Ranking-based Semantics for Incomplete Argumentation Frameworks

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Abstract

In this paper, we introduce ranking-based semantics for incomplete argumentation frameworks (IAFs). These frameworks are extensions of Dung's abstract argumentation framework, with qualitative uncertainty about the existence of arguments and attacks. Up to this point, reasoning on incomplete argumentation frameworks has only followed Dung's extension-based approach, where a set of arguments is either jointly accepted or not. We discuss several approaches on how to rank the individual arguments. The first two approaches rely on the acceptance status of the arguments with respect to an extension-based semantics. Another family of approaches utilises ranking-based semantics on the individual completions of an IAF (i.e. AFs where the uncertainty is "resolved") and then aggregates the resulting rankings with *voting rules*. In addition, we propose a number of principles the ranking-based semantics for IAFs should follow, some adapted from existing principles out of the formal argumentation literature, others novel principles tailored to IAFs. We determine which of these properties are satisfied by our approaches.

Keywords

Abstract Argumentation, Ranking, Uncertainty

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1. Introduction

Dung's abstract argumentation frameworks (AFs) [1] are a simple approach to model rational decision-making based on representations of arguments and their relations. Here, argumentative scenarios are represented as directed graphs, where *arguments* are identified by vertices, and an *attack* from one argument to another is represented as a directed edge.

AFs have been widely studied and extended over the years. One of these extensions is the introduction of qualitative uncertainty regarding the existence of arguments and attacks, inducing the so-called *incomplete argumentation frameworks* [2, 3, 4]. In this framework the sets of arguments and attacks are split into *certain* and *uncertain* elements. When reasoning with an IAF, certain arguments or attacks appear in every possible scenario, while uncertain arguments and attacks sometimes appear in the framework and sometimes they do not. Such doubts about the actual existence of arguments and attacks can be explained, e.g., by uncertainty in the knowledge the IAF is constructed on. All the reasoning methods for IAFs follow Dung's original approach and use *extensions* to evaluate the (collective) acceptability of a set of arguments. There are other frameworks handling uncertainty, like *weighted argumentation frameworks* [5] or various *probabilistic argumentation frameworks* [6]. But, these approaches all tackle uncertainty as quantitative, thus they need additional numerical values as input.

The binary classification of extension-based reasoning is overly restrictive [7]. Thus, another topic of interest in the argumentation community has been to study the individual evaluation of argument's acceptability. For that purpose, *ranking-based semantics* (and the related *gradual semantics*) [7, 8, 9, 10, 11]

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have been proposed, some focusing on the topological structure of the argumentation frameworks [7], others utilising rankings over sets of arguments to rank the individual arguments [11]. Until now, this kind of semantics has been ignored in the context of IAFs. Studying the acceptability of the individual arguments is even more important in the case of uncertainty than for AFs. As soon as arguments and attacks are uncertain, their acceptability has to be questioned. Thus, not only the topological structure of the argumentation scenario is significant in the reasoning process, but also the availability of each argument and each attack plays a role. Hence, a simple binary classification of arguments as either *acceptable* or *not acceptable* is not enough, and more fine-grained approaches are necessary.

We establish a formal setting for ranking arguments in casees of qualitative uncertainty. We introduce ranking-based semantics (and the related gradual semantics) for IAFs. We follow a principle-based approach for determining which properties a ranking-based semantics for IAFs should follow, by adapting existing properties from the literature [7, 8, 9], but also introducing novel properties. We propose several approaches for defining ranking-based semantics for IAFs. These approaches rely on the set of completions of an IAF, i.e., AFs that “solve” the uncertainty in the IAF, where first we apply either an extension-based or a ranking-based semantics on the individual completions, and then combine the results into an overall ranking. We determine the principles satisfied by the proposed semantics.

The structure of the remaining paper is as follows: In Section 2, we recall the necessary background information on abstract argumentation frameworks, ranking-based semantics, and incomplete argumentation frameworks. We formally introduce *ranking-based semantics for IAFs* in Section 3, as well as the corresponding principles. Section 4 contains two approaches for defining ranking-based semantics for IAFs, based on the acceptance statuses of the arguments with respect to an extension-based semantics. An approach utilising ranking-based semantics is shown in Section 5, where we apply ranking-based semantics on the individual completions of an IAF, and then aggregate the resulting vector of preorders with voting rules. Section 6 concludes the paper by discussing related work and future work avenues. The proofs for the propositions can be found in the supplementary material.

2. Preliminaries

Abstract Argumentation Frameworks An abstract argumentation framework is a directed graph $F = (A, R)$ where A is a (finite) set of *arguments* and $R \subseteq A \times A$ is an *attack relation* among them [1]. An argument a is said to *attack* an argument b if $(a, b) \in R$. We say that an argument a is *defended by a set* $E \subseteq A$ if every argument $b \in A$ that attacks a is attacked by some $c \in E$. For $a \in A$, we define $a^-_F = \{b \mid (b, a) \in R\}$ and $a^+_F = \{b \mid (a, b) \in R\}$ as the set of arguments attacking a and the set of arguments that are attacked by a in $F = (A, R)$. For a set of arguments $E \subseteq A$, we extend these definitions to E^-_F and E^+_F via $E^-_F = \bigcup_{a \in E} a^-_F$ and $E^+_F = \bigcup_{a \in E} a^+_F$, respectively. If the AF is clear in the context, we will omit the index. For two AFs $F = (A, R)$ and $F' = (A', R')$ we denote with $F \oplus F' = (A \cup A', R \cup R')$ the union between F and F'. Most semantics for abstract argumentation rely on two basic concepts: *conflict-freeness* and *admissibility*.

Definition 1. *Given $F = (A, R)$, a set $E \subseteq A$ is:* conflict-free *iff $\forall a, b \in E$, $(a, b) \notin R$;* admissible *iff it is conflict-free, and every element of E is defended by E.*

For an AF F, we use $cf(F)$ and $ad(F)$ to denote the sets of conflict-free and admissible sets, respectively. We define the remaining semantics (denoted as *extension semantics*) as proposed by [1].

Definition 2. *For an AF $F = (A, R)$ and a set of arguments $E \subseteq A$. An admissible set $E \subseteq A$ is: a* complete *extension (co) iff it contains every defended argument; a* preferred *extension (pr) iff it is a $\subseteq$-maximal complete extension; the unique* grounded *extension (gr) iff it is the $\subseteq$-minimal complete extension. With $\sigma(F)$ for $\sigma \in \{co, pr, gr\}$, we denote the set of σ-extensions.*

We call an argument a, that is part of at least one σ-extension of AF F, *credulous accepted* with respect to extension semantics σ in F ($a \in \mathsf{cred}_\sigma(F)$). An argument a is *skeptical accepted* with respect to extensions semantics σ in AF F, if a is part of every σ extension of F ($a \in \mathsf{skep}_\sigma(F)$).

