Postulates Here, Postulates Everywhere: A Discussion of Contraction Postulates

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Abstract

Modifying a knowledge base to remove an unwanted consequence is a difficult task, since it is not obvious how to change it so that the unwanted consequence is no longer implied while avoiding unnecessary loss of information. This problem has been widely researched in belief change under the name of contraction and in ontology engineering under the name of ontology repair. The extension of belief change theory to description logic ontologies represents an important development for both areas, as it allows repair techniques to benefit from advances in belief change and enables contraction operations to preserve more consequences than classical contractions based on subsets of the belief base by utilizing advances in repair techniques. Consequently, different variations of these postulates have emerged in the literature, seeking to preserve the same intuitions as the original sets in different ways, as is the case with the use of Description Logics. This paper discusses the existing contraction postulates and their adequacy across different scenarios, exploring the intuition behind them and the use of different types of contraction postulates in the literature.

Keywords

Belief Change, AGM Theory, Contraction Postulates

1. Introduction

Belief change is a research area that aims to understand the rationality of an agent's belief changes in the face of possibly conflicting new information [1, 2].

The seminal work of Alchourrón, Gärdenfors, and Makinson [3], known as the AGM model after the authors' initials, is a milestone in the field, establishing a set of operations and rationality criteria for how rational agents change their beliefs. In the AGM model, beliefs are represented by logically closed sets of formulas, called *belief sets*, and the change occurs through three basic operations applied directly to the belief sets: expansion, contraction, and revision [2]. We focus on contraction in this paper, an operation that gives up as many beliefs as necessary so that the belief set no longer implies an unwanted sentence. Alchourrón et al. [3] established a set of postulates capturing the intuition behind this operation and the properties it should satisfy.

From a computational perspective, identifying belief states as belief sets has disadvantages: a belief set can be very large, and logical closure blurs the distinction between basic and inferred beliefs [2]. For these reasons, Hansson [4] proposed an extension of the AGM model in which an agent's beliefs are represented as a set of sentences that need not be closed under logical consequence, referred to as a *belief base*. Belief bases distinguish basic from derived beliefs and thus have greater expressive power; changes are made to basic beliefs, and derived beliefs change only in response [2]. The operations and postulates for contraction and revision were correspondingly adapted to belief bases.

Alchourrón et al. [3] introduced a constructive definition of contraction, together with a representation theorem demonstrating that the AGM postulates for contraction precisely characterize the corresponding class of contraction operations. Constructive definitions for formulations of contraction for belief bases were also subsequently proposed by Hansson [5]. *Partial meet contraction* is a construction based on the idea of selecting some maximal subsets of the beliefs (belief set or belief base) and building their intersection.

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Another alternative construction, proposed by Hansson [6], is called *kernel contraction.* In kernel contraction, unlike partial meet contraction, the minimal subsets of the belief base that imply the sentence to be contracted, called *kernels*, are identified, and at least one element from each kernel is removed from the belief base [6].

There exists a trade-off between representing an agent's beliefs as a belief set or as a belief base. Belief sets enjoy representation theorems characterizing behavior at the knowledge level, but sacrifice expressive power and tractability since they may be infinite; belief bases offer expressive power and tractability but lack such theorems. The gap can be bridged: applying contraction to a finite base and closing the result under logical consequence yields, for every base contraction operator, a corresponding *base-generated* belief set operator [2, 7].[1]

Although the AGM theory and its extensions were originally formulated for logics close to classical propositional logic, many adaptations to other logics were considered in the last twenty years [8]. Consequently, several variations of these postulates have emerged in the literature, seeking to preserve the same intuitions as the original sets in different ways, as is the case with the use of Description Logics [9].

Given an ontology represented in Description Logics, we can use belief change operations to repair the ontology [10]. Ontology repair is an operation that aims to correct logical inconsistencies or make modifications to the knowledge base so that an unwanted consequence is not inferred. Repairing an ontology implies modifying the ontology in a way that removes unwanted consequences while ensuring minimal changes, that is, preserving as many consequences as possible [11, 12].

The field of belief change plays an important role in the ontology evolution process, although contraction operations and repair techniques primarily address the problem of removing undesirable implications from the knowledge base [13, 14]. Recent works (see [15, 16, 14, 17]) have established and demonstrated the connection between belief change and ontology repair by proposing repair methods based on contraction operations and, in some cases, adaptations of contraction for Description Logics. This extension benefits both areas: repair techniques gain from advances in belief change, while contraction operations can preserve more consequences than classical contractions based on subsets of the belief base by utilizing advances in repair techniques [13, 16]. Contraction postulates, however, are not always suitable for practical applications. This paper discusses the existing contraction postulates and their adequacy across different scenarios, exploring the intuition behind them and their use throughout the literature [2].

This paper is organized as follows: Section 2 discusses postulates related to the principle of minimal change, Section 3 addresses syntax independence postulates. Sections 4, 5, and 6 cover closure, inclusion, and success postulates, respectively. Section 7 examines the supplementary postulates for conjunction-based contraction. Section 8 collects additional postulates from the literature that do not fall neatly into the previous categories.

Notation: In the rest of the paper, we will use K, K' to refer to belief sets, B, B' for belief bases and lowercase Greek letters for formulas. Cn stands for closure under logical consequence. $K{-}\alpha$ stands for the result of contracting K by α and $K{+}\alpha$ stands for expansion.

2. Minimal Change

The principle of *minimal change* requires that when changing beliefs, as much as possible should be retained, so that changes in belief states should be kept to the minimum necessary [18, 1]. In this section, we will discuss the postulates proposed to enforce minimal change.

Although the principle itself is undisputed, there is no consensus about which metric should be used to establish what counts as a minimal change (see [18] for a discussion).

