Tenability and Non-Uniform Defense (Extended Abstract)*

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Abstract

We introduce *tenability*, a dialogue-based semantics for Dung-style Argumentation Frameworks that formalizes when a designated argument (or a set of arguments) can be maintained in debate by a proponent against any legitimate (conflict-free) attack which the opponent may present. The approach naturally corresponds to one-shot debate scenarios in which a uniform defense against all possible counterarguments is not required, and is robust with respect to further constraints placed on the rules of the dialogue game.

Keywords

Non-monotonic Reasoning, Dung-style Argumentation Frameworks, Weak Semantics

1. Introduction

Dung's *Argumentation Frameworks* (AFs) [2] are an influential formalism for investigating the logic of non-monotonic reasoning. In particular, there are clean, elegant, and computationally efficient ways to identify *acceptable* sets of arguments with respect to various standards. Most semantics (sets or criteria for identifying acceptable sets of arguments of an AF) are defined in terms of their defense against competing arguments. One of the most fundamental semantics is *admissibility* which requires that a set of arguments counterattacks each of its attackers. Despite its central role, admissibility can be too skeptical in its acceptance of arguments, even when their only counterarguments are spurious. For example, consider the case of the AF $\mathcal{S} = (\{a, b\}, \{(b, b), (b, a)\})$ consisting of a self-attacking argument b which also attacks another argument a. Admissibility does not permit a to be accepted, because it does not provide a counterattack to b, even though the argument b is self-defeating. *Weak semantics*, such as *cogency* and *cyclic cogency* [3], *weak admissibility* [4], and *weak completeness* [5] relax the requirements of admissibility to obtain more liberal notions of acceptability. For one example, a set of arguments is *cogent* if and only if it is admissible with respect to arguments which are not self-attacking [6, Prop 8.9]. Under cogent and other weak semantics, a is indeed acceptable in $\mathcal{S}$.

Many of these semantics (e.g. admissibility, cogency) are motivated by and equivalently expressed as winning strategies in certain dialogue games [7, 8]. However, in many cases the dialogues corresponding to the acceptability of a set often demand too much from the *Proponent* (Pro) of a set of arguments and not nearly enough from the *Opponent* (Opp). Some of the dialectical asymmetry lies in what arguments either player is allowed to put forward (e.g. Opp may advance conflicting arguments against Pro), or in their obligations to previous arguments (Pro must defend the arguments she has committed to, while Opp need not defend anything). Thus, we define a dialogue-based semantics called *tenability*, wherein the responsibilities of Pro and Opp are as symmetrical as possible insofar as: both must advance conflict-free collections of arguments, both must defend their played arguments in future plays, and the conditions for acceptability are predicated upon one player or the other having a *winning strategy*. This last ingredient, a strategy-based reading, allows tenability semantics to capture the *non-uniformity* of

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many one-shot debate contexts: the defense one gives in one debate need not be the same defense given in a separate debate. For example, in a negotiation, the development of arguments in the negotiation depend seriously upon the previously stated arguments. In legal scenarios, the defense's next choice of witness or evidence may seriously depend on the prior arguments brought by the plaintiff. In one-shot scenarios, where a proponent of X need defend against only one line of attack instead of all possible lines of attack, a uniform response isn't necessary. By defining tenability in terms of a winning strategy, we capture a distinct standard of acceptance attuned to non-uniform defense. An argument, or set of arguments which manages to clear these standards will be called tenable, indicating that it is an argument which is *maintainable in debate.*