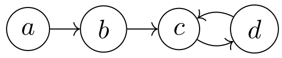

Figure 1: AF F from Example 1.

Example 1. *Consider the AF $F = (A, R)$ with $A = \{a, b, c, d\}$ and $R = \{(a, b), (b, c), (c, d), (d, c)\}$ depicted as a direct graph in Figure 1. The admissible sets and complete extensions are $\{a\}$, $\{a, c\}$, and $\{a, d\}$, thus the preferred extensions are $\{a, c\}$ and $\{a, d\}$, while $\{a\}$ is the grounded extension. Therefore are a, c and d credulously accepted with respect to admissible, complete and preferred semantics and a is also skeptically accepted with respect to these semantics.*

Ranking-based and Gradual Semantics Instead of reasoning based on the acceptance of sets of arguments, *gradual semantics* and *ranking-based semantics* [7] rank the individual arguments based on their strength. See [12] for a overview of this topic.

Definition 3. *A* ranking-based semantics ρ *is a function which maps an AF $F = (A, R)$ to a preorder*[1] $\succeq_F^\rho$ *on A.*

Intuitively $a \succeq_F^\rho b$ means that a is at least as strong as b in F. We define the usual abbreviations as follows; $a \succ_F^\rho b$ denotes *strictly stronger*, i.e. $a \succeq_F^\rho b$ and $b \not\succeq_F^\rho a$. $a =_F^\rho b$ denotes *equally strong*, i.e. $a \succeq_F^\rho b$ and $b \succeq_F^\rho a$. $a \bowtie_F^\rho b$ denotes *incomparability* so neither $a \succeq_F^\rho b$ nor $b \succeq_F^\rho a$.

A subfamily of ranking-based semantics are *gradual semantics* [13], where each argument is assigned a numerical strength value called *acceptability degree*, that can be used to induce an argument ranking. We consider every gradual semantics to be *positive* and *increasing*, this means that every strength value is non-negative, and the higher the strength value is the better an argument is ranked.

Definition 4. *A* gradual semantics *is a function $\mathcal{S}$ which associates to any AF $F = (A, R)$ a function* $\mathsf{Deg}_F^{\mathcal{S}} : A \to [0, 1]$.

With $\mathsf{Deg}_F^{\mathcal{S}}(a)$ we denote the acceptability degree of $a \in A$. While there are a number of different approaches to define argument-ranking semantics (see [8, 9] for recent surveys), we will use the *h-categoriser ranking-based semantics* by Besnard and Hunter (2001) representative for any ranking-based semantics. The h-categoriser ranking-based semantics considers the strength of the direct attackers of an argument to calculate its acceptability value.

Definition 5. *Let $F = (A, R)$ be an AF and $a \in A$. The* h-categoriser function $\mathsf{cat} : A \to]0, 1]$ *is:*

$$\mathsf{cat}(a) = \begin{cases} 1 & \text{if } a_F^- = \emptyset \\ \frac{1}{1+\sum_{b \in a_F^-} \mathsf{cat}(b)} & \text{otherwise} \end{cases}$$

The h-categoriser ranking-based semantics Cat *for $a, b \in A$ is defined as: $a \succeq_F^{Cat} b$ iff* $\mathsf{cat}(a) \geq \mathsf{cat}(b)$.

Pu et al. (2014) have shown that the h-categoriser ranking-based semantics is well-defined.

Example 2. *Consider AF F from Example 1. Argument a is unattacked, thus $\mathsf{cat}(a) = 1$ and $\mathsf{cat}(b) = 0.5$. Using these two strength values, we can compute $\mathsf{cat}(c) = 0.46$ and $\mathsf{cat}(d) = 0.69$. The induced ranking is then: $a \succ_F^{Cat} d \succ_F^{Cat} b \succ_F^{Cat} c$.*

Ranking-based semantics are evaluated and compared with a principle-based approach. For a longer discussion on principles for ranking-based semantics, we refer to [8]. We present a small list of principles.

Definition 6. *A ranking-based semantics ρ satisfies the respective principle iff for all AFs $F = (A, R)$ and any $a, b \in A$:*

Abstraction (Abs). *The names of the arguments do not affect the ranking. For a pair of AFs $F = (A, R)$ and $F' = (A', R')$ and every isomorphism $\gamma : A \to A'$, we have $a \succeq_F^\rho b$ iff $\gamma(a) \succeq_{F'}^\rho \gamma(b)$.*

[1] A preorder is a (binary) relation that is *reflexive* and *transitive*.

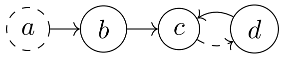

Figure 2: IAF I from Example 3.

Independence (In). *Unconnected arguments should not influence a ranking. For every* $F' = (A', R') \in cc(F)$ *and for all* $a, b \in A'$: $a \succeq^{\rho}_{F} b$ *iff* $a \succeq^{\rho}_{F'} b$*, where* $cc(F)$ *are the* connected components *of* F.

Non-attacked Equivalence (NaE). *Two unattacked arguments should be equally ranked. If* $a^-_F = b^-_F = \emptyset$*, then* $a =^{\rho}_{F} b$.

Void Precedence (VP). *Unattacked arguments should be ranked better than attacked ones. If* $a^-_F = \emptyset$ *and* $b^-_F \neq \emptyset$ *then* $a \succ^{\rho}_{F} b$.

Self-Contradiction (SC). *Self-attacking arguments should be ranked worse than any other argument. If* $(a, a) \notin R$ *and* $(b, b) \in R$ *then* $a \succ^{\rho}_{F} b$.

Cardinality Precedence (CP). *An argument with fewer attackers is ranked better than one with more attackers. If* $|a^-_F| < |b^-_F|$ *then* $a \succ^{\rho}_{F} b$.

Quality Precedence (QP). *If there is an attacker of an argument which is ranked better than every attacker of an other argument, the latter is ranked better than the former. If there is* $c \in b^-_F$ *such that for all* $d \in a^-_F$ *it holds that* $c \succ^{\rho}_{F} d$ *then* $a \succ^{\rho}_{F} b$.

Counter-Transitivity (CT). *If the attackers of an argument are as numerous and as strong as the attackers of another argument, then the former is as least as strong as the latter. If some injective* $f : a^-_F \to b^-_F$ *exists such that* $f(x) \succeq^{\rho}_{F} x$ *for all* $x \in a^-_F$*, then* $a \succeq^{\rho}_{F} b$.

Incomplete Argumentation Frameworks Dung's AFs have been extended in various ways, including some frameworks that take into account uncertainty regarding the existence of arguments and/or attacks. While some approaches equip arguments and attacks with a probability of existence [6, 16, 17], in this work we focus on incomplete argumentation frameworks [18, 2], where uncertainty is qualitative. This means that the existence of some of the arguments and attacks is uncertain, but there is no quantitative measure of this uncertainty. Formally,

Definition 7. *An incomplete argumentation framework is a tuple* $I = (A, A^?, R, R^?)$ *where* A *and* $A^?$ *are disjoint sets of arguments, and* $R, R^? \subseteq (A \cup A^?) \times (A \cup A^?)$ *are disjoint sets of attacks.*

Elements of A and R are *certain* arguments and attacks, while $A^?$ and $R^?$ are *uncertain* arguments.