[1] This holds for compact logics; there are logics in which a belief set has no finite base.

[2] We assume the reader has some familiarity with Horn logic and Description Logics. A basic acquaintance with these formalisms is sufficient to follow the discussion.

In AGM theory [3], the recovery postulate is an attempt to define the notion of minimal change for contraction [1]. According to this postulate, all beliefs from the original belief set must be recovered after being contracted and then expanded with respect to the same belief:

(recovery) $K \subseteq (K - \varphi) + \varphi$

Recovery is the most debated and controversial AGM postulate within the belief change community and is criticized for presenting counterintuitive behavior and consequences [19, 8, 20]. The counterexample below illustrates the undesirable behavior resulting from the recovery postulate:

Example 2.1. Hansson [2] *I believe that "Cleopatra had a son" (φ) and "Cleopatra had a daughter" (ψ), and thus also that "Cleopatra had a child" ($\varphi \vee \psi$, shortly κ). Then I receive information that makes me give up my belief in κ and contract my belief set accordingly. Soon afterwards I learn from a reliable source that Cleopatra had a child. It seems perfectly reasonable for me to then add κ to my set of beliefs without reintroducing either φ or ψ.*

In addition, recovery is a very restrictive postulate and does not apply to belief base contraction [2, 21]. For this reason, Hansson [22] proposed the relevance postulate to ensure that contractions in belief bases follow the principle of minimal change and avoid some non-intuitive aspects of recovery. In the relevance postulate, the beliefs that are removed during contraction must contribute in some way to inferring the contracted belief [8].

(relevance) if $\psi \in B$ and $\psi \notin B - \varphi$, then there is a set B' such that $B - \varphi \subseteq B' \subseteq B$ and that $\varphi \notin Cn(B')$ but $\varphi \in Cn(B' \cup \{\psi\})$.

Relevance is usually used in the literature in contraction approaches on belief bases that use both classical and non-classical logic [8]. Relevance has a weaker version called core-retainment, which is similar to relevance, except that it does not require the inclusion of the belief base and its subset. In the core-retainment postulate, there is only the requirement that B' be a subset of B [22].

(core-retainment) if $\psi \in B$ and $\psi \notin B - \varphi$, then there is a set B' that $B' \subseteq B$ and that $\varphi \notin Cn(B')$ but $\varphi \in Cn(B' \cup \{\psi\})$.

Ribeiro and Wassermann [21] relaxed the recovery postulate to require only that the initial belief base be contained in the contraction result and then expanded by the same formula, resulting in a weakened version of recovery called *logical recovery*. When the logical closure is Tarskian, the recovery postulate implies logical recovery.

(logical recovery) $B \subseteq Cn(B - \varphi + \varphi)$.

Moreover, in [21], another alternative for weakening the relevance postulate is proposed, in which logical inclusion ([23]) is considered rather than the inclusion of AGM theory adapted to belief bases:

(logical relevance) if $\psi \in B \setminus (B - \varphi)$, then there is some B' such that $B - \varphi \subseteq B' \subseteq Cn(B), \varphi \notin Cn(B')$ and $\varphi \in Cn(B' \cup \{\psi\})$

A special case of partial meet contraction, *maxichoice contraction*, is when a single maximal subset of the belief state not implying the input is selected as the result of contraction. The fullness postulate proposed in [3] is a stronger version of recovery and characterizes maxichoice contraction:

(fullness) if $\psi \in K$ and $\psi \notin K - \varphi$, then $\varphi \in (K - \varphi) + \psi$.

Maxichoice contraction is known to produce belief sets that are "too large" in the AGM setting [3]. However, for some non-classical logics as well as for belief bases, this does not occur.

Delgrande and Wassermann [24] propose characterizations of maxichoice and partial meet contraction for the case in which the belief set and belief base are composed of Horn clauses. In the case of belief sets, both maxichoice and partial meet contraction are based on the notion of weak remainder sets. A weak remainder set, unlike a remainder set of a Horn belief set H (which consists of the maximal subsets of H that do not imply the unwanted sentence), admits only one counter-model of φ in addition to the models of H and is closed under intersection.

To avoid unnecessary loss of information, instead of using recovery, the authors develop a new form of relevance for partial meet contraction in Horn belief sets called *weak relevance*. The weak relevance postulate does not require that witness H' be included in the original set of beliefs.

(weak relevance) [3] if $\beta \in H \backslash (H - \varphi)$, then there is some H' such that $H - \varphi \subseteq H'$, $\varphi \notin Cn^h(H')$ and $\varphi \in Cn^h(H' \cup \{\beta\})$.

And, for the case of maxichoice contraction of Horn belief sets, the following postulate (in the spirit of fullness) was proposed [24]:

(maximality^h) if $H \neq H \dot{-}_w \varphi$, then exists a β belong the Horn language s.t. $\{\varphi, \beta\}$ is inconsistent, $H \dot{-}_w \varphi \subseteq Cn^h(\{\beta\})$ and for all H' s.t $H \dot{-}_w \varphi \subset H' \subseteq H$ we have $H' \nsubseteq Cn^h(\{\beta\})$.

In Horn belief bases, the authors use the *e-remainder* [4], which, unlike in the case of belief sets, can be applied directly without causing undesirable behavior. Regarding minimal change to belief bases, Delgrande and Wassermann [24] formulate an adaptation of the fullness postulate for maxichoice in the context of Horn logic:

(fullness^h) if $\beta \in B \backslash (B \dot{-}_e \varphi)$, then $\varphi \notin Cn^h(B \dot{-}_e \varphi)$ and $\varphi \in Cn^h(B \dot{-}_e \varphi \cup \{\beta\})$.

