \mathcal{K}\}.$$

These derived sets are inspired by the sets constructed in the algorithm `StandpointSplit`, given by Leisegang et al. [4]. As a result, it is known that computing the sets  $\mathcal{K}_s$ ,  $\mathcal{K}_s^\phi$  and  $\mathbf{PropKB}_{\mathcal{K}}(s)$  is in PTIME. We then show how the sets defined above allow us to check for preferential entailment in the DRSL case.

**Proposition 1.** Consider a DRSL knowledge base  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$ , and a statement  $\Box_s \psi \in \mathcal{L}_S^\sim$ , where  $\psi \in \mathcal{L}^\sim$ . Then,  $\mathcal{K} \models_P \Box_s \psi$  iff  $\mathcal{K}_s \models_{P,prop} \psi$ .

Note that this also allows us to check whether  $\mathcal{K} \models_P \psi$  for  $\psi \in \mathcal{L}^\sim$ , since we can equivalently check  $\mathcal{K} \models_P \Box_* \psi$ . For diamond-bound statements, we cannot reduce DRSL preferential entailment-checking to a single propositional entailment check, but rather to a set of propositional entailment checks.

**Proposition 2.** Consider a DRSL knowledge base  $\mathcal{K} \subseteq \mathcal{L}_S^\sim$ , and a statement  $\Diamond_s \psi \in \mathcal{L}_S^\sim$ , where  $\psi \in \mathcal{L}^\sim$ . Then  $\mathcal{K} \models_P \Diamond_s \psi$  iff  $X \models_{P,prop} \psi$  for some  $X \in \mathbf{PropKB}_{\mathcal{K}}(s)$ .

We can then, in general, reduce DRSL preferential entailment-checking to propositional preferential entailment-checking. Each DRSL formula can be rewritten in normal form. Then, in order to check such a conjunction in normal form, we just check each of the conjuncts using the corresponding propositional entailment checks from Propositions 1 and 2. Using this, we are able to analyse the complexity of preferential reasoning in DRSL, and show that it is within the same complexity class as preferential entailment for KLM propositional logic.

**Theorem 3.** Preferential entailment-checking in DRSL is coNP-complete.

## 4.2. Entailment Based on Single Ranked Models

Next, we consider systematically a class of non-monotonic entailment relations considered in KLM-style defeasible reasoning. In particular, these are the defeasible entailment relations where the order on the preferential interpretation can be expressed by a *ranking function*. This is an important class of relations, as many well-known non-monotonic entailment relations fall within this class, such as rational closure [8, 9], lexicographic closure [10] and any inference relation based on a single  $c$ -representation [11].

**Definition 18.** A ranking function  $r$  is a function  $r : 2^P \rightarrow \mathbb{N} \cup \{\infty\}$ , satisfying the following property: if  $r(u) < \infty$ , then for every  $0 \leq j < r(u)$  there exists  $v \in 2^P$  such that  $r(v) = j$ .

Each ranking function can be associated to a preferential interpretation by choosing  $\mathcal{I} = (W, l, <_r)$  where  $W = 2^P \setminus r^{-1}\{\infty\}$ ,  $l$  is the identity and  $u <_r v$  iff  $r(u) < r(v)$ . We write  $r \Vdash \alpha \sim \beta$  iff  $\min_{<_r} \llbracket \alpha \rrbracket \subseteq \llbracket \beta \rrbracket$ . That is, if the minimally ranked  $\alpha$ -valuations satisfy  $\beta$ . Given a knowledge base  $\mathcal{K} \in \mathcal{L}^\sim$  and a formula  $\phi \in \mathcal{L}^\sim$ , we consider the non-monotonic entailment relations  $\approx$  such that  $\mathcal{K} \approx \phi$  iff  $r \Vdash \phi$ , where  $r$  is a single ranking function which satisfies  $\mathcal{K}$ . This builds on the work of Casini et al. [7], who analyse algorithmic and semantic approaches to this general class of entailments. In particular, they introduce the `DefeasibleEntailment` algorithm which takes a ranking function, and provides an algorithmic means for computing the defeasible entailment relation which is based on this ranking. Our work can be seen as a significant extension of the work of Leisegang et al. [4] who consider Rational Closure in the context of DRSL. In the following section, we provide a systematic means for lifting both the semantics and algorithms for entailments within this class. Furthermore, we

will show that the proposed algorithms and semantics are equivalent definitions for determining such an entailment. Finally, we show that entailment-checking for this class of relations in the DRSL case stays within the same complexity class as entailment checking in the propositional case. In our analysis of complexity, we note that `DefeasibleEntailment` may require exponentially many SAT calls in the size of  $\mathcal{K}$  [7]. This is under the assumption that  $r_{\mathcal{K}}$  is already computed, and in general there are no known complexity bounds for the construction  $r_{\mathcal{K}}$  from  $\mathcal{K}$ . In the well-studied cases of rational closure and lexicographic closure, entailment is  $P_{\parallel}^{\text{NP}}$ -complete and  $P^{\text{NP}}$ -complete, respectively [8, 12]. In this section, we make the following two assumptions about a given propositional defeasible entailment relation  $\approx$ :

