A Many-Valued Multi-Preferential Propositional Typicality Logic and a Conditional Interpretation for Gradual Argumentation (Extended Abstract)

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Abstract
In the extended abstract we report about a many-valued and multi-preferential conditional logic with typicality developed in [1], which is based on a multi-preferential semantics and is proven to be a generalization of KLM preferential semantics to the many-valued case. The multi-preferential semantics provides a preferential interpretation to gradual argumentation, considering both weighted and non-weighted argumentation graphs. The approach allows for conditional reasoning over arguments, with respect to some gradual semantics, through the verification of graded (strict or defeasible) implications over argumentation graphs.

In this extended abstract, we describe a many-valued and multi-preferential propositional conditional logic with typicality developed in [1], which features a typicality operator in a many-valued logic, along the lines of the Propositional Typicality Logic (PTL) by Booth et al. [2] and of the Description Logics (DLs) with typicality by Giordano et al. [3, 4]. The many-valued multi-preferential semantics for conditionals is a generalization of the KLM preferential semantics of conditionals [5, 6] (which exploits a single preference relation), and, under some conditions, it satisfies the KLM properties of a preferential consequence relation, which can be properly reformulated for the many valued case. Many-valued extensions of conditionals have also been explored, e.g., for Lewis' system of counterfactuals [7].

The many-valued multi-preferential semantics of conditionals is exploited to develop a general approach for providing a preferential interpretation of argumentation graphs in gradual semantics, under some conditions on the semantics, thus allowing for *conditional reasoning over the argumentation graph*, by formalizing *conditional properties* of the graph in the many-valued multi-preferential logic with typicality.

The typicality operator allows to single out the typical situations in which some formula holds: "a sentence of the form $\mathbf{T}(\alpha)$ is understood to refer to the *typical situations in which α holds*" [2]. The typicality operator allows for the definition of *conditional implications* $\mathbf{T}(\alpha) \rightarrow \beta$, meaning that "normally if α holds then β holds", in the sense that "in the typical situations where α holds, β also holds" [2]. Such implications are intended to correspond to conditional implications $\alpha \mid\sim \beta$ in the KLM approach [5, 6], but the typicality operator can occur elsewhere (provided it is not nested).

The paper deals with the many-valued case: we introduce *graded implications* of the form $\alpha \rightarrow \beta \geq l$ (resp., $\alpha \rightarrow \beta \leq u$), and, more specifically, *graded conditionals* of the form $\mathbf{T}(\alpha) \rightarrow \beta \geq l$ (resp., $\mathbf{T}(\alpha) \rightarrow \beta \leq u$), the last two meaning that "normally α implies β with degree at least l (resp., at most u) ", where α cannot contain the typicality operator. Graded implications and graded conditionals are inspired to graded inclusion axioms in fuzzy DLs [8] and in fuzzy DLs with typicality [9, 10].

The semantics of the language is based on a notion of multi-preferential interpretation, which exploits *multiple preference relations* $<_{\alpha_i}$ on worlds (propositional interpretations), each one associated with a formula α_i. The relation $w <_{\alpha_i} w'$ between two worlds w and w' means that world w represents a more typical situation with respect to w'. For instance, the world w may describe the properties of

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an employee, who is as well a student, and is very typical as an employee (e.g., she has an income, a boss, etc.), while rather atypical as a student (e.g., she is old and does not have classes). On the other hand, w' may describe the properties of an employee, who is also a student, is atypical as an employee (e.g., she has no boss and no fixed working time) but is rather typical as a student (e.g., she has classes and is young). In this case, we would say that w describes a more typical employee with respect to w' (i.e., $w <_{employee} w'$) and, vice-versa, that w' describes a more typical student than w (i.e., $w' <_{student} w$). Note that, in this example, worlds w and w' would be hardly comparable by considering a single preference relation $<$.

To define a *many-valued semantics* for the language, as usual in propositional many-valued logics [11], we consider a *truth degree set* $\mathcal{D}$, so that, in an interpretation, each formula will be given a value in $\mathcal{D}$. We assume that $\mathcal{D}$ is equipped with a preorder relation $\leq^{\mathcal{D}}$, a bottom element $0^{\mathcal{D}}$, and a top element $1^{\mathcal{D}}$. We denote by $<^{\mathcal{D}}$ and $\sim^{\mathcal{D}}$ the related strict preference relation and equivalence relation.