Delgrande and Wassermann [24] propose an adaptation of relevance for the case of partial meet contraction of Horn belief bases:

(relevance^h) if $\beta \in B \backslash (B \dot{-}_e \varphi)$, then exists a B' such that $B \dot{-}_e \varphi \subseteq B' \subseteq B$, $\varphi \notin Cn^h(B')$ and $B \dot{-}_e \varphi = B \dot{-}_e \beta$.

Given that belief base inclusion and recovery cannot be simultaneously satisfied, Santos et al. [26] investigate the effects of weakly closed sets in partial meet contraction of a belief base, called Cn^*pseudo-contraction. The weak version consists of sets closed under a consequence relation Cn^* that generates some consequences of a set of formulas, for instance $Cn^*(B) = \{\varphi \vee \psi \mid \varphi \in B\}$, but yields fewer consequences than the classical consequence [21, 26]. As a result, it is necessary to define weaker versions of some postulates to include Cn^* in the formulation, the starred versions [26]. In this way, minimal change is ensured by *relevance**:

(relevance*) if $\psi \in Cn^*(B) \backslash (B - \varphi)$, then there is a B' such that $B - \varphi \subseteq B' \subseteq Cn^*(B), \varphi \notin Cn^*(B')$, but $\varphi \in Cn(B' \cup \{\psi\})$.

In an attempt to apply belief change to Description Logics, the contraction of axioms in an ontology based on the Description Logic $\mathcal{EL}$ was considered Zhuang and Pagnucco [27]. The authors define the notion of logical difference between two $\mathcal{EL}$ TBoxes, *Diff*(T, T'), as the subsumptions that follow from T, but not from T'[5]. Zhuang and Pagnucco [27] define a variation of recovery which is actually a version of the classical fullness postulate:

(TBox recovery) there exists no T' such that $T \models T'$ and $T \not\models \varphi$ and *Diff*$(T, T - \varphi) \models$ *Diff(T,T')* and $T' \not\equiv T - \varphi$.

Zhuang et al. [28] adapted another well-known construction for contraction, *epistemic entrenchment*, for logically closed TBoxes in the Description Logic $\mathcal{DL}$-Lite$_\mathcal{R}$. To capture the principle of minimal change, Zhuang et al. introduced the *critical retention (cr)* postulate. The authors define the *critical axioms* of ψ with respect to φ (denoted by $C^\varphi(\psi)$) as those axioms whose rankings in the epistemic entrenchment are critical in deciding whether to retain ψ in the contraction of φ. The intuition is that if an axiom ψ belongs to a TBox $\mathcal{T}$ and there exists a critical axiom σ such that σ is retained in $\mathcal{T} - \varphi$, then ψ it should also be retained in $\mathcal{T} - \varphi$ [28].

($\mathcal{T}$-critical retention) if $\psi \in \mathcal{T}$ and there is $\sigma \in C^\varphi(\psi)$ such that $\sigma \in \mathcal{T} - \varphi$ then $\psi \in \mathcal{T} - \varphi$.

[3] $Cn^h(A)$ denotes the deductive closure of a set of Horn formulas A under Horn consequence.

[4] In short, the *e-remainder* is an adaptation of remainder sets from AGM theory to Horn logic. For further details, see [25].

[5] The definition of *Diff* in [27] is more complicated than that, but this is the general idea.

Rienstra et al. [29] proposed to use an AGM like contraction operation to solve the following problem: given two $\mathcal{EL}$ concepts, C and D, find a concept $C \ominus D$ which is more general than C, is not subsumed by D, but is as similar as possible to C. AGM contraction postulates were adapted to this scenario [29].

Recovery is not appropriate for the context of concept contraction, as is usually the case with logics like $\mathcal{EL}$, which are not closed under negation. Thus, just as with contraction in belief bases, in concept contraction Rienstra et al. [29] adapted the fullness and relevance postulates to ensure the principle of minimal change. In the concept contraction similar to partial meet contraction, the relevance postulate means that if X is removed when D is removed from C, then there exists an intermediate concept Y between C and $C \ominus D$ such that Y is not subsumed by D, but is subsumed by D when X is to Y [29].

($\mathcal{T}$-relevance) if $C \sqsubseteq^{\mathcal{T}} X$ and $C \ominus D \not\sqsubseteq^{\mathcal{T}} X$ then there is a Y such that $C \sqsubset^{\mathcal{T}} Y \sqsubseteq^{\mathcal{T}} C \ominus D$ and $Y \not\sqsubseteq^{\mathcal{T}} D$ and $Y \sqcap X \sqsubseteq^{\mathcal{T}} D$.

For a maxichoice-like concept contraction, the fullness postulate guarantees that if X is lost after contracting D from C, adding X to the result yields D again:

($\mathcal{T}$-fullness) if $C \sqsubseteq^{\mathcal{T}} X$ and $(C \ominus D) \not\sqsubseteq^{\mathcal{T}} X$ then $(C \ominus D) \sqcap X \sqsubseteq^{\mathcal{T}} D$.

Baader and Wassermann [16] propose a generalization of partial meet contraction that combines the advantages of syntax independence found in belief sets with the finite representation of belief bases. In this work, the authors defined the sum and product operations based on the union and intersection of belief sets, but which operate on general knowledge bases. The proposed generalization, named *Partial Product Contractions* (PPC) [6], replaces the partial meet contraction intersection with the product operation and the idea of maximal subsets not implying a formula with optimal repairs [30]. For minimal change, the PPC operation *ctr* satisfies the following version of the relevance postulate, where $Con(K)$ is the set of consequences of K and $Rep(K, \varphi)$ are repairs of K for φ [16].