1. We assume that, given a KB  $\mathcal{K} \in \mathcal{L}^{\sim}$ , there exists a deterministic means to construct our ranking function  $r_{\mathcal{K}}$  such that  $\mathcal{K} \approx \phi$  iff  $r_{\mathcal{K}} \Vdash \phi$ . That is, there exists some known mapping  $r : 2^{\mathcal{L}^{\sim}} \rightarrow \mathcal{R}$  which defines  $\approx$ , where  $\mathcal{R}$  is the set of ranking functions. We refer to such a mapping as a *ranking strategy*, and denote  $r(\mathcal{K})$  as  $r_{\mathcal{K}}$ .
2. We assume that  $\phi \in \mathcal{K}$  implies  $\mathcal{K} \approx \phi$  and if  $\phi$  is a Boolean formula  $\mathcal{K} \models_{P,prop} \phi$  iff  $\mathcal{K} \approx \phi$ . These properties are known as *Inclusion* and *Classical Preservation*, and are considered necessary for a well-defined defeasible entailment [7].

For such an entailment relation  $\approx_{prop}$ , our goal is to define an entailment relation  $\approx_{DRSL}$  which faithfully lifts the entailment relation from  $\mathcal{L}^{\sim}$  to  $\mathcal{L}_S^{\sim}$ . In order to understand this, we once again consider the approach we take in the case of preferential reasoning. In Definition 17,  $\mathbf{Prop}_{\mathcal{K}}(s)$  refers to a number of different propositional knowledge bases which represent salient sets of beliefs which ought to be considered for each standpoint. Here,  $\mathcal{K}_s$  provides the basic necessary set of knowledge that this standpoint is required to have, while each  $\mathcal{K}_s^{\phi}$  is required in order to combine the basic beliefs of  $s$  with additional possibilities which are *made distinct* through their inclusion in the knowledge base. Furthermore, if we let  $r$  be the ranking strategy associated with  $\approx_{prop}$ , we are able to semantically realise the sets of beliefs in one of the knowledge bases  $X \in \mathbf{Prop}_{\mathcal{K}}(s)$  through the associated ranking function  $r_X$ . Using this motivation, we define a ranked standpoint structure in which each precisification is exactly the ranking function associated to some knowledge base in  $\mathbf{Prop}_{\mathcal{K}}(*)$ , and the set of precisifications assigned to each standpoint  $s$  is exactly those precisifications which model the different sets of beliefs in  $\mathbf{Prop}_{\mathcal{K}}(s)$ .

**Definition 19.** Consider a DRSL knowledge base  $\mathcal{K} \subseteq \mathcal{L}_S^{\sim}$ , a propositional defeasible entailment  $\approx$  and its associated ranking strategy  $r$ . We define the standpoint  $\approx$ -model for  $\mathcal{K}$  as  $M_{\mathcal{K}}^{\approx} = (\Pi_{\mathcal{K}}, \sigma_{\mathcal{K}}, \tau_{\mathcal{K}}^r)$  where

- $\Pi_{\mathcal{K}} := \{\pi_X \mid X \in \mathbf{Prop}_{\mathcal{K}}(*) \setminus \{\mathcal{K}_*\}\}$ .
- $\sigma_{\mathcal{K}}(s) = \{\pi_X \in \Pi_{\mathcal{K}} \mid X \in \mathbf{Prop}_{\mathcal{K}}(s) \setminus \{\mathcal{K}_*\}\}$  for  $s \in \mathcal{S}$ .
- $\tau_{\mathcal{K}}^r(\pi_X) = r_X$ , where  $r_X$  is the ranking function determined by the ranking strategy  $r$ .

Note here that the definitions of  $\Pi_{\mathcal{K}}$  and  $\sigma_{\mathcal{K}}$  depends only on  $\mathcal{K}$  and not on the particular entailment or ranking strategy which we consider, and therefore we omit  $r$  from their indices. The first basic results to note is that  $M_{\mathcal{K}}^{\approx}$  is a model of  $\mathcal{K}$ .