Example 3. *We extend the AF* F *from Example 1 to an IAF by considering* a *and the attack* (c, d) *as uncertain. The corresponding IAF* $I = (A, A^?, R, R^?)$ *with* $A = \{b, c, d\}$, $A^? = \{a\}$, $R = \{(a, b), (b, c), (d, c)\}$*, and* $R^? = \{(c, d)\}$ *is depicted in Figure 2, where uncertain arguments and attacks are drawn dashed.*

To propose reasoning approaches adapted to this kind of framework, most works in the literature use the notion of *completions*, that are (standard) AFs corresponding to all "possible worlds" encoded in the IAFs. For instance, in a debate, uncertain elements can be used to represent what the agent's opponent may use. Then, completions are all possible evolutions of the debate, depending on what the opponent will actually claim during the next steps of the debate.

Definition 8. *Given IAF* $I = (A, A^?, R, R^?)$*, a* completion *of* I *is an AF* $F_I = (A', R')$*, s.t.* $A \subseteq A' \subseteq A \cup A^?$ *and* $R_{|A'} \subseteq R' \subseteq (R \cup R^?)_{|A'}$*, where* $X_{|A'} = X \cap (A' \times A')$ *for* $X \in \{R, R \cup R^?\}$.

The set of completions of an IAF I is denoted by $\text{comp}(I)$. With $\text{comp}(I, a) = \{F = (A', R') \mid F \in \text{comp}(I) \text{ and } a \in A'\}$ we denote the set of completions for IAF I, where argument a is present with $a \in A \cup A^?$. Naturally, if $a \in A$ then $\text{comp}(I, a) = \text{comp}(I)$ for IAF I.

Example 4. *Consider IAF* I *from Example 3, the corresponding completions* $comp(I) = \{C_1, C_2, C_3, C_4\}$ *are depicted in Figure 3, where in* C_1 *both* a *and* (c, d) *are excluded, in* C_2 *the attack* (c, d) *is excluded, argument* a *is excluded in* C_3*, and finally both* a *and* (c, d) *are present in* C_4.

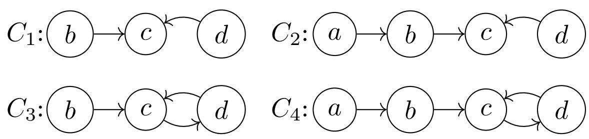

Figure 3: Completions for the I from Example 4.

For the completion-based reasoning approaches for IAFs, standard decision problems (e.g. verifying the credulous acceptability of a given argument) are adapted in two versions: the possible view (is the argument credulously accepted in *at least one* completion?) and the necessary view (is the argument credulously accepted in *all* completions?).

3. Ranking-based Semantics for IAFs

In this section, we define *ranking-based semantics* (and the corresponding *gradual semantics*) for IAFs.

Definition 9. *A* ranking-based semantics for IAFs ψ *is a function which maps an IAF* $I = (A, A^?, R, R^?)$ *to a preorder* $\succeq_I^\psi$ *on* $A \cup A^?$.

A gradual semantics for IAFs *is a function* $\mathcal{G}$ *which associates to any IAF* $I = (A, A^?, R, R^?)$ *a function* $\mathsf{Deg}_I^{\mathcal{G}} : A \cup A^? \to [0, 1]$.

As for AFs, any gradual semantics $\mathcal{G}$ induces a ranking-based semantics s.t. for two arguments $a, b \in A \cup A^?$ in an IAF $I = (A, A^?, R, R^?)$ if $\mathsf{Deg}_I^{\mathcal{G}}(a) \geq \mathsf{Deg}_I^{\mathcal{G}}(b)$ then $a \succeq_I^{\mathcal{G}} b$. For an IAF I, we denote with $a \succeq_I^\psi b$ that argument a is at least as strong as b with respect to ranking-based semantics ψ. We use the usual abbreviations: $a \succ_I^\psi b$ for $a \succeq_I^\psi b$ and $b \not\succeq_I^\psi a$ and $a =_I^\psi b$ for $a \succeq_I^\psi b$ and $b \succeq_I^\psi a$.

The ranking-based semantics for IAF can be understood in terms of the arguments and their acceptability in different possible worlds. For example, when IAFs are used to model an opponent in a debate [4], each completion represents a possible "state of mind" of the opponent. In this context, a "strong" argument is one that is likely to be accepted by the opponent, with respect to these possible states of mind.

Principles Before we discuss principles, we introduce a notion of *attacker sets vectors* for an argument a. These vectors contain the sets of attackers for a grouped by their types of certainty.

Definition 10. *Let* $I = (A, A^?, R, R^?)$ *be an IAF and* $a \in A \cup A^?$*, we define the* attacker sets *of* a *in* I *as follows:*

- $a_I^{c,c} := \{b \in A \mid (b, a) \in R\}$ *is the set of certain arguments attacking* a *via a certain attack.*
- $a_I^{c,u} := \{b \in A \mid (b, a) \in R^?\}$ *is the set of certain arguments attacking* a *via an uncertain attack.*
- $a_I^{u,c} := \{b \in A^? \mid (b, a) \in R\}$ *is the set of uncertain arguments attacking* a *via a certain attack.*
- $a_I^{u,u} := \{b \in A^? \mid (b, a) \in R^?\}$ *is the set of uncertain arguments attacking* a *via an uncertain attack.*

We denote the set of attackers of a *in* I *via:* $a_I^- = a_I^{c,c} \cup a_I^{c,u} \cup a_I^{u,c} \cup a_I^{u,u}$*. We define the* attacker sets vector *(*ASV*) of* a *by:* $a_{\mathsf{vec},I}^- = \langle a_I^{c,c}, a_I^{c,u}, a_I^{u,c}, a_I^{u,u} \rangle$.

We denote the deletion of a set $E \subseteq A \cup A^?$ from the attacker sets vector of an argument $a \in A \cup A^?$ as follows: $a_{\mathsf{vec},I}^- \setminus E = \langle a_I^{c,c} \setminus E, a_I^{c,u} \setminus E, a_I^{u,c} \setminus E, a_I^{u,u} \setminus E \rangle$.

Naturally, we can compare two arguments with respect to the attacker sets vectors. We utilise these vectors to define a *similarity* relation between two arguments.