(PPC-relevance) if $K' \in Con(K)$ and $K' \notin Con(ctr(K, \varphi))$, then there is K'' such that $K \models_s K'' \models ctr(K, \varphi)$, $K'' \in Rep(K, \varphi)$, and $K'' \oplus K' \notin Rep(K, \varphi)$.

Maxichoice PPC operations satisfy the strong version of relevance:

(PPC-fullness) if $K' \in Con(K)$ and $K' \in Con(ctr(K, \varphi))$, then $ctr(K, \varphi) \oplus K' \notin Rep(K, \varphi)$.

An extreme case in which the principle of minimal change also aims to avoid unnecessary loss of information is when the belief to be contracted is not part of the belief state. In this scenario, the minimal change is to leave the original belief set unchanged [1, 7]. The vacuity postulate captures the intuition behind this extreme case for belief set and belief base (denoted here by B-Vacuity):

(vacuity) if $\varphi \notin K$, then $K - \varphi = K$.

(B-vacuity) if $\varphi \notin Cn(B)$, then $B - \varphi = B$.

Delgrande and Wassermann [24] adapt the vacuity postulate to Horn belief sets for both maxichoice and partial meet contraction:

(vacuity^h) if $\varphi \notin H$, then $H \dot{-}_w \varphi = H$.

Santos et al. [26] proposed a weakened version of the vacuity postulate that included the new logical closure operator proposed in their work:

(vacuity*) if $\varphi \notin Cn(B)$, then $B - \varphi = Cn^*(B)$.

In a similar way, Zhuang and Pagnucco [27] adapted the AGM postulate vacuity for the contraction operator in $\mathcal{EL}$:

(TBox vacuity) if $\varphi \notin T$, then $T - \varphi = T$.

[6]Although [16] illustrate PPC using $\mathcal{EL}$ description logic, the framework is general and applies to any logic.

The entrenchment-based contraction presented in [28] follows the same intuition as vacuity for sets and belief bases. Thus, if the axiom to be contracted is not part of the TBox, the TBox remains unchanged.

($\mathcal{T}$-vacuity) if $\varphi \notin \mathcal{T}$, then $\mathcal{T}-\varphi = \mathcal{T}$.

In Rienstra et al. [29], the vacuity postulate is adapted to the concept contraction in an acyclic TBox, such that if a concept C is not subsumed by D, then there is no need to generalize C:

($\mathcal{T}-$vacuity) if $C \not\sqsubseteq^{\mathcal{T}} D$ then $C \ominus D \equiv^{\mathcal{T}} C$.

Baader and Wassermann [16] also adapt the vacuity postulate for PPC contraction:

(PPC-vacuity) if $K \in Rep(K, \varphi)$, then $ctr(K, \varphi) \equiv K$.

The postulates surveyed here show that no single postulate captures minimal change in every context, independently of how the epistemic state is represented or how expressive the underlying logic is. For belief sets, recovery has been contested since the early days of AGM [19], as Example 2.1 illustrates. For belief bases it is inapplicable: presupposing a logically closed state, it is satisfied by neither partial meet nor maxichoice contraction [22] and is incompatible with inclusion [22, 21]. The relevance postulate [22] emerged as its primary alternative for belief bases, restricting contraction to what is necessary and characterizing partial meet contraction; since the latter may intersect several remainders, it permits moderate information loss.

Core-retainment [22] relaxes relevance by not requiring the witnessing set to contain the contraction result, making it suitable for kernel contraction [2]. Fullness [2], a stronger version of recovery requiring every removed element to suffice for re-deriving the contracted sentence, characterizes maxichoice contraction and is thus typical of belief base contraction. Logical relevance and logical recovery [21] weaken their standard counterparts by replacing the inclusion and recovery conditions with logical ones, characterizing partial meet pseudo-contraction.

In non-classical logics, minimal-change postulates are adapted to the underlying logic while retaining the same contraction constructions. In Description Logics, $\mathcal{T}$-relevance and $\mathcal{T}$-fullness [29] redefine minimal change via subsumption; PPC-relevance and PPC-fullness [16] generalize the witness conditions through syntactic independence while preserving finite AGM counterparts; TBox recovery [27] uses logical difference to avoid the original postulate's syntactic commitments; and the critical retention postulate [28] trades characterization completeness for a logic-independent sufficient condition. In Horn logic, recovery fails because Horn clauses cannot express the required disjunctive information: weak relevance [24] adapts the witness condition to Horn consequence, maximalityh^h [24] characterizes maxichoice contraction, and [26] weaken the closure operator to admit weaker forms of relevance and inclusion in belief bases.

Vacuity, by contrast, requires minimal reformulation across representations and logic's, making it the most stable of the minimal-change postulates. Though the simplest, it fixes the boundary case in which the contracted sentence is not entailed, where minimal change collapses to no change, the only substantive departure being vacuity* [26], which under pseudo-contraction forces weak closure even in this case.

3. Syntax Independence

In the AGM setting, a belief change operation should not depend on the syntactic form of the belief state, but only on its semantic content and the input [2, 1]. The equivalence postulate, also known as *extensionality*, captures syntax independence, stating that contracting by logically equivalent beliefs makes the same changes to a belief set [1].

(equivalence) if $\vdash \varphi \leftrightarrow \psi$, then $K-\varphi = K-\psi$.

The representation of an agent's beliefs as belief sets occurs at the level of knowledge, such that this type of representation is syntactically independent. However, using belief bases brings in the possibility of syntax dependence. The extensionality postulate has also been used for for belief bases contraction [2].

(extensionality) if $\vdash \varphi \leftrightarrow \psi$, then $B{-}\varphi = B{-}\psi$.