A *(multi-)preferential interpretation* over the truth degree set $\mathcal{D}$ is a triple $\mathcal{M} = \langle \mathcal{W}, \{<_{\alpha_i}\}, v\rangle$ where: $\mathcal{W}$ is a non-empty set of worlds; each $<_{\alpha_i} \subseteq \mathcal{W} \times \mathcal{W}$ is a strict partial order relation on $\mathcal{W}$; the valuation function $v : \mathcal{W} \times Prop \longrightarrow \mathcal{D}$ assigns a truth value $v(w, p)$ in $\mathcal{D}$ to any propositional variable p at each world $w \in \mathcal{W}$. For evaluating complex formulas, each connective $\wedge, \vee, \rightarrow, \neg$ is associated with a *truth degree function* in $\mathcal{D}$, while the truth degree of a typicality formula is defined as follows:

$$v(w, \mathbf{T}(\alpha_i)) = \begin{cases} v(w, \alpha_i) & \text{if } \nexists w' \in \mathcal{W} \text{ s.t. } w' <_{\alpha_i} w \\ 0_{\mathcal{D}} & \text{otherwise} \end{cases}$$

The interpretation of a typicality formula $\mathbf{T}(\alpha_i)$ in $\mathcal{M}$, at a world w, provides the degree of typicality of α_i in w. When there is no world which is preferred to w with respect to formula α_i, then the value of $\mathbf{T}(\alpha_i)$ in w is taken to be the value of α_i in w; otherwise it is set to $0_{\mathcal{D}}$. When $v(w, \mathbf{T}(\alpha)) \neq 0_{\mathcal{D}}$, we say that w is a typical/normal α-world in $\mathcal{M}$.

Notions of *coherent*, *faithful* and *weakly faithful* multi-preferential interpretations are introduced to impose an agreement between how the preference relations $<_{\alpha_i}$ are defined in an interpretation $\mathcal{M}$ and how the valuation function is defined in $\mathcal{M}$. For instance, a many-valued (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{\alpha_i}\}, v\rangle$ is *coherent with respect to the preference relation* $<_{\alpha_i}$ if, for all $w, w' \in \mathcal{W}$, $v(w, \alpha_i) >_{\mathcal{D}} v(w', \alpha_i) \iff w <_{\alpha_i} w'$. That is, in a coherent interpretation, the preference relation $<_{\alpha_i}$ between worlds reflects *exactly* the relationship between the valuations of α_i in the different worlds. A notion of coherence was first introduced in the multi-preferential semantics for DLs with typicality in [12, 10].

Suppose that, in the student example above, the truth value assigned to proposition *student* in w is greater than the truth value assigned to proposition *student* in w' (e.g., $v(w, student) = 0.9$ and $v(w', student) = 0.4$), that is, $v(w, student) >_{\mathcal{D}} v(w', student)$. In this case, we may expect that world w represents a more typical situation describing a student with respect to the situation described by world w' (i.e., $w <_{student} w'$). In a coherent interpretation, the vice-versa also holds, and the preference among the worlds w and w' with respect to proposition $student$ must be reflected in the truth degrees of proposition $student$ at the worlds w and w'.

The last requirement is strong and, in the two-valued case, it would lead to trivial preference relations, were the worlds in which $student$ is true are all incomparable, the worlds in which $student$ is false are all incomparable, and the worlds in which $student$ is true are all preferred to the worlds in which $student$ is false. This motivates the introduction of weaker notions of *faithfulness* and *weak faithfulness*. Let us consider the notion of weak faithfulness introduced in [1]. A many-valued (multi-)preferential interpretation $\mathcal{M}$ is *weakly faithful with respect to a preference relation* $<_{\alpha_i}$ if, for all $w, w' \in \mathcal{W}$,

$$w' <_{\alpha_i} w \;\Rightarrow\; v(w, \alpha_i) \leq_{\mathcal{D}} v(w', \alpha_i).$$

Note that, by contraposition, when $v(w, \alpha_i) >_{\mathcal{D}} v(w', \alpha_i)$, it must be the case that $w' \not<_{\alpha_i} w$, i.e., that either $w <_{\alpha_i} w'$ or $w \sim_{\alpha_i} w'$ or w and w' are incomparable wrt $<_{\alpha_i}$. The notion of weak faithfulness does not impose that worlds (as w) in which formula α_i has a higher valuation must be preferred to worlds (as w') in which α_i has a lower one. The two worlds w and w' may as well be incomparable with respect to α_i. In [1] it is proven that, under some condition on the preference relations, weakly-faithful