Definition 11. *Let $I = (A, A^?, R, R^?)$ be an IAF and $E, E' \subseteq A \cup A^?$ and a ranking-based semantics ψ, we say E and E' are* similar *(denoted $E \sim_I^\psi E'$), iff there is a bijection θ from E to E' s.t. for each $a \in E$, $a =_I^\psi \theta(a)$.*

For two arguments $a, b \in A \cup A^?$, we can extend this definition to $a^-_{\mathsf{vec},I} \sim b^-_{\mathsf{vec},I}$ iff $a_I^{c,c} \sim_I^\psi b_I^{c,c}$, $a_I^{c,u} \sim_I^\psi b_I^{c,u}$, $a_I^{u,c} \sim_I^\psi b_I^{u,c}$ and $a_I^{u,u} \sim_I^\psi b_I^{u,u}$.

Two arguments are similar if, for each attacker x of the first argument, there is an attacker y of the second argument, s.t. x and y have the same nature, with respect to (un)certainty, and x and y are equally strong.

Next, we introduce a number of principles, ranging from simple ones relying solely on the graph structure, and others where the certainty of the arguments is important. We adapt some principles defined for ranking-based semantics in the context of AFs or other extensions of AFs (for an overview see [9]), however we extend our list of principles by some tailored to IAFs and especially uncertainty.

Definition 12. *A ranking-based semantics for IAF ψ satisfies the respective principles iff for all IAFs $I = (A, A^?, R, R^?)$ and $a, b \in A \cup A^?$:*

incomplete-based Abstraction (i-Abs). *The names of arguments should not affect the ranking. For a pair for IAFs $I = (A, A^?, R, R^?)$ and $I' = (A', A'^?, R', R'^?)$ and a isomorphism $\gamma : (A \cup A^?) \to (A' \cup A'^?)$, we have $a \succeq_I^\psi b$ iff $\gamma(a) \succeq_{I'}^\psi \gamma(b)$.*

incomplete-based Independence (i-In). *Unconnected arguments should not influence a ranking. For a pair for IAFs $I = (A, A^?, R, R^?)$ and $I' = (A', A'^?, R', R'^?)$ s.t. $(A \cup A^?) \cap (A' \cup A'^?) = \emptyset$, if $a \succeq_I^\psi b$ then $a \succeq_{I \oplus I'}^\psi b$ for $a, b \in A \cup A^?$ with $I \oplus I' = (A \cup A', A^? \cup A'^?, R \cup R', R^? \cup R'^?)$.*

incomplete-based weak Void Precedence for Certain Arguments (i-wVPC). *Unattacked certain arguments should not be ranked worse than any attacked argument. For all $a \in A$ s.t. $a_I^- = \emptyset$ it holds for every $b \in A \cup A^?$ with $b_I^- \neq \emptyset$, $a \succeq_I^\psi b$.*

incomplete-based weak Void Precedence for Uncertain Arguments (i-wVPU). *Unattacked uncertain arguments should not be ranked worse than any attacked uncertain argument. For all $a \in A^?$ s.t. $a_I^- = \emptyset$ it holds for every $b \in A^?$ with $b_I^- \neq \emptyset$, $a \succeq_I^\psi b$.*

incomplete-based Self-Contradiction (i-SC). *Self-attacking arguments should not be ranked better than any other argument. If $(a, a) \notin R$ and $(b, b) \in R$ then $a \succeq_I^\psi b$.*

Certain Argument Precedence (CAP). *For two similar arguments, certain arguments not should not be weaker than uncertain ones. If $a \in A$, $b \in A^?$ and $a^-_{vec,I} \sim_I^\psi b^-_{vec,I}$, then $a \succeq_I^\psi b$.*

Uncertain Attacker (UArg). *Uncertain attackers influence a ranking less than certain ones. If it holds that:*

1. *$a, b \in A$ (reciprocally $a, b \in A^?$),*
2. *$\exists z \in A^?$ and $\exists z' \in A$ s.t. $(z, a) \in R$ and $(z', b) \in R$ (or $(z, a) \in R^?$ and $(z', b) \in R^?$),*
3. *$a^-_{vec,I} \setminus \{z\} \sim_I^\psi b^-_{vec,I} \setminus \{z'\}$,*

then $a \succeq_I^\psi b$.

incomplete-based Attack Equivalence (i-AE). *Two arguments with the same certainty and similar attackers are equally strong. If $a, b \in A$ (reciprocally $a, b \in A^?$) and $a^-_{vec,I} \sim_I^\psi b^-_{vec,I}$, then $a =_I^\psi b$.*

incomplete-based Cardinality Precedence (i-CP). *An argument with fewer attackers is not weaker than one with more attackers. If there is a non-empty set $E \subseteq b_I^-$ s.t. $a^-_{vec,I} \sim_I^\psi b^-_{vec,I} \setminus E$, then $a \succeq_I^\psi b$.*

Reinforcement (RF). *If the attacker sets of two arguments are similar except for two arguments, then the stronger attacker weakens more. For $a, b \in A$ (reciprocally $a, b \in A^?$) if there are $x, y \in A \cup A^?$ s.t. $a^-_{vec,I} \setminus \{x\} \sim_I^\psi b^-_{vec,I} \setminus \{y\}$ with $x \in a_I^{i,j}$ and $y \in b_I^{i,j}$ for $i, j \in \{c, u\}$ and $y \succeq_I^\psi x$, then $a \succeq_I^\psi b$.*

Raising Certainty (RC). *Changing the certainty of an argument from uncertain to certain should not weaken it. For every pair for IAFs $I = (A, A^?, R, R^?)$ and $I' = (A', A'^?, R', R'^?)$ s.t. $a \in A^?$, $A' = A \cup \{a\}$, $A'^? = A^? \setminus \{a\}$, $R = R'$ and $R^? = R'^?$, then for all $b \in A \cup A^?$ with $a \succeq_I^\psi b$ then $a \succeq_{I'}^\psi b$.*

Lowering Certainty (LC). *Changing the certainty of an argument from certain to uncertain should not strengthen it. For every pair for IAFs* $I = (A, A^?, R, R^?)$ *and* $I' = (A', A'^?, R', R'^?)$ *s.t.* $a \in A$, $A' = A \setminus \{a\}$, $A'^? = A^? \cup \{a\}$, $R = R'$ *and* $R^? = R'^?$, *then for all* $b \in A \cup A^?$ *with* $b \succeq^{\psi}_{I} a$ *then* $b \succeq^{\psi}_{I'} a$.

These principles describe how ranking-based semantics should behave in specific scenarios. For instance, the two void precedence principles i-wVPC and i-wVPU state that unattacked arguments should be ranked highly among their peers. Note that satisfaction of these principles is not always desired. Already for AFs, the desirability of ranking-based principles was discussed. [19] questioned Void Precedence by arguing that unattacked arguments have not yet proven their strength against counterattacks. Thus, the principles should be seen as guidelines to find suitable ranking-based semantics for a specific context. In some scenarios, unattacked arguments should be the strongest; in others, they shall not. Nevertheless, the principles are not necessarily independent of each other.