A stronger version of extensionality is needed to characterize partial meet base contraction. In *uniformity*, if two sentences φ and ψ are implied by exactly the same subsets of B, then the result of contracting B by φ and ψ must be identical:

(uniformity) if for all subsets B' of B that $\varphi \in Cn(B')$ iff $\psi \in Cn(B')$, then $B{-}\varphi = B{-}\psi$.

Although these postulates aim to ensure syntactic independence for belief bases, they are not sufficient to guarantee that contraction yields a syntactically independent result. While two different belief bases may generate the same belief set, they may behave differently under transformation operations [2]. Consequently, some researchers ([26, 16]) have proposed new postulates, or variations on existing ones, with the aim of achieving some degree of syntactic independence for belief bases.

Santos et al. [26] aimed to clarify specific syntactic distinctions and examined how these could be addressed by employing a weakened version of the uniformity postulate in Cn^*-pseudo-contraction:

(uniformity*) if for all $B' \subseteq Cn^*(B), \varphi \in Cn(B')$ iff $\psi \in Cn(B')$, then $B{-}\varphi = B{-}\psi$.

In addition, the authors propose the postulate *enforced closure**, which fixes the degree of syntactic independence of the operation (between the syntax dependence of base contraction and the syntax independence of belief set contraction) at the level imposed by Cn^*. The postulate forces the resulting belief base to be weakly closed even if the input was not implied by the initial base, going against the intuition of vacuity [26].

(enforced closure*) $B{-}\varphi = Cn^*(B{-}\varphi)$.

The postulates used for belief sets and belief bases in Horn logic by Delgrande and Wassermann [24] are basically the same as for the classical case, both for maxichoice and partial meet contraction. The adaptations for sets and belief bases, respectively, are defined below:

(extensionality^h) if $\varphi \leftrightarrow \psi$, then $H \dot{-}_w \varphi = H \dot{-}_w \psi$.

(uniformity^h) if for all subsets B' of B $\varphi \in Cn^h(B')$ iff $\beta \in Cn^h(B')$, then $B \dot{-}_e \varphi = B \dot{-}_e \beta$.

For the case of Description Logics, the postulates for syntactic irrelevance are essentially the same as extensionality. Zhuang and Pagnucco [27] define for contraction in $\mathcal{EL}$:

(TBox equivalence) if $Cn(\varphi) = Cn(\psi)$, then $T{-}\varphi = T{-}\psi$.

Zhuang et al. [28] define for TBox contraction in $\mathcal{DL}$-Lite$_\mathcal{R}$:

($\mathcal{T}$-equivalence) if $\varphi \equiv \psi$, then $\mathcal{T}{-}\varphi = \mathcal{T}{-}\psi$.

For concept contraction, Rienstra et al. [29], propose that if two concepts are semantically equivalent with respect to the TBox, the contraction of concept C by either of them must produce equivalent results:

($\mathcal{T}$-preservation) if $D \equiv^\mathcal{T} D'$ then $C \ominus D \equiv^\mathcal{T} C \ominus D'$.

For PPC contraction, Baader and Wassermann [16] propose that if two repair requests φ and φ' are equivalent with respect to the knowledge base, they induce the same set of repairs:

(preservation) if $\varphi \equiv_\mathcal{K} \varphi'$, then $ctr(\mathcal{K}, \varphi) \equiv ctr(\mathcal{K}, \varphi')$.

The postulates discussed in this section reflect one of the key differences between belief sets and belief bases: the ability to perform belief change operations independently of syntax. For belief sets, syntactic independence is inherent in the representation; equivalence postulates capture this directly. Belief bases, on the other hand, are not independent of syntax, since two bases with the same closure may differ syntactically and contract differently [2]. Uniformity strengthens extensionality and characterizes partial meet, yet constrains only the input's syntactic form, not the base's structure [2]. Santos et al. [26] parametrize this residual dependence rather than removing it: uniformity* and enforced closure* fix the operator's degree of independence at the level set by Cn^*, placing Cn^*-pseudo-contraction base contraction's syntax dependence and set contraction's syntax independence.

4. Closure Postulates

When performing contraction on belief sets, we want the result to be a belief set, that is, a set closed under logical consequence. This condition is covered by the closure postulate.

(closure) $K - \varphi = Cn(K - \varphi)$

For Horn belief sets, Delgrande and Wassermann [24] use the Horn closure operator for maxichoice and partial meet contraction:

(closure^h) $H \dot{-}_w \varphi = Cn^h(H \dot{-}_w \varphi)$

In belief base contraction, we also want the contraction of that belief base by a sentence to result in a new belief base. However, we may require that, if the original belief base is logically closed, the new belief base be logically closed as well [2]:

(B-closure) if B is logically closed, then for all φ, $B - \varphi$ is logically closed.

An interesting property is to require that the consequences of the contraction of B by a formula that were already in B are retained:

(relative closure) $B \cap Cn(B - \varphi) \subseteq B - \varphi$.

Zhuang and Pagnucco [27] works with logically closed TBoxes, and hence, require closure of the contraction:

(TBox closure) $T - \varphi = Cn(T - \varphi)$.

Zhuang et al. [28] only substitute the classical logical consequence operator for the finite closure *cl* of the TBox:

($\mathcal{T}$-closure) $\mathcal{T} - \varphi = cl(\mathcal{T} - \varphi)$.