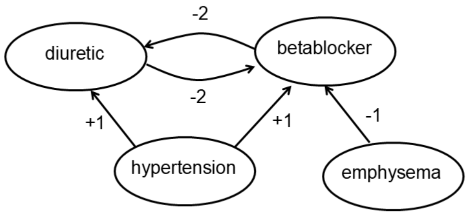

Figure 1: Example argumentation graph G

multi-preferential interpretations can be regarded as a generalization of the two-valued KLM preferential interpretations [5, 6].

In [1] a preferential interpretation of gradual argumentation semantics [13, 14, 15, 16, 17, 18, 19] is also proposed. The arguments in the argumentation graph become the propositional variables of the typicality logic and the domain of argument interpretation (in the gradual semantics under consideration) becomes the truth degree set. This allows typicality properties and defeasible properties of an argumentation graph to be validated with respect to some given argumentation semantics. In [1], examples are considered for the φ-coherent semantics [20] for weighted argumentation graphs, and for the h-categorizer semantics [21] for non-weighted argumentation graphs.

For two arguments A_1 and A_2, the intended meaning of a conditional implication $\mathbf{T}(A_1) \rightarrow A_2$ is that "normally argument A_1 implies argument A_2", in the sense that "in the typical situations where A_1 holds, A_2 also holds". *Graded implications over arguments* of the form $\alpha \rightarrow \beta \geq l$ (resp., $\alpha \rightarrow \beta \leq u$) are allowed, where α and β may be *boolean combination of arguments*.

Consider the weighted argumentation graph in Figure 1, adapted from [22], here using both attacks and supports, with a weight (a negative or positive number, respectively); $hypertension$ is an argument supporting possible prescriptions $diuretic$ and $betablocker$, attacking each other (assuming that a single treatment is sufficient). The additional argument $emphysema$ attacks $betablocker$, being a contraindication for such a treatment. We may be interested in checking, for a given semantics, whether, for example, the conditional $\mathbf{T}(emphysema) \rightarrow \neg betablocker \geq v$, or the corresponding strict version $emphysema \rightarrow \neg betablocker \geq v$, hold for some grade v. Similarly, we may consider graded implications with boolean combinations of arguments, such as $\mathbf{T}(hypertension \wedge emphysema) \rightarrow \neg betablocker \geq v$, or $\mathbf{T}(hypertension \wedge emphysema) \rightarrow diuretic \geq v$. All such implications regard the strength of a (contra)indication, in typical situations with single or multiple medical conditions. An implication $\mathbf{T}(\neg betablocker) \rightarrow \mathbf{T}(diuretic) \geq v$ can also be considered, to verify to what degree the typical situations in which *betablocker* is not prescribed are typical situations for *diuretic*.

The idea exploited in [1] is that a *multi-preferential* interpretation $\mathcal{M}_G^S$ can be constructed for an argumentation graph G with respect to some gradual semantics S, by considering the strength of the arguments in G, according to the chosen (gradual) argumentation semantics. The interpretation $\mathcal{M}_G^S$ exploits multiple preference relations $<_{A_i}$ (resp., $<_{\alpha_i}$) over the many-valued labellings (or strength functions) of the graph G, where the preferences are associated with arguments A_i (resp., with boolean combinations of arguments α_i). The validity of graded implications can then be verified over the argumentation graph G in the argumentation semantics S by checking their satisfiability in the interpretation $\mathcal{M}_G^S$. The satisfiability of a graded conditional $\mathbf{T}(\alpha) \rightarrow \beta \geq k$ in a *finite* preferential interpretation $\mathcal{M}_G^S$ (with α and β boolean combination of arguments) can be decided in polynomial time in the size of the interpretation $\mathcal{M}_G^S$ and in the size of the conditional formula (which is usually much smaller than the size of $\mathcal{M}_G^S$).

The definition of a preferential interpretation $\mathcal{M}_G^S$ associated with an argumentation graph G and a gradual semantics S also sets the ground for the definition of a *probabilistic extension of gradual argumentation*. We refer to [1] for details.