Proposition 1. *The following implications hold:*

1. *i-CP implies i-wVPC.*
2. *i-CP implies i-wVPU.*
3. *UArg implies i-AE.*
4. *RF implies i-AE.*

In Proposition 1, we see that, for the principles which are adapted from ranking-based semantics for AFs such as i-CP, i-wVPC, and i-wVPU, we inherit the implication relationship (for AFs, CP implies VP [7]), however we do not inherit the incompatibilities. For instance, for AFs, the principles SC and CP are incompatible, but the principles i-SC and i-CP are compatible, since these two principles do not enforce a strict relation between arguments. Arguments excluded from a completion cannot be acceptable in that completion, and self-attacking arguments are never acceptable under any extension-based semantics (satisfying conflict-freeness), thus uncertain non self-attacking arguments can be equally strong to certain self-attacking arguments in some semantics (see Section 4).

4. Approaches based on Acceptance Status

In this section, we define families of ranking-based semantics for IAFs, relying on an extension-based semantics and an acceptance mode (credulous or skeptical). Our process assigns each argument a score in $[0, 1]$ (formally, inducing a gradual semantics), and then rank-orders arguments using these scores.

Our intuition is that an argument a that is (credulously or skeptically) accepted in more completions than an argument b should be considered as more acceptable, at the level of the IAF. Our approach is thus parametric, in the sense that a ranking-based semantics can be obtained from an extension-based semantics σ and an acceptance mode $x \in \{\mathrm{cr}, \mathrm{sk}\}$. Moreover, we provide two variants of our approach. In the first one, the score of an argument is based on its acceptability in all completions of the IAF, while in the second approach, we only consider the completions where the argument exists.

Acceptance Degree Based on All the Completions Let us start with a first family of semantics, where all completions are considered for evaluating the acceptability of arguments. This means that an uncertain argument $a \in A^?$ is penalised by the completions where it is absent.

Definition 13. *Given IAF* $I = (A, A^?, R, R^?)$, σ *an extension-based semantics, and* $x \in \{\mathrm{cr}, \mathrm{sk}\}$, *the acceptability degree of an argument* $a \in A \cup A^?$ *w.r.t.* σ *and* x *is* $Deg^{\nu^{\sigma,x}}_{I}(a) = \frac{|\mathrm{comp}^{\sigma,x}(I,a)|}{|\mathrm{comp}(I)|}$ *where* $\mathrm{comp}^{\sigma,x}(I, a)$ *is the set of completions of* I *where* a *is* x*-accepted for the semantics* σ, i.e. $\mathrm{comp}^{\sigma,x}(I, a) = \{C \in \mathrm{comp}(I) \mid a \in x_{\sigma}(C)\}$

Example 5. *Consider again IAF from Figure 2, the completions are given in Figure 3, and* $x = sk$ *and* $\sigma = \mathrm{pr}$. *The extensions of each completions are:* $\mathrm{pr}(C_1) = \{\{b, d\}\}$, $\mathrm{pr}(C_2) = \{\{a, d\}\}$, $\mathrm{pr}(C_3) = \{\{b, d\}\}$ *and* $\mathrm{pr}(C_4) = \{\{a, c\}, \{a, d\}\}$. *We obtain* $Deg^{\nu^{\mathrm{pr},sk}}_{I}(a) = Deg^{\nu^{\mathrm{pr},sk}}_{I}(b) = \frac{2}{4} = 0.5$, $Deg^{\nu^{\mathrm{pr},sk}}_{I}(c) = \frac{0}{4} = 0$ *and* $Deg^{\nu^{\mathrm{pr},sk}}_{I}(d) = \frac{3}{4} = 0.75$.

If only the relative acceptability is of interest, then the ranking extracted from $\nu^{\sigma,x}$ can be used.

Definition 14. *Given* $I = (A, A^?, R, R^?)$*,* σ *an extension-based semantics, and* $x \in \{cr, sk\}$*, the acceptability ranking of arguments* $A \cup A^?$ *w.r.t.* $\nu^{\sigma,x}$ *is* $a \succeq_I^{\nu^{\sigma,x}} b$ *iff* $Deg_I^{\nu^{\sigma,x}}(a) \geq Deg_I^{\nu^{\sigma,x}}(b)$

Example 6. *From the acceptability degrees given in Example 5, we obtain the following ranking associated with* I*:* $d \succ_I^{\nu^{\mathrm{pr},sk}} a =_I^{\nu^{\mathrm{pr},sk}} b \succ_I^{\nu^{\mathrm{pr},sk}} c$

In this example, we observe that, although it is not attacked, the uncertain argument a is equally acceptable to b and strictly less acceptable than d. This comes from the "penalty" mentioned previously, regarding the completions where a does not appear.

Acceptance Degree Based on the Argument Existence The acceptability degree assigned to argument a in Example 6 can be debatable. Indeed, we have observed that although a is not attacked, it has the same acceptability degree/ranking as b, which is attacked in two completions. This can be interesting when one wants to indirectly weaken the uncertain arguments compared to the certain arguments. Indeed, as uncertain arguments appear in fewer completions, their score can never be maximal. In order to fairly evaluate and compare certain and uncertain arguments, we define another family of semantics, where only the completions that actually contain an argument are taken into account for computing this argument's acceptability degree.

Definition 15. *Given IAF* $I = (A, A^?, R, R^?)$*,* σ *an extension-based semantics, and* $x \in \{\mathrm{cr}, \mathrm{sk}\}$*, the acceptability degree of an argument* $a \in A \cup A^?$ *w.r.t.* σ *and* x *is* $Deg_I^{\mu^{\sigma,x}}(a) = \frac{|\,\mathrm{comp}^{\sigma,x}(I,a)|}{|\,\mathrm{comp}(I,a)|}$

Notice that $Deg_I^{\mu^{\sigma,x}}(a) = Deg_I^{\nu^{\sigma,x}}(a)$ for a certain argument $a \in A$ because it appears in every completion. Let us define the ranking corresponding to this gradual semantics.

Definition 16. *Given an IAF* $I = (A, A^?, R, R^?)$*,* σ *an extension-based semantics, and* $x \in \{cr, sk\}$*, we define the acceptability ranking of arguments* $A \cup A^?$ *w.r.t.* $\mu^{\sigma,x}$ *as* $a \succeq_I^{\mu^{\sigma,x}} b$ *iff* $Deg_I^{\mu^{\sigma,x}}(a) \geq Deg_I^{\mu^{\sigma,x}}(b)$*.*

Example 7. *We continue the previous example, with* I *the IAF from Figure 2, and* $x = sk$ *and* $\sigma = \mathrm{pr}$*. The extensions of each completion are given at Example 5. Recall that, for the argument* a*, only two completions are used for computing* $\mu^{\sigma,x}(a)$*. All the other arguments appear in each completion. We obtain* $Deg_I^{\mu^{\mathrm{pr},sk}}(a) = \frac{2}{2} = 1$*,* $Deg_I^{\mu^{\mathrm{pr},sk}}(b) = \frac{2}{4} = 0.5$*,* $Deg_I^{\mu^{\mathrm{pr},sk}}(c) = 0$ *and* $Deg_I^{\mu^{\mathrm{pr},sk}}(d) = 0.75$*. We obtain the following ranking associated with* I*:* $a \succ_I^{\mu^{\mathrm{pr},sk}} d \succ_I^{\mu^{\mathrm{pr},sk}} b \succ_I^{\mu^{\mathrm{pr},sk}} c$ *If we compare this ranking with the one from Example 6, we see that argument* a *is the best ranked argument, since it is unattacked. The relationship between the remaining arguments coincide,* d *is still ranked above* b *and* c*.*

Principle-based Analysis Next, we identify the principles satisfied or violated by $\nu^{\sigma,x}$ and $\mu^{\sigma,x}$.