**Proposition 3.** Consider  $\mathcal{K} \subseteq \mathcal{L}_S^{\sim}$ , a defeasible entailment  $\approx$  and the associated ranking strategy  $r$ :

1. If  $\mathcal{K}$  is satisfiable, then there is no  $\pi_X \in \Pi_{\mathcal{K}}$  such that  $\tau_{\mathcal{K}}^r(\pi_X) = \emptyset$ .
2. If  $\phi \in \mathcal{K}$  then  $M_{\mathcal{K}}^{\approx} \Vdash \phi$ .

This model defines a means for lifting a propositional defeasible entailment  $\approx_{prop}$  to the DRSL case.

**Definition 20.** For  $\mathcal{K} \subseteq \mathcal{L}_S^{\sim}$  and  $\phi \in \mathcal{L}_S^{\sim}$ , we say that  $\mathcal{K} \approx_{DRSL} \phi$  iff  $M_{KB}^{\approx_{prop}} \Vdash \phi$ .

For the rest of the section we abuse notation by using  $\approx$  to refer to both the propositional entailment and its lifting to DRSL. An important design choice to discuss here is the fact that in our model, we remove  $\mathcal{K}_*$  from  $\mathbf{Prop}_{\mathcal{K}}(*)$ . We refer to this as the *closed world assumption for standpoints*:

The *closed world assumption for standpoints* is the assumption that there exist no novel standpoints outside those that occur in our vocabulary. This is what allows us to faithfully construct  $M_{\mathcal{K}}^{\approx}$  as a representative model for entailment, since we can create only precisifications that occur in standpoints that are named in our knowledge base. In particular, in both the algorithmic and semantic lifting of  $\approx$  to the propositional case, we remove  $\mathcal{K}_*$  from the considered propositional knowledge bases. This is because  $\mathcal{K}_*$  only contains those statements which we know are universally true, and therefore, emulates a standpoint where nothing except universal beliefs are known. We make this design choice since when we are working with defeasible entailment, we are reasoning non-monotonically about the beliefs of known standpoints, and so we believe this assumption is a valid one. In fact, the alternative may lead us to disregard conclusions which seem reasonable in a setting of prototypical reasoning. Consider the knowledge base  $\mathcal{K} = \{\Box_s(p \sim q), \Box_t(p \sim q)\}$ . It seems reasonable here to say that all considered standpoints believe that  $p \sim q$  holds. That is  $\mathcal{K} \approx \Box_*(p \sim q)$ . However, if we were to require that  $\mathcal{K}_*$  is considered, then we must consider whether the empty set entails  $p \sim q$ , which it usually does not for well-known definitions of  $\approx$ . Hence,  $\mathcal{K} \not\approx \Box_*(p \sim q)$ , simply because  $\mathcal{K}_*$  is empty. This seems undesirable, since our logic ultimately looks to increase possible agreements between standpoints, if possible, and withdraw upon learning new information which contradicts this (for example, by adding  $v$  to  $\mathcal{S}$  and  $\Box_v(p \sim \neg q)$  to  $\mathcal{K}$ ). The same assumption is not made when  $\Diamond_*\psi$  is introduced to the knowledge base. In this case, we still consider  $\mathcal{K}_*^\psi$ , since our knowledge informs us that some standpoint holds  $\psi$  possible, but we cannot be more specific about which standpoint that is. A result of this is that it is possible to emulate the open world setting using our framework by simply adding  $\Diamond_*\top$  to a given DRSL knowledge base. This necessitates the consideration of the knowledge base  $\mathcal{K}_*^\top$  in the semantics, which is deductively equivalent to  $\mathcal{K}_*$ . That is, it introduces a “place-holder” precisification, whose only beliefs are those we know to be universal across our whole standpoint domain, and therefore includes the possibility of unnamed standpoints where no details of their beliefs are known.

One corollary of this, is that the entailment for DRSL is not well-defined when our knowledge base is propositional. If  $\mathcal{K} \subseteq \mathcal{L}^\sim$ , then for any defeasible entailment  $\Pi_{\mathcal{K}} = \emptyset$  and so  $M_{\mathcal{K}}^{\approx}$  is no longer a well-defined standpoint structure. However, we note that in cases where the knowledge base is entirely propositional, we can refer to well-known propositional approaches to KLM, or we can avoid this by simulating the open world scenario by adding  $\Diamond_*\top$  to our knowledge base. With this addition, we get the expected result that  $\approx_{DRSL}$  collapses into  $\approx_{prop}$  when considering propositional statements.