The postulates in this section share a single goal, the principle of *categorical matching* [31]: contraction must preserve the kind of object it operates on. A belief set is closed by definition, and $K = Cn(K)$ expresses the equilibrium of the epistemic state [1]. Thus, closure, closure^h, TBox closure, and $\mathcal{T}$-closure are one requirement re-instantiated under each logic's consequence operator, indifferent to the underlying construction. Belief bases need not be closed, so closure becomes graded: B-closure imposes it only on already-closed inputs, and relative closure retains only original beliefs still entailed by the result. This closed/non-closed distinction matters because recovery presupposes closure and fails for maxichoice and partial meet over non-closed sets [2, 19].

5. Inclusion Postulates

The role of the inclusion postulate is to ensure that no new information is added to the belief state during contraction [1]. This means that the contracted set must be a subset of the original set of beliefs:

(inclusion) $K - \varphi \subseteq K$.

The inclusion postulate for belief bases looks exactly the same:

(B-inclusion) $B - \varphi \subseteq B$.

However, B-inclusion is a very strong assumption. When representing the state of belief as belief bases, we would like to retain parts of certain beliefs that will be removed, to replace them with their consequences [26]. Thus, Hansson [23] proposed a weakened version of the inclusion postulate, called logical inclusion, which allows the addition of some kinds of formulas. Logical inclusion is considered to be far more adequate than the inclusion postulate for belief bases [26].

(logical inclusion) $Cn(B - \varphi) \in Cn(B)$.

Santos et al. [26] adapted the inclusion postulate for belief bases to use the weak logical consequence operator Cn^*:

(inclusion*) $B - \varphi \subseteq Cn^*(B)$.

Baader and Wassermann [16] also use a variation of logical inclusion as a postulate for PPC contraction:

(PPC-inclusion) $ctr(\mathcal{K}, \varphi) \in Con(\mathcal{K})$.

Delgrande and Wassermann [24] use the traditional inclusion postulate for maxichoice and partial meet contraction in both Horn belief sets and Horn belief bases. In the case of Horn belief set H denotes a set closed under Horn consequence.

(inclusion^h) $H \dot{-}_w \varphi \subseteq H$.

(B-inclusion^h) if $B \dot{-}_e \varphi \subseteq B$.

The same happens in [27] and [28], where the only change is in the underlying logic:

(TBox inclusion) $T - \varphi \subseteq T$.

($\mathcal{T}$-inclusion) $\mathcal{T} - \varphi \subseteq \mathcal{T}$.

In Rienstra et al. [29], the inclusion postulate ensures that the result of a concept contraction constitutes a generalization of the original concept under the semantics imposed by the background TBox:

($\mathcal{T}-$inclusion) $C \sqsubseteq^{\mathcal{T}} C \ominus D$.

These inclusion postulates share the same intuition: no new information should be added after a contraction. Syntactic variants (inclusion, B-inclusion, and their adaptations for Horn logic and description logics) require the result to be a subset of the original, which suits belief sets but is too restrictive for bases. Semantic variants (logical inclusion, inclusion* and PPC-inclusion) relax this to the consequence level, while $\mathcal{T}-$inclusion reads containment via subsumption. The choice among them depends on the representation formalism and on whether one prioritises syntactic or epistemic preservation.

6. Success Postulate

The success and inclusion postulates together form the minimum requirements for a contraction operation [2]. Success ensures that, unless the belief to be contracted is a tautology, the contraction operation will always eliminate the unwanted belief [1].

(success) if $\nvdash \varphi$, then $\varphi \notin K - \varphi$.

To capture the idea of success for belief bases, one needs to look at the logical closure of contraction:

(B-success) if $\nvdash \varphi$, then $\varphi \notin \mathrm{Cn}(B - \varphi)$.

Delgrande and Wassermann [24] adapt the two versions of the success postulate to Horn belief sets and Horn belief bases, respectively. In both representations, the adaptation applies uniformly to maxichoice and partial meet contraction, with no further modification needed depending on the operation.

(success^h) if $\nvdash \varphi$, then $\varphi \notin H \dot{-}_w \varphi$.

(success^h) if $\nvdash \varphi$, then $\varphi \notin Cn^h(B \dot{-}_e \varphi)$.

For $\mathcal{EL}$ contraction [27] and TBox contraction in $\mathcal{DL}$-Lite$_\mathcal{R}$ [28], the proposed success postulates look exactly the same:

(TBox success) if $\nvdash \varphi$, then $\varphi \notin T - \varphi$.

($\mathcal{T}$-success) if $\nvdash \varphi$, then $\varphi \notin \mathcal{T} - \varphi$.

The $\mathcal{T}-$success postulate presented in Rienstra et al. [29] ensures that if D is not equivalent to the universal concept $\top$ with respect to the TBox, then the result of contracting C by D is not subsumed by D:

($\mathcal{T}-$success) if $D \not\equiv^{\mathcal{T}} \top$ then $C \ominus D \not\sqsubseteq^{\mathcal{T}} D$.

The intuition behind the success postulate presented by Baader and Wassermann [16] is that, given a repair request φ, if there is at least one possible repair for the knowledge base $\mathcal{K}$, then the result of the contraction operation must be a valid repair:

(PPC success) $ctr(\mathcal{T}, \varphi) \in Rep(\mathcal{K}, \varphi)$ if $Rep(\mathcal{K}, \varphi) \neq \emptyset$.

In [26], we find a weakened version of the success postulate, known as success*. The difference lies in the fact that contractions that satisfy only success* may allow the sentence to be contracted to be logically inferred from the set of sentences obtained after the contraction. Santos et al. [26] emphasize that in several applications, contraction should satisfy success in belief bases, rather than its weakened version success*.

(success*) If $\nvdash \varphi$, then $\varphi \notin Cn^*(B - \varphi)$.