Proposition 2. *Given* $\alpha \in \{\mu, \nu\}$*,* $\sigma \in \{\mathrm{gr}, \mathrm{pr}, \mathrm{co}\}$ *and* $x \in \{\mathrm{cr}, \mathrm{sk}\}$*,*

1. $\alpha^{\sigma,x}$ *satisfies i-Abs,*
2. $\alpha^{\sigma,x}$ *satisfies i-In,*
3. $\alpha^{\sigma,x}$ *satisfies i-wVPC,*
4. $\nu^{\sigma,x}$ *violates i-wVPU*
5. $\mu^{\sigma,x}$ *satisfies i-wVPU,*
6. $\alpha^{\sigma,x}$ *satisfies i-SC,*
7. $\alpha^{\sigma,x}$ *satisfies i-CAP,*
8. $\alpha^{\sigma,x}$ *violates UArg,*
9. $\alpha^{\sigma,x}$ *violates i-AE,*
10. $\nu^{\sigma,x}$ *violates i-CP,*
11. $\alpha^{\sigma,x}$ *violates RF,*
12. $\alpha^{\sigma,x}$ *satisfies RC,*
13. $\alpha^{\sigma,x}$ *satisfies LC.*

Proposition 2 shows that the $\nu^{\sigma,x}$ violates i-wVPU, which is satisfied by $\mu^{\sigma,x}$, thus these two semantics do not only produce different rankings as discussed in Example 7, they differ in the satisfied principles as well. Interestingly, that there is no difference in satisfied principles for the acceptance modes, nor in the extension-based semantics considered in this paper.

5. Approaches based on Ranking-based Semantics and Voting Rules

In the previous section, we utilised the acceptance statuses of the arguments in the completions of an IAF. Besides the acceptance status of each argument, we can also determine the strength of each argument with ranking-based semantics. These semantics give us a preorder over the arguments with respect to an AF, thus we can apply ranking-based semantics to each of our completions and get a list of preorders. In order to receive our final preorder for an IAF, we still need to aggregate the list of preorders. In this paper, we use *voting rules* from the area of *Computational Social Choice* to aggregate the induced preorders. Voting rules are used by voters to collectively choose the best alternative among a set of alternatives. This process aims to be fair by reflecting the voters' preferences.

Definition 17. *Let $\mathcal{N} = \{v_1, \dots, v_n\}$ be a finite set of voters and a finite set of alternatives $\mathcal{A} = \{a_1, \dots, a_m\}$. A* vote *cast by voter v_i is a preorder*[2] $\succeq_{v_i}$ *of $\mathcal{A}$.*

A voting rule *τ maps a vector of votes $\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle$ to a single preorder $\succeq^{\tau(\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle)}$ over the alternatives $\mathcal{A}$.*

A number of voting rules can be found in the literature, such as the *plurality voting rule*, where the alternative with the most top votes wins. For an overview, we refer the reader to [20, 21]. Because of its simplicity, we use the *Borda rule* in our work.

Definition 18. *Let $\mathcal{A}$ be a set of alternative, $\mathcal{N} = \{v_1, \dots, v_n\}$ a set of voters and $\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle$ the votes of $\mathcal{N}$. The* Borda count *$Borda(x)$ for alternative $x \in \mathcal{A}$ is defined as:*

$$Borda_{\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle}(x) = \sum_{i=1}^{n} |\mathcal{A}| - |\{y \mid y \succ_{v_i} x\}|$$

The Borda rule *for two alternatives $x, y \in \mathcal{A}$ is then:*

$$x \succeq^{Borda(\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle)} y \quad \textit{iff} \quad Borda_{\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle}(x) \geq Borda_{\langle \succeq_{v_1}, \dots, \succeq_{v_n} \rangle}(y)$$

Each alternative receive points according to their position in each vote, and the one with the highest total points wins the election.

Example 8. *Consider the voters $\mathcal{N} = \{v_1, v_2, v_3, v_4\}$ and alternatives $\mathcal{A} = \{a, b, c, d\}$ with votes:*

$$\succeq_{v_1}: b \succ_{v_1} d \succ_{v_1} c \succ_{v_1} a \qquad \succeq_{v_2}: a \succ_{v_2} d \succ_{v_2} b \succ_{v_2} c$$
$$\succeq_{v_3}: b \succ_{v_3} d \succ_{v_3} c \succ_{v_3} a \qquad \succeq_{v_4}: a \succ_{v_4} d \succ_{v_4} b \succ_{v_4} c$$

The Borda counts *for the alternatives are:*

$$Borda_{(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)}(a) = 0 + 3 + 0 + 3 = 6 \qquad Borda_{(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)}(b) = 3 + 1 + 3 + 1 = 8$$
$$Borda_{(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)}(c) = 1 + 0 + 1 + 0 = 2 \qquad Borda_{(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)}(d) = 2 + 2 + 2 + 2 = 8$$

The resulting preorder is: $d \simeq^{Borda(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)} b \succ^{Borda(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)} a \succ^{Borda(\langle \succeq_{v_1}, \dots, \succeq_{v_4} \rangle)} c$

We consider the completions of an IAF as the voters and the arguments are the alternatives, thus the votes are the preorders induced by a ranking-based semantics applied to a completion.

Definition 19. *Let $I = (A, A^?, R, R^?)$ be an IAF, ρ a ranking-based semantics and τ a voting rule. Consider the completions $C_1, \dots, C_n \in \mathrm{comp}(I)$ of I, we denote the rankings induced by τ for $C_1, \dots, C_n$ with the vector $\langle \succeq^{\tau}_{C_1}, \dots, \succeq^{\tau}_{C_n} \rangle$. The corresponding ranking-based semantics $\psi^{\rho,\tau}$ for IAFs is defined as with $a, b \in A \cup A^?$: $a \succeq^{\psi^{\rho,\tau}}_{I} b$ iff $a \succeq^{\tau(\langle \succeq^{\rho}_{C_1}, \dots, \succeq^{\rho}_{C_n} \rangle)} b$*

In other words, we first compute a preorder over the arguments in every completion. Note that we add all arguments that are excluded from the completion to the end of the ranking, to ensure that the ranking is total. This decision is motivated by the fact that excluded arguments are never considered acceptable in an completion, therefore these arguments are the worst ranked ones. We receive a vector of preorders, which we aggregate with a voting rule.

