Solving variants of the Multi-Mode Resource Constrained Project Scheduling Problem with quantum annealing

(Special Track: Quantum Computing)

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Abstract

This paper describes an ongoing research project for using *quantum annealing* to solve variants of one of the most challenging *NP-hard* scheduling problem: the Multi-Mode Resource Constrained Project Scheduling Problem (MRCPSP). The relevance of this problem arises both from its broad applicability in projects' makespan minimization (or in the minimization of other objective functions, for example the overall project's cost), and in the non-trivial nature of its formulation as an input of the quantum machines. In the MRCPSP each activity can be executed in one of several modes under limited resources. In addition, each execution mode represents an alternative combination of resource requirements and activity duration. In this work we consider the classical MRCPSP setting by introducing both renewable and nonrenewable resources. In general, our future plan is to: (i) consider several Mixed Integer Linear Programming (MILP) formulations of each considered MRCPSP variant; (ii) compare their conversion into a Quadratic Unconstrained Binary Optimization (QUBO) model with respect to both the number of used qubits (i.e.,the number of decision variables) and the number of constraints; (iii) choose the most suited QUBO model for embedding/mapping the original scheduling problem in the target quantum hardware and running the annealing process.

Keywords

Resource-constrained project scheduling, Multi-mode, Renewable and nonrenewable resources, Quantum annealing, MILP

1. Introduction

In recent years, quantum computing has evolved from a theoretical concept to a practical paradigm for tackling complex optimization problems. Among the available approaches, quantum annealing [1] has gained attention for its ability to address combinatorial problems by leveraging quantum effects to explore solution spaces efficiently. The availability of commercial quantum annealers, such as those by D-Wave Systems, has further stimulated research on encoding real-world problems into suitable quantum formulations.

Quantum annealing operates on problems expressed as Quadratic Unconstrained Binary Optimization (QUBO) models or equivalent Ising Hamiltonians. The systematic transformation of combinatorial problems into QUBO form has been widely studied, notably by Lucas [2], while the foundational principles of quantum annealing were introduced in [3] and experimentally demonstrated in [4]. These developments have opened new perspectives for addressing NP-hard problems in operations research and related fields. Within this context, scheduling problems are of central importance due to their relevance across multiple application domains.

The Resource-Constrained Project Scheduling Problem (RCPSP), a well-known NP-hard problem [5], models the scheduling of activities under precedence and resource constraints, with the objective of

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minimizing the makespan. Its complexity arises from the interplay between precedence relations, limited renewable resources, and activity-specific requirements.

A key extension is the Multi-Mode RCPSP (MRCPSP), where each activity can be executed in one of several modes, each characterized by different durations and resource consumptions. While increasing modeling flexibility, this feature introduces an additional combinatorial dimension, making the problem significantly more challenging. Foundational works [6, 7, 8, 9] highlight both its practical relevance and computational difficulty. Despite substantial progress in classical solution methods, including exact algorithms and metaheuristics, the application of quantum annealing to the MRCPSP remains largely unexplored. Existing contributions have mainly focused on simpler scheduling problems, leaving a gap for more expressive and realistic models.

This paper addresses this gap by proposing a preliminary QUBO formulation of the base MRCPSP variant suitable for quantum annealing that includes both renewable and consumable resources, with the intention of outlining future extensions that enhance modeling realism, such as setup times between activities (MRCPSP/ST), generalized precedence constraints with time lags (MRCPSP/Max), and time-dependent resource costs and capacities. These features enable the representation of complex temporal and resource interactions, which are often difficult to handle with classical approaches but may benefit from the parallelism inherent in quantum annealing. Given the rapid advancement of quantum hardware, particularly in terms of qubit count and connectivity, exploring such formulations represents a promising research direction. This work thus provides an initial step toward leveraging quantum annealing for advanced scheduling problems and establishes a foundation for future developments at the intersection of quantum computing and operations research.

The remainder of the paper is organized as follows. Section 2 reviews the QUBO framework and quantum annealing principles. Section 3 formally introduces the MRCPSP. Section 4 presents the proposed QUBO formulation, and Section 5 discusses future research directions.