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Declaration on Generative AI

The authors have not employed any Generative AI tools.

References

[1] M. Alviano, L. Giordano, D. Theseider Dupré, A many-valued multi-preferential propositional typicality logic and a conditional interpretation for gradual argumentation, J. Log. Comput. 36 (2026). doi:10.1093/LOGCOM/EXAF063.

[2] R. Booth, G. Casini, T. Meyer, I. Varzinczak, On rational entailment for propositional typicality logic, Artif. Intell. 277 (2019).

[3] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, ALC+T: a preferential extension of Description Logics, Fundamenta Informaticae 96 (2009) 1–32.

[4] L. Giordano, V. Gliozzi, N. Olivetti, G. L. Pozzato, Semantic characterization of rational closure: From propositional logic to description logics, Art. Int. 226 (2015) 1–33.

[5] S. Kraus, D. Lehmann, M. Magidor, Nonmonotonic reasoning, preferential models and cumulative logics, Artificial Intelligence 44 (1990) 167–207.

[6] D. Lehmann, M. Magidor, What does a conditional knowledge base entail?, Artificial Intelligence 55 (1992) 1–60.

[7] L. Běhounek, O. Majer, A graded semantics for counterfactuals, Synthese 199 (2021) 11963–11994.

[8] T. Lukasiewicz, U. Straccia, Managing uncertainty and vagueness in description logics for the semantic web, J. Web Semant. 6 (2008) 291–308.

[9] L. Giordano, D. Theseider Dupré, An ASP approach for reasoning on neural networks under a finitely many-valued semantics for weighted conditional knowledge bases, Theory Pract. Log. Program. 22 (2022) 589–605.

[10] M. Alviano, F. Bartoli, M. Botta, R. Esposito, L. Giordano, D. Theseider Dupré, A preferential interpretation of multilayer perceptrons in a conditional logic with typicality, Int. Journal of Approximate Reasoning 164 (2024).

[11] S. Gottwald, A Treatise on Many-valued Logics, Research Studies Press, 2001.

[12] L. Giordano, D. Theseider Dupré, Weighted defeasible knowledge bases and a multipreference semantics for a deep neural network model, in: Proc. JELIA 2021, LNCS, vol. 12678, 225–242, 2021.

[13] C. Cayrol, M. Lagasquie-Schiex, Graduality in argumentation, J. Artif. Intell. Res. 23 (2005) 245–297.

[14] J. Janssen, M. D. Cock, D. Vermeir, Fuzzy argumentation frameworks., in: IPMU 2008, 2008, pp. 513–520.

[15] P. E. Dunne, A. Hunter, P. McBurney, S. Parsons, M. J. Wooldridge, Weighted argument systems: Basic definitions, algorithms, and complexity results, Artif. Intell. 175 (2011) 457–486.

[16] S. Egilmez, J. G. Martins, J. Leite, Extending social abstract argumentation with votes on attacks, in: TAFA 2013, Beijing, China, Aug. 3-5, LNCS 8306, Springer, 2013, pp. 16–31.

[17] L. Amgoud, J. Ben-Naim, D. Doder, S. Vesic, Acceptability semantics for weighted argumentation frameworks, in: IJCAI 2017, Melbourne, Australia, 2017, pp. 56–62.

[18] P. Baroni, A. Rago, F. Toni, How many properties do we need for gradual argumentation?, in: Proc. AAAI 2018, New Orleans, Louisiana, USA, February 2-7, 2018, pp. 1736–1743.

[19] L. Amgoud, D. Doder, Gradual semantics accounting for varied-strength attacks, in: Proceedings AAMAS '19, Montreal, QC, Canada, May 13-17, 2019, 2019, pp. 1270–1278.

[20] M. Alviano, L. Giordano, D. Theseider Dupré, Weighted knowledge bases with typicality and defeasible reasoning in a gradual argumentation semantics, Intelligenza Artificiale 18 (2024).

[21] P. Besnard, A. Hunter, A logic-based theory of deductive arguments, Artif. Intell. 128 (2001) 203–235.

[22] K. Atkinson, P. Baroni, M. Giacomin, A. Hunter, H. Prakken, C. Reed, G. R. Simari, M. Thimm, S. Villata, Towards artificial argumentation, AI Mag. 38 (2017) 25–36.