Across the different logics (propositional, Horn, and DLs) and representations (belief sets and bases), the success postulate expresses the same intuition: unless the contracted formula is a tautology, it no longer belongs to the resulting set or base. Santos et al. [26] draw an interesting distinction, however: under the weakened success* for belief bases, a contraction may still allow the contracted sentence to be inferred from its result.

Among the DLs adaptations, only [29] and [16] depart from the AGM success criterion. Rienstra et al. [29] formulate success through concept subsumption, grounding it in the semantics of $\mathcal{EL}$, where subsumption plays the role implication has in AGM. Baader and Wassermann [16], in turn, define PPC-success so that, whenever an optimal repair exists, the contraction yields a valid repair, grounding success in the repair structure and making it directly applicable to ontology evolution.

Success attracts fewer criticisms than recovery, but in both contraction and revision it is faulted for prioritizing new information over the agent's prior beliefs, risking the loss of more valuable information [20].

7. Postulates for Conjunction-Based Contraction

In AGM theory, we have the *supplementary postulates* for contractions by conjunctions, named *conjunctive overlap* and *conjunctive inclusion*. In conjunctive overlap, beliefs that are both in the contraction of K by φ and in the contraction of K by ψ will also be in the contraction of belief set by $\varphi \wedge \psi$ [1, 7].

(conjunctive overlap) $(K - \varphi) \cap (K - \psi) \subseteq K - (\varphi \wedge \psi)$.

In conjunctive inclusion, if φ is not in the contraction by the conjunction, then beliefs excluded when removing φ will also be removed when excluding $\varphi \wedge \psi$:

(conjunctive inclusion) if $\varphi \notin K - (\varphi \wedge \psi)$, then $K - (\varphi \wedge \psi) \subseteq K - \varphi$.

The conjunctive overlap and conjunctive inclusion postulates, in presence of the AGM basic postulates for contraction, are equivalent to *conjunctive factoring* [7].

(conjunctive factoring) $K{-}(\varphi \wedge \psi) = \begin{cases} K{-}\varphi, \text{ or,} \\ K{-}\psi, \text{ or,} \\ K{-}\varphi \cap K{-}\psi. \end{cases}$

In [32], Baader and Wassermann extend the work proposed in [16] and investigate under what conditions the partial product contraction satisfies the supplementary AGM postulates. To demonstrate that the supplementary postulates apply to PPC, a new operator $\boxplus$ is needed so that $Rep(\mathcal{K}, \varphi \boxplus \psi) = Rep(\mathcal{K}, \varphi) \cup Rep(\mathcal{K}, \psi)$ The supplementary postulates of conjunctive overlap and conjunctive inclusion in partial product contraction can be written as:

(PPC conjunctive overlap) $ctr(\mathcal{K}, \varphi \boxplus \psi) \models ctr(\mathcal{K}, \varphi) \otimes ctr(\mathcal{K}, \psi)$.

(PPC conjunctive inclusion) if $ctr(\mathcal{K}, \varphi \boxplus \psi) \in Rep(\mathcal{K}, \varphi)$, then $ctr(\mathcal{K}, \varphi) \models ctr(\mathcal{K}, \varphi \boxplus \psi)$.

Postulates based on conjunctions govern how the contraction of a set of beliefs by a conjunction relates to the contractions by the individual conjuncts [1]. Notably, the basic AGM postulates already restrict contraction operations, and many contraction functions, such as partial meet contraction, may fail to satisfy conjunctive overlap or conjunctive inclusion. Satisfying the conjunction-based postulates amounts to assuming a stronger preference structure on the agent's part, and their adoption must be justified by the domain of application, as they impose substantive restrictions that go beyond minimal rationality. When these postulates are adapted to non-classical scenarios, such as PPC, satisfying them requires explicit structural conditions within the repair method, which may or may not be available depending on the representation in question [32].

8. Additional Literature Postulates

In this section, we will discuss postulates for sets and belief bases that have not been grouped into categories like the others, but which exhibit behavior and characteristics worth analyzing.

The φ-core of a belief base B is a set of sentences in B that do not contribute at all to B implying φ. *Core identity* reduces the belief base by removing all sentences from the base except for its φ-core [2]:

(core identity) $\psi \in B{-}\varphi$ iff $\psi \in B$ and for no $B' \subseteq B$, $\varphi \notin Cn(B')$ and $\varphi \notin Cn(B' \cup \{\psi\})$.

In the context of belief sets, there is a simpler version of core identity, also known as *intersection* [3]:

(meet identity) if $\varphi, \psi \in K$, then $(K{-}\varphi) \cap (K{-}\psi) = K{-}(\varphi \wedge \psi)$.

The next postulate is finitude, a postulate for base-generated contraction. In the finitude postulate we have that for every sentence φ, there is some finite set B such that $\mathbf{K} - \varphi = Cn(B)$. Furthermore, there may be infinitely many sentences by which the belief set can be contracted, and there exists only a finite number of belief sets that can be obtained through contraction [2].

(finitude) there is a finite set B such that for every φ, $K{-}\varphi = Cn(B')$ for some $B' \subseteq B$.

The following postulate captures the intuition that if two beliefs stand or fall together, it means they share the same justifications, and that it is not possible to remove the justification for one without removing the justification for the other, and the result of the contraction operation is the same [2].

(symmetry) if for all β, $\varphi \in Cn(K{-}\beta)$ iff $\psi \in Cn(K{-}\beta)$, then $K{-}\varphi = K{-}\psi$.

The next postulate addresses cases in which an agent is unable to choose between different ways of contracting a belief [2].

(indecisiveness) if there is δ such that $K{-}\delta \nvdash \varphi$ and $(K{-}\psi) \cup (K{-}\delta) \vdash \varphi$, then $K{-}\psi \nsubseteq K{-}\varphi$.