2. Tenability

Our formal context is within *Argumentation Frameworks* (AFs) which are merely directed graphs: $\mathcal{F} = (A_{\mathcal{F}}, \rightarrowtail)$. The set $A_{\mathcal{F}}$ is a collection of arguments and the binary relation $\rightarrowtail$ on $A_{\mathcal{F}}$ represents attack between arguments. A set of arguments $X \subseteq A_{\mathcal{F}}$ is *conflict-free* if there is no $a, b \in X$ such that $a \rightarrowtail b$. A set of arguments $X \subseteq A_{\mathcal{F}}$ is *admissible* if for every attacker b of an argument $x \in X$, there is an argument $y \in X$ where $y \rightarrowtail b$. To facilitate some technical needs in the definition of tenability, we say that a set $E \subseteq A_{\mathcal{F}}$ is *at least as cogent as* $D \subseteq A_{\mathcal{F}}$, written $E \succeq D$, if E is *admissible* in the restriction $\mathcal{F}\restriction_{E \cup D}$. We write $E \succ D$ if $E \succeq D$ and $D \not\succeq E$, and say that E *is more cogent than* D.

Definition 2.1 (Tenability). A *tenability dispute* on $\mathcal{F}$ is a sequence of subsets $(X_0, X_1, ..., X_n)$ of $A_{\mathcal{F}}$, of which the even moves (X_{2i}) are Pro moves, and the odd moves (X_{2i+1}) are Opp moves, such that:

(1) each X_i is conflict-free. (*Consistency)*
(2) $X_i \subseteq X_{i+2}$ for each i. (*Monotonicity*)
(3) X_1 and each $(X_{i+2} \setminus X_i)$ are finite.
(4) $X_{2i} \prec X_{2i+1}$. (*Opp maintains its prior moves against Pro's attacks, and progresses its attack.*)
(5) $X_{2i+1} \preceq X_{2i+2}$. (*$Pro$ maintains its prior moves against Opp's attacks*)

A tenability dispute is *won* by a player Pro/Opp if they have made the last move and there are no further legal moves for the other player. If the dispute is infinitely long (as may happen in infinite AFs), the dispute is a win for Pro. A set $A \subseteq \mathcal{F}$ is *tenable* if there is a winning strategy[1] for Pro beginning with $X_0 = A$.

An informal description of a tenability dispute goes something like this: Pro has a set of arguments X_0 she would like to maintain. *Opp* has the burden of showing that X_0 is not a reasonable position, so he puts forward some arguments X_1 that challenge X_0. Because tenability is an inherently defensive notion, it is enough for Pro to counter every attack from *Opp*'s X_1. Thus, she plays a set X_2 which extends X_0 and defends against (is as cogent as) X_1. Because of the symmetry of the dispute, *Opp*'s subsequent plays must similarly extend his prior plays and be as cogent as Pro's. However, it does nothing to progress the case against the tenability of X_0 if *Opp*'s next play is merely as cogent as Pro's prior play. Instead, *Opp*'s next play X_{2n+1} must give a set of arguments to disbelieve Pro's position X_{2n}; a set of arguments X_{2n+1} which is more cogent than X_{2n}.

We now consider an example $\mathcal{U}$, an AF described in Figure 1. We show that $\{a\}$ is tenable by describing Pro's winning strategy in the dispute that begins with $\{a\}$. There are four sets of arguments of $\mathcal{U}$ which are at least as cogent as $\{a\}$. They are: $\{b_1\}$, $\{b_1, c_2\}$, $\{b_2\}$, and $\{b_2, c_1\}$. If *Opp* plays either $\{b_1\}$ or $\{b_1, c_2\}$, then Pro responds with the set $\{a, c_1\}$ which is at least as cogent as either of the two. There are no sets containing b_1 (or c_2 for that matter) which are more cogent than $\{a, c_1\}$. Symmetrically, if *Opp* plays either $\{b_2\}$ or $\{b_2, c_1\}$, then Pro responds with the set $\{a, c_2\}$, which is as cogent as any set containing b_2. In either case, Pro has the last laugh, and wins the dispute.

[1] Informally, Pro having a winning strategy means that she always has a way of winning the dispute, even if that depends on *Opp*'s choice of moves. See, e.g., [9] for other argument games which necessarily employ winning strategies.