**Proposition 4.** Suppose  $\mathcal{K} = \mathcal{K}' \cup \{\Diamond_*\top\}$  where  $\mathcal{K}' \subseteq \mathcal{L}^\sim$ . Then for  $\phi \in \mathcal{L}^\sim$ , we have  $\mathcal{K} \approx_{DRSL} \phi$  iff  $\mathcal{K}' \approx_{prop} \phi$ .

**Example 3.** We show here the model construction of the rational closure of  $\mathcal{K}_T$  in Example 2. This is given by the model  $M_{RC} = (\Pi, \sigma, \tau)$  where  $\Pi = \{\pi_B, \pi_C, \pi_L\}$ ;  $\sigma(B) = \{\pi_B\}$ ,  $\sigma(C) = \{\pi_C, \pi_L\}$ ,  $\sigma(L) = \{\pi_L\}$ ;  $\tau(\pi_B)$  is given by a ranking function where  $r(v) = 0$  if  $v \Vdash (t \rightarrow f) \wedge (f \rightarrow v)$  and  $r(v) = \infty$  otherwise;  $\tau(\pi_C)$  and  $\tau(\pi_L)$  are defined as ranking functions in the table below:

| rank     | $\tau(\pi_C)$                                   | $\tau(\pi_L)$                              |
|----------|-------------------------------------------------|--------------------------------------------|
| $\infty$ | all other valuations                            | all other valuations                       |
| 1        | $\{ts_avs_wf\}, \{ts_wf\}, \{t\}, \{s_avs_wf\}$ | $\{ts_wf\}, \{t\}$                         |
| 0        | $\{ts_av\}, \{s_av\}, \{s_wf\}, \emptyset$      | $\{ts_av\}, \{s_av\}, \{s_wf\}, \emptyset$ |

We can see that  $M \Vdash \Box_C(t \sim \neg f)$  and so  $\mathcal{K} \approx_{RC} \Box_C(t \sim \neg f)$ , where  $\approx_{RC}$  denotes rational closure entailment. Moreover, we can see by the counter-model in Example 2 that  $\mathcal{K}_T \not\approx_P \Box_C(t \sim \neg f)$ , showing that  $\approx_{RC}$  is strictly stronger than  $\approx_P$ .

Now that we have defined a semantic means for extending ranking-based defeasible entailment from the propositional to the DRSL case, we analyse algorithmic approaches to DRSL, and show that we can lift entailment algorithms from the propositional case to the DRSL case. The construction of such algorithms is linked closely to the semantic structures previously defined. Since our basis for lifting

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**Algorithm 1** StdptRankEntail

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**Input:** A DRSL knowledge base  $\mathcal{K}$  in normal form, a ranking strategy  $r$  and a (non-standpoint sharpening) DRSL  $\phi$  in normal form.

**Output:** True if  $\mathcal{K} \models \phi$ ; False otherwise.

```
1: if  $\phi = \phi_1 \wedge \phi_2$  then
2:   if StdptRankEntail( $\mathcal{K}, r, \phi_1$ ) = True and
     StdptRankEntail( $\mathcal{K}, r, \phi_2$ ) = True then
3:     return True;
4:   else
5:     return False;
6: else if  $\phi = \Box_s \psi$  then
7:   for  $X \in \text{PropKB}_{\mathcal{K}}(s) \setminus \{K_*\}$  do
8:     if DefeasibleEntail( $X, r_X, \psi$ ) = False then
9:       return False;
10:  return True;
11: else if  $\phi = \Diamond_s \psi$  then
12:  for  $X \in \text{PropKB}_{\mathcal{K}}(s) \setminus \{K_*\}$  do
13:    if DefeasibleEntail( $X, r_X, \psi$ ) = True then
14:      return True;
15: else
16:  return False;
```

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**Figure 4:** Ranked Entailment Algorithm for DRSL