The core-addition postulate proposed by Ribeiro and Wassermann [21] ensures that if a new sentence is added when performing a contraction in a belief base, it is necessary for some belief that was previously given up to be recovered. This postulate prevents the unnecessary addition of new beliefs, and is a counterpart of core-retainment, that prevents the unnecessary loss of beliefs.

(core-addition) if $\psi \in (B - \varphi) \setminus B$, then there exists $\psi' \in B \setminus (B - \varphi)$ and $B' \subseteq B - \varphi$ such that $\varphi \rightarrow \psi' \notin Cn(B')$ but $\varphi \rightarrow \psi' \in Cn(B' \cup \{\psi\})$.

The core identity, meet identity and indecisiveness postulates all help characterize *full meet contraction*, i.e., a contraction in which no choice is made and all the maximal subsets not implying the input are intersected.

The variety of postulates presented in this section demonstrates the diversity of rationality constraints that have been explored in the literature on belief contraction. Postulates such as core identity and core-addition act as regulators between the syntactic structure of the base and the result of the contraction. Similarly, symmetry and indecisiveness impose conditions on how the operation must respond to epistemic equivalences or the absence of a principled criterion for choosing between different contraction strategies. Finiteness, in turn, restricts the space of possible outcomes.

Different combinations of these postulates give rise to operators with distinct behaviors, and while some of these properties are mutually compatible, others impose conflicting requirements, highlighting that the choice among them depends on the intended application and the underlying logic.

9. Ontology Repair - a Belief Contraction View

After looking at the different contraction postulates in the literature, let us get back to the problem of repairing an ontology formalized in Description Logics.

Baader et al. [33] define a classical repair [7] of an ontology $\mathcal{O}$ with respect to a sentence α to be a maximal subset of $\mathcal{O}$ not entailing α. In terms of belief contraction, this amounts to a maxichoice base contraction. The characterization of base contraction given by Hansson [4] only requires the underlying logic to be compact and monotonic [34], which means that the same postulates can be used for Description Logics. However, Baader et al. defend that this approach can be too brittle, in the sense that it removes too much information. Ideally, one should use repairs that are not necessarily subsets of the original ontology, but are contained in the consequences of the ontology. Basically, the authors argue that, instead of the inclusion postulate, the operation satisfies logical inclusion (as discussed in Section 5).

Ribeiro et al. [8] adapted AGM contraction for sets of DL formulas closed under logical consequences. In practice, for most DLs it is not possible to work computationally with closed sets. This was one of the motivations of the pseudo-contractions proposed in [26] and [17]. Instead of working with full logical closure, the Cn^* operator returns only some ("the easiest") consequences.

An optimal repair [33] of an ontology $\mathcal{O}$ with respect to a sentence α is an ontology $\mathcal{O}'$ such that $Cn(\mathcal{O}') \subseteq Cn(\mathcal{O} \setminus \{\alpha\})$ and there is no repair $\mathcal{O}'' \subset \mathcal{O}$. An optimal repair does not always exist, but when it does, it can be used to produce a useful version of pseudo-contraction [15, 16].

10. Conclusion and Future Work

Although the AGM rationality postulates form an elegant logical theory together with the constructions for contraction and revision, they are not always adequate for practical applications [20], which has motivated several variations in the literature [2, 20]. We survey contraction postulates for belief sets and belief bases under different underlying logics, organizing them into categories that reflect the rationality criteria they aim to capture. Within each category, the AGM postulate captures a clear intuition, but its formulation rests on assumptions such as logical closure, the availability of disjunction, and classical negation, which fail once the representation shifts to belief bases or the logic moves from propositional to Horn [24] or Description Logics [29, 16, 27, 28]; the postulates must then be adapted to each setting.

Minimal change is the clearest example: recovery is subject to well-known counterintuitive consequences [20, 8], cannot be satisfied together with inclusion in belief bases [22, 21], and fails for concept contraction in Description Logics because existential restrictions do not distribute over intersections

[7] In the paper they call this an *optimal* classical repair.

[29]. The remaining postulates exhibit the same pattern: syntax independence requires uniformity [2] for belief bases and domain-specific notions of equivalence for Description Logics [29, 16, 28]; closure must be weakened in belief bases to avoid conflicts with inclusion [2]; inclusion ranges from strict set containment to logical [23] and consequence-based formulations [16, 26]; success admits a weakened variant (success*) that balances effective contraction against informational value [26]; and the supplementary postulates require additional structural conditions in the generalized setting of partial product contractions [32].

This investigation makes evident that no single postulate captures minimal change across all contraction contexts: the choice of postulate is linked to (i) the use of belief sets or belief bases, (ii) the expressiveness of the underlying logic, and (iii) the scope and type of problem. Moreover, extending belief change theory to Description Logics not only requires new formulations but also reveals limitations of the original postulates that were not apparent in the propositional setting, while the interaction with ontology repair gives rise to repair-based constructions that satisfy postulates classical subset-based contractions cannot, fostering the mutual development of both areas.

As future work, we intend to translate the postulates into a common logical framework, identifying when and under what conditions a postulate cannot be transferred to a given logic; to provide a mapping between postulates and their application contexts, such as ontology repair; to incorporate the computational complexity of computing contractions and of checking postulate satisfaction; and, in the context of pseudo-contraction, to investigate replacing the consequence operator Cn^* with Horn closure, examining which postulates are preserved under this restriction.

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Declaration on Generative AI

During the preparation of this work, the author(s) used Claude Opus 4.6 and Grammarly in order to: Grammar and spelling check. After using these tool(s)/service(s), the author(s) reviewed and edited the content as needed and take(s) full responsibility for the publication's content.

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