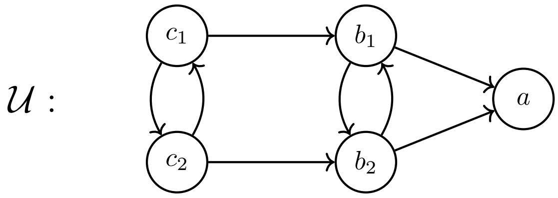

Figure 1: An AF $\mathcal{U}$ representing a situation of disjunctive reinstatement. The argument c_1 (respectively c_2) may reinstate a from the attack by b_1 (respectively b_2), but there is no conflict-free set of arguments reinstating a from the joint attacks by $\{b_1, b_2\}$. Tenability remedies this by disallowing $\{b_1, b_2\}$ as a legitimate challenge to $\{a\}$.

Deciding whether a set $X \subseteq \mathcal{A}_{\mathcal{F}}$ is tenable *prima facie* requires checking a game tree of height at most $|A_{\mathcal{F}}|$ where each game state has at most $2^{|A_{\mathcal{F}}|}$ possible next plays. It follows that the problem of determining if a set is tenable is $PSPACE$-complete [1, Theorem 4.6], putting it on a par with weak admissibility [10, Theorem 3.16]. This computational barrier inspires a natural question of whether restricting disputes to plays where all newly introduced arguments are *relevant* (i.e., provide a counterattack or pose an immediate threat) changes the complexity or results in a new semantics.

Definition 2.2. A *relevant tenability dispute* on $\mathcal{F}$ is a tenability dispute in which we further require every argument $a \in X_{i+1} \setminus X_0$ to attack an argument $b \in X_i$. The *relevant tenability game*, its conclusions, and its winners are defined analogously as in Definition 2.1. A set $A \subseteq \mathcal{F}$ is *relevantly tenable* if there is a winning strategy for Pro beginning with $X_0 = A$.

Relevant tenability eliminates the possibility of "trap arguments" which might be advanced early on, but only later impact the dispute. Relevance is a natural constraint placed on participants in debates, reflecting scenarios where irrelevant arguments are dismissed as inconsequential or distracting. In legal contexts, there are embedded requirements where evidence must be relevant in order to be admissible. As it turns out, this extra constraint does not perturb the tenability of an argument, but it does simplify identifying if an argument is tenable and facilitates technical arguments regarding tenability.

Theorem 2.3. *A set of arguments $X \subseteq A_{\mathcal{F}}$ is tenable if and only if it is relevantly tenable.*

While Theorem 2.3 does not lower the complexity of verifying tenability, it does narrow searches through the game tree for particular instances. Furthermore, it shows that tenability is naturally aligned with our intuitive dialectical standards, where participants in a debate are only required to address relevant, immediate threats. This alignment with the dynamics of debate leads directly into our broader observations about tenability and non-uniform defense.

3. Conclusion and Future Work

Tenability semantics, motivated by non-uniform defense and a more symmetric form of debate, capture a distinct perspective on the acceptability of arguments. Such a perspective is a break from the extension-based Dung-style semantics where the acceptance of an argument is tied to the existence of an extension which provides a defense of the argument. Such a correspondence is inherently uniform. On the other hand, regarding a's tenability in $\mathcal{U}$ of Figure 1, there is no extension of $\mathcal{U}$ which properly corresponds to $\{a\}$'s non-uniform defense. Because of this divergence from the uniform extension-based semantics, existing principles for the evaluation of weak semantics [11, 12, 13] tend to fail for reasons that are orthogonal to the intended reading of tenability. Two ways to make such a study informative are to analyze maximal variants of the tenability notions, and to extract from tenability new principles that isolate non-uniformity as a design principle applicable beyond our specific games.

While the current presentation of tenability is formulated only for argumentation frameworks, it seems plain that the spirit of non-uniform defense and symmetric debate will be readily adaptable to more descriptive formalisms such as Bipolar Argumentation Frameworks [14], Quantitative Bipolar Argumentation Frameworks [15], SETAFs [16], and beyond.

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Declaration on Generative AI

The authors have not employed any Generative AI tools.

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