entailment to a DRSL knowledge base involves utilising the propositional knowledge bases in  $\text{Prop}_{\mathcal{K}}(*)$ , we can similarly use an algorithm which queries defeasible entailment in DRSL by using the original propositional algorithms applied to appropriate knowledge bases in  $\text{Prop}_{\mathcal{K}}(*)$ . This is defined by the algorithm `StdptRankEntail` in Figure 4, which is a generalization of the DRSL algorithm for rational closure proposed by Leisegang et al. [4]. The algorithm works by directly checking a query with respect to the underlying propositional algorithm given for  $\models$ , where we check a query of the form  $\Box_s \phi$  or  $\Diamond_s \phi$  by checking the knowledge bases in  $\text{Prop}_{\mathcal{K}}(s)$ . Note here that unlike the case of  $\models_P$ , we cannot check  $\Box_s \phi$  queries by simply checking with respect to the knowledge base  $\mathcal{K}_s$ . This is due to the non-monotonicity of  $\models$ , which means that there might be some  $\mathcal{K}_s^\psi$  and  $\alpha \in \mathcal{L}^\sim$  such that  $\mathcal{K}_s \models \alpha$  and  $\mathcal{K}_s^\psi \not\models \alpha$ . We also note that in its most general form, our algorithm takes the ranking strategy  $r$  as an input. For known entailment relations such as rational and lexicographic closure, we replace the call to `DefeasibleEntail` with calls to the propositional rational and lexicographic closure algorithms respectively. Lastly, we note the abuse of notation here that in the original case, `DefeasibleEntail` is only defined for single defeasible implications “ $\alpha \vdash \beta$ ”, while in our case we allow conjunctions of such implications as inputs. In this case, we treat computing `DefeasibleEntail` for a conjunction of implications as equivalent to computing `DefeasibleEntail` for each of its conjuncts. The algorithm structures itself similarly to the semantic model in Definition 19: we split the model into a series of precisifications whose valuations are determined by propositional knowledge bases. On the other hand we split the algorithm into a series of calls to algorithms defined in the propositional case based on the same set of knowledge bases. This leads us to the following correspondence result.

**Theorem 4.** *Given DRSL knowledge base  $\mathcal{K}$  with  $\mathcal{K} \not\subseteq \mathcal{L}^\sim$ , a non-sharpening formula  $\phi \in \mathcal{L}_S^\sim$  and a defeasible entailment  $\models$  with selection strategy  $r$ , we have  $\mathcal{K} \models \phi$  iff `StdptRankEntail`( $\mathcal{K}, r, \phi$ ) = True.*

For standpoint sharpening statements, we once again only entail those which are noted in Lemma 7. This is clear from the construction of  $M_{\mathcal{K}}^\sim$ . Lastly, we consider the complexity of the algorithm. The algorithm above performs polynomially many calls to an underlying propositional algorithm, and the knowledge bases input in these calls are no bigger than the original knowledge base. Furthermore,

these calls do not depend on outcomes of previous calls and can be parallelised. Therefore we obtain the following complexity result.

**Theorem 5.** *For any propositional defeasible entailment  $\approx_{prop}$ , we have that if  $\approx_{prop}$  is computable in a complexity class  $C$  such that  $P_{||}^{NP} \subseteq C$ , then entailment-checking for  $\approx_{DRSL}$  remains in  $C$ . In particular, entailment checking for rational closure in DRSL is  $P_{||}^{NP}$ -complete and entailment checking for lexicographic closure in DRSL is  $P^{NP}$ -complete.*

Therefore, we are able to lift ranking based entailments from the propositional case to DRSL without increasing complexity in the cases of rational and lexicographic closure. More generally, if the underlying propositional entailment is in a class no better than  $P_{||}^{NP}$ , the complexity is preserved in the DRSL case.

## 5. Related Work and Conclusions

DRSL and its semantics were originally considered by Leisegang et al. [4]. Similar notions of non-monotonic standpoint logics with default-style beliefs are considered by Gorczyca and Straß [5, 13] who consider standpoint modalities in the non-monotonic modal logic S4F. Another method of integrating standpoint logics with KLM defeasibility is given by Leisegang et al. [14], who introduce Propositional Defeasible Standpoint Logic (PDSL) in which a defeasible notion of standpoint modalities and standpoint sharpenings are introduced. However, while defeasible implications occur within PDSL, they act as outer-level implications between modal statements, rather than defeasible beliefs held by standpoints. Hence, the semantics for PDSL and DRSL are non-equivalent. KLM-style defeasible reasoning has been employed in other modal logics such as **K** [15] and linear temporal logic (LTL) [16]. The addition of standpoint modalities to monotonic modal logics, such as LTL, has also been considered [17, 18, 19]. In this paper, we provided a study of integrating standpoint modalities and KLM-style defeasible beliefs, extending the work by Leisegang et al. [4] on DRSL. In particular, this paper contributes a KLM-style representation result for a set of proof-theoretic modal and KLM-style postulates which are sound and complete with respect to the semantics of preferential standpoint structures. Moreover, any set of DRSL statements closed under our postulates can be represented by a unique preferential standpoint structure. We then characterised preferential entailment in DRSL, as well as providing a systematic means for lifting the class of single ranking function based defeasible entailment relations from propositional to standpoint logics. This includes well-known relations such as rational and lexicographic closure. Lastly, for all defeasible entailments considered, we showed that entailment-checking in the DRSL case falls within the same complexity class as the propositional case.

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## Declaration on Generative AI

The authors have not employed any Generative AI tools.

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