2. QUBO and quantum annealing

The Quadratic Unconstrained Binary Optimization (QUBO) problem has become a key framework in combinatorial optimization, largely due to its compatibility with quantum computing technologies. It provides a compact way to model discrete optimization problems as the minimization of a quadratic function over binary variables, and its relevance has grown with the rise of quantum annealing, which exploits quantum effects to solve such problems. Formally, a QUBO problem is expressed as

$$\min_{x \in \{0,1\}^n} x^T Q x, \tag{1}$$

where x is a vector of binary variables and Q encodes linear and pairwise interactions. Despite its simple structure, this formulation captures a broad class of NP-hard problems, including graph, routing, and scheduling tasks. A major strength of QUBO is its universality: constrained problems can be reformulated by embedding constraints into the objective function through penalty terms, yielding a unified unconstrained representation.

This formulation is especially important in quantum annealing, which seeks optimal solutions by evolving a quantum system toward its ground state. In this setting, problems are mapped to energy functions naturally represented by the Ising model. A straightforward transformation allows QUBO problems to be converted into equivalent Ising Hamiltonians by mapping binary variables to spin variables. This equivalence enables direct implementation on quantum annealing hardware, where QUBO coefficients correspond to interactions between qubits. While practical limitations remain, quantum annealing represents a promising approach for complex optimization. At the same time, QUBO formulations are also effectively addressed by classical techniques, including metaheuristics and specialized hardware, making them suitable for hybrid approaches. Building on this flexibility, this work focuses on the Multi-Mode Resource Constrained Project Scheduling Problem (MRCPSP). The richness of the MRCPSP makes it an ideal candidate for QUBO modeling, where the aspects of the problem can be encoded within a unified quadratic formulation. The following section presents a detailed QUBO formulation of the MRCPSP, suitable for both classical and quantum annealing methods.

3. MRCPSP definition

The Multi-Mode Resource-Constrained Project Scheduling Problem (MRCPSP) can be formally described as follows. We consider a set of n activities, indexed by $1, 2, \ldots, n$, and define the extended set $\mathcal{A} = \{0, 1, \ldots, n, n+1\}$, where the dummy activities 0 and $n+1$ represent, respectively, the start and the completion of the project. Each activity $i \in \mathcal{A}$ is associated with a decision variable S_i denoting its starting time; the dummy activities have zero duration, require no resources, and we fix $S_0 = 0$.

The project execution is subject to both renewable and non-renewable resource constraints. Let $\mathcal{R}$ denote the set of renewable resources, each with a limited capacity B_k available at every time instant, and let $\mathcal{N}$ denote the set of non-renewable resources, each with a total budget B_k over the entire project horizon. Each activity i can be performed in one of several execution modes $m \in \mathcal{M}_i$, where each mode is characterized by a processing time $p_{i,m}$ and by resource consumptions $b_{i,k,m}$ for all $k \in \mathcal{R} \cup \mathcal{N}$. The value $b_{i,k,m}$ denotes the amount of resource k required by activity i when it is executed in mode m. If $k \in \mathcal{R}$, this amount is used during the activity execution. If $k \in \mathcal{N}$, it is consumed over the project. Precedence relations are represented by a directed acyclic graph $(\mathcal{A}, \mathcal{E})$, where each arc $(i, j) \in \mathcal{E}$ enforces that activity i must be completed before activity j starts.

The objective is to determine both the start times S_i and the selected modes $m_i \in \mathcal{M}_i$ for all activities so as to minimize the overall project makespan S_{n+1}, while satisfying the following constraints. Precedence feasibility requires that

$$S_i + p_{i,m_i} \leq S_j \quad \forall (i,j) \in \mathcal{E},$$

ensuring that no activity starts before all its predecessors have finished. Renewable resource constraints enforce, at each time instant t and for each $k \in \mathcal{R}$,

$$\sum_{i \in \mathcal{A}_t} b_{i,k,m_i} \leq B_k,$$

where $\mathcal{A}_t = \{i \in \mathcal{A} : S_i \leq t < S_i + p_{i,m_i}\}$ denotes the set of activities in execution at time t, thus guaranteeing that resource usage never exceeds available capacity. Finally, non-renewable resources impose global budget constraints of the form

$$\sum_{i \in \mathcal{A}} b_{i,k,m_i} \leq B_k \quad \forall k \in \mathcal{N},$$

which limit the total consumption of such resources over the entire project.