[2]Note we require preorders instead of linear order as typical for voting rules, thus we adapt all needed definitions.

Example 9. *Recall I from Example 3 and its four completions* $\mathrm{comp}(I) = \{C_1, C_2, C_3, C_4\}$ *(Figure 3). The argument rankings with respect the* h-categoriser ranking-based semantics *for each completion is:*

$$b \simeq^{Cat}_{C_1} d \succ^{Cat}_{C_1} c \succ^{Cat}_{C_1} a \qquad\qquad a \simeq^{Cat}_{C_2} d \succ^{Cat}_{C_2} b \succ^{Cat}_{C_2} c$$
$$b \succ^{Cat}_{C_3} d \succ^{Cat}_{C_3} c \succ^{Cat}_{C_3} a \qquad\qquad a \succ^{Cat}_{C_4} d \succ^{Cat}_{C_4} b \succ^{Cat}_{C_4} c$$

Applying the Borda rule gives us the following ranking: $d \succ^{\psi^{Cat,Borda}}_I b \succ^{\psi^{Cat,Borda}}_I a \succ^{\psi^{Cat,Borda}}_I c$. *Note that the resulting argument ranking differs from all rankings generated from the completions.*

Next, we discuss the satisfied and violated principles in relation to the satisfied properties of the underlying ranking-based semantics ρ and fix the Borda rule as the voting rule to be investigated.

Proposition 3. *Let ρ be a ranking-based semantics then following statements hold:*

1. *If ρ satisfies Abs, then $\psi^{\rho,Borda}$ satisfies i-Abs.*
2. *If ρ satisfies Abs, In, NaE and VP, then $\psi^{\rho,Borda}$ violates i-In.*
3. *If ρ satisfies NaE and VP, then $\psi^{\rho,Borda}$ satisfies i-wVPC.*
4. *If ρ satisfies Abs, NaE and VP, then $\psi^{\rho,Borda}$ violates i-wVPU.*
5. *If ρ satisfies Abs and SC, then $\psi^{\rho,Borda}$ violates i-SC.*
6. *If ρ satisfies Abs, NaE and VP, then $\psi^{\rho,Borda}$ violates CAP.*
7. *If ρ satisfies Abs, VP and QP, then $\psi^{\rho,Borda}$ violates UArg.*
8. *If ρ satisfies Abs, NaE and VP, then $\psi^{\rho,Borda}$ violates i-AE.*
9. *If ρ satisfies Abs, NaE and CP, then $\psi^{\rho,Borda}$ violates i-CP.*
10. *If ρ satisfies Abs, NaE and CT, then $\psi^{\rho,Borda}$ violates RF.*
11. *$\psi^{\rho,Borda}$ satisfies RC and LC.*

Proposition 3 shows that the ranking-based semantics only needs to satisfy a small set of principles in order for $\psi^{\rho,Borda}$ to satisfy or to violate principles for IAFs. We will now discuss the counterexample for i-AE in more detail to demonstrate the behaviour of $\psi^{\rho,Borda}$.

Example 10. *Consider the IAF $I_2 = (A_2, A_2^?, R_2, R_2^?)$ with $A_2 = \{a, b, c\}$, $A_2^? = \{z, z'\}$, $R_2 = \{(z', b), (z, a), (z, c)\}$ and $R_2^? = \emptyset$ as depicted in Figure 4. We see that arguments a and c are isomorphic to each other, we can even consider them copies of each others. I_2 has four completions C_1, C_2, C_3 and C_4 (also depicted in Figure 4). If a ranking-based semantics ρ satisfies Abs, NaE and VP then we obtain the following four rankings:*

$$C_1 : a =^{\rho}_{C_1} c =^{\rho}_{C_1} b \succ^{\rho}_{C_1} z =^{\rho}_{C_1} z' \qquad\qquad C_2 : z' =^{\rho}_{C_2} a =^{\rho}_{C_2} c \succ^{\rho}_{C_2} b \succ^{\rho}_{C_2} z'$$
$$C_3 : z =^{\rho}_{C_3} b \succ^{\rho}_{C_3} a =^{\rho}_{C_3} c \succ^{\rho}_{C_3} z' \qquad\qquad C_4 : z =^{\rho}_{C_4} z' \succ^{\rho}_{C_4} a =^{\rho}_{C_4} c =^{\rho}_{C_4} b$$

We see that b is ranked strictly better than a only in completion C_3, because of argument c, the score of b raises disproportionally to a and c. Thus, by copying an argument, we can disproportionally reward an independent argument, which lead to the violation of i-AE in this case, i.e., $b \succ^{\psi^{\rho,Borda}}_{I_2} a$.

For voting rules, the behaviour in Example 10 is known as *dependence of clones* [22] and the Borda rule is vulnerable to clone attacks. Clone attacks are a voting manipulation approach, where (exact) copies of alternatives are added into the voting and this addition may change the outcome of the entire voting. It is interesting to note that, to satisfy RC and LC the underlying ranking-based semantics is not relevant, here the Borda rule helps us. Thus, the properties of the Borda rule influences the satisfied and violated principles significantly.

The h-categoriser ranking-based semantics satisfies Abs, In, NaE, VP and CT [14, 15, 8], thus we can directly derive the properties of $\psi^{Cat,Borda}$ except for i-SC, UArg and i-CP (see Table 1).

Proposition 4. *$\psi^{Cat,Borda}$ violates i-SC, UArg and i-CP.*

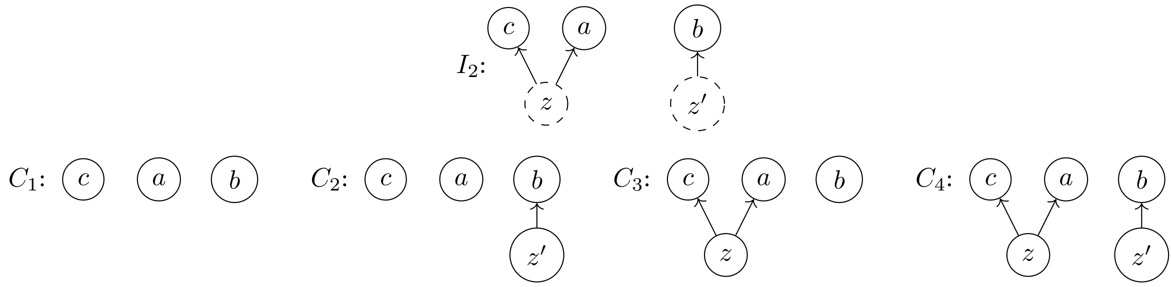

Figure 4: IAF I_2 including the four completions C_1, C_2, C_3 and C_4 from Example 10.

Principles	$\nu^{\sigma,x}$	$\mu^{\sigma,x}$	$\psi^{Cat,Borda}$
i-Abs	✓	✓	✓
i-In	✓	✓	✗
i-wVPC	✓	✓	✓
i-wVPU	✗	✓	✗
i-SC	✓	✓	✗
CAP	✓	✓	✗
UArg	✗	✗	✗
i-AE	✗	✗	✗
i-CP	✗	?	✗
RF	✗	✗	✗
RC	✓	✓	✓
LC	✓	✓	✓

Table 1
Principles satisfied (✓) and violated (✗) by the studied ranking-based semantics for $\sigma \in \{\mathrm{gr}, \mathrm{pr}, \mathrm{co}\}$ and $x \in \{\mathrm{cr}, \mathrm{sk}\}$. ”?” indicates a remaining open question.

6. Related work and Conclusion

In this paper, we introduce ranking-based semantics for incomplete argumentation frameworks, so argumentation frameworks with qualitative uncertainty on the existence of the arguments and attacks. With these functions, we can establish a ranking over the arguments according to their individual strength. We propose a number of principles to evaluate different semantics. Some of these principles are adapted from ranking-based semantics for AF, such as i-Abs, I-In or i-SC, others are tailored to the IAF case, like CAP, RC and LC. We also discuss three families of ranking-based semantics. The first one is based on the number of possible worlds an argument can be accepted in, with respect to a Dung-style extension-based semantics. The second semantics is a variation of the first one, where the non-existence of arguments is not punished. For the final semantics, we first compute a ranking over the arguments in every possible world, with a ranking-based semantics, and then aggregate the resulting list of preorders to a single preorder with a voting rule. We evaluate our approaches with our proposed principles and show that these semantics behave differently. A summary of all satisfied and violated principles can be found in Table 1. Recall that extension-based semantics σ used to define the families $\nu^{\sigma,x}$ and $\mu^{\sigma,x}$ belong to $\{\mathrm{gr}, \mathrm{pr}, \mathrm{co}\}$. Other extension-based semantics, including the stable semantics, may be investigated in future work. Based on the satisfied principles $\nu^{\sigma,x}$ and $\mu^{\sigma,x}$ differ slightly, while $\psi^{Cat,Borda}$ satisfies significantly fewer principles. This behaviour can also be observed by comparing the three rankings from Example 5, Example 7 and Example 9. While all three rankings agree on the fact that argument c is the least ranked argument, for the other three arguments they differ. For $\mu^{\mathrm{pr},\mathrm{sk}}$ argument a is the strongest argument, while for $\nu^{\mathrm{pr},\mathrm{sk}}$ and $\psi^{Cat,Borda}$ d is the strongest one. Further, $\nu^{\mathrm{pr},\mathrm{sk}}$ and $\psi^{Cat,Borda}$ disagree on the order of a and b.

Related work Besides incomplete argumentation frameworks, we can find a number of extensions of Dung’s abstract argumentation frameworks, for instance *argumentation frameworks with collective* *attacks* (SETAFs) [23], where attacks are not only between two arguments, but rather a set of arguments collectively attacks an argument, or *weighted argumentation frameworks* (WAFs) [24], where arguments and attacks receive weights already in the construction phase of the framework. Originally, reasoning in both these extensions of AFs was done utilising an extension-based approach. Similar to our work, ranking-based approaches were proposed for these extensions. Yun et al. (2020) discussed ranking-based semantics for SETAFs by generalising the h-categoriser ranking-based semantics, where for each attacking set only the weakest attacker is considered. Amgoud and Doder (2019) introduced a family of gradual semantics for WAFs by presenting evaluation methods for the individual arguments. These gradual semantics are a generalisation of the gradual semantics for AFs and build a bridge to gradual semantics for *weighted bipolar argumentation frameworks* [26]. While the methodology between our work and the works by Yun et al. (2020) and Amgoud and Doder (2019) is similar, both SETAFs and WAFs are unable to handle any kind of uncertainty, thus our results are novel.

A research direction different to abstract argumentation in the area of computational argumentation are the structured approaches such as *assumption-based argumentation frameworks* (ABA frameworks) [27]. The frameworks are based on deductive systems over a formal language and rules. Part of this language are the so-called *assumptions*, which are used to derive further pieces of information. Similar to other argumentation approaches, reasoning is done by identifying sets of assumptions that can be considered acceptable. As we already argued, such a binary classification is also too restrictive for structured argumentation, thus, a number of papers discussing ranking-based semantics or gradual semantics in the context of structured argumentation can be found. In the works [28, 29], the authors propose a family of ranking-based semantics to rank assumptions within an ABA framework by transforming an (flat) ABA framework into the corresponding AF and then applying well-known ranking-based semantics, such as h-categoriser to receive a ranking over the assumptions. The paper [30] extended the works by Skiba et al. to general ABA frameworks. Here, the authors utilise *quantitative bipolar argumentation frameworks* [31] instead of AFs to compute the individual strength values to the assumptions. Other works discussing ranking-based semantics for structured argumentation are for instance, [32] where *deductive argumentation* is investigated, *statement graphs* by [33], or *ASPIC*$^+$ by [34] and [35].

In the paper [36], the authors discuss ranking-based semantics for statement graphs with incomplete statements. These are statements, which are not facts nor are supported by facts, thus there is no evidence on these statements. Similar to [30], the authors utilise ranking-based semantics for quantified bipolar argumentation frameworks for their evaluations.

Future work The three families of ranking-based semantics we introduce follow a similar methodology. First, for an IAF we compute every completion and apply a semantics on each of these completions, either extension-based semantics like for $\nu^{\sigma,x}$ and $\mu^{\sigma,x}$ or ranking-based semantics like for $\psi^{\rho,\tau}$. Then we aggregate these results to a single preorder. While we discussed the different semantics in detail, the different preference aggregation methods received less attention. Particularly axiomatically investigating different preference aggregation methods applied to the context of IAFs would be interesting.

Recently, IAFs have been extended in various directions, like adding a support relation [37, 38] or higher-order attacks [39]. Ranking-based (or gradual) semantics have already been studied in bipolar AFs [40] and AFs with higher-order attacks [41], so a natural question for investigation for us would be to study ranking (or gradual) semantics in the context of bipolar or higher-order IAFs.

A point of criticism of IAFs is their reliance on completions [42, 43]. The exponential number of completions restricts the usage of IAFs in a real world scenario. Thus, there is a need for an approach that computes rankings over arguments within an IAF without relying on *all* completions. Such *direct approaches* have been discussed for extension-based cases, first for *Partial AFs* (a subclass of IAFs s.t. $A^? = \emptyset$) [44] and recently adapted to IAFs [42, 43]. We want to continue our work by investigating direct approaches for ranking-based semantics for IAFs.

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Declaration on Generative AI

During the preparation of this work, the author(s) used Grammarly in order to: Grammar and spelling check, Paraphrase and reword. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the publication's content.

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