4. Towards a QUBO encoding for the MRCPSP

The MILP formulation we start from is inspired by [10], and uses a single set of binary decision variables $y_{i,t,m}$, where $i \in \mathcal{A} \setminus \{0\}$, $t = 0, \ldots, H$ and $m \in \mathcal{M}_i$, with the intended meaning that $y_{i,t,m} = 1$ if and only if the activity i starts at time t and will be executed in the mode m. The time H, called horizon, is the maximum value for the end of the project, i.e., the start time of activity $n+1$. H can be computed as

$$H = \sum_{i=1}^{n} \max\{p_{i,m} : m \in \mathcal{M}_i\}.$$

The size of the $y_{i,t,m}$ set of variables is $(H+1) \sum_{i \in \mathcal{A}} |\mathcal{M}_i|$.

The makespan (i.e., the objective function to be minimized) is defined as follows:

$$\sum_{t=1}^{H} t y_{n+1,t,1} \tag{2}$$

and is subject to the following constraints:

1. $\sum\limits_{m \in \mathcal{M}_i} \sum\limits_{t=0}^{H-1} y_{i,t,m} = 1$ for each activity $i \in \mathcal{A}$
2. $\sum\limits_{m \in \mathcal{M}_i} (\sum\limits_{t=0}^{H-1} t y_{i,t,m} + p_{i,m}) \leq \sum\limits_{m' \in \mathcal{M}_j} \sum\limits_{t=1}^{H} t y_{j,t,m'}$, for each $(i,j) \in \mathcal{E}$
3. $\sum\limits_{i \in \mathcal{A}} \sum\limits_{m \in \mathcal{M}_i} b_{i,k,m} \sum\limits_{s=t-p_{i,m}+1}^{t} y_{i,s,m} \leq B_k$ for each $k \in \mathcal{R}$
4. $\sum\limits_{i \in \mathcal{A}} \sum\limits_{m \in \mathcal{M}_i} b_{i,k,m} \sum\limits_{t=0}^{H-1} y_{i,t,m} \leq B_k$ for each $k \in \mathcal{N}$

The previous constraints are translated in the following four penalty functions, leading us to the QUBO formulation:

$$P_1(y) = \sum_{i \in \mathcal{A}} \sum_{m \in \mathcal{M}_i} \left(\sum_{t=0}^{H} y_{i,t,m} - 1 \right)^2$$

$$P_2(y) = \sum_{(i,j) \in \mathcal{E}} \sum_{m \in \mathcal{M}_i} \sum_{m' \in \mathcal{M}_j} \sum_{t=0}^{H} \sum_{t'=0}^{t+p_{i,m}-1} y_{i,t,m} \cdot y_{j,t',m'}$$

$$P_3(y) = \sum_{t=1}^{H} \sum_{k \in \mathcal{R}} \left(\sum_{i \in \mathcal{A}} \sum_{m \in \mathcal{M}_i} \sum_{t'=t-p_{i,m}+1}^{t} (b_{i,k,m} \cdot y_{i,t',m}) + \sum_{j=0}^{\ell_k - 1} 2^j z_{t,k,j} - B_k \right)^2$$

$$P_4(y) = \sum_{k \in \mathcal{N}} \left(\sum_{i \in \mathcal{A}} \sum_{m \in \mathcal{M}_i} \sum_{t=1}^{H} (b_{i,k,m} \cdot y_{i,t,m}) + \sum_{j=0}^{\ell_k - 1} 2^j \zeta_{k,j} - B_k \right)^2$$

where ℓ_k is the number of bits needed to represent B_k in binary form.

To encode the inequality constraint 3. a second set of binary variables is used (the $z_{t,k,j}$ variables), with size $(H+1)\sum_{k \in \mathcal{R}} \ell_k$. Similarly, the constraint 4. requires another set of binary variables (the $\zeta_{k,j}$ variables), whose size is $\sum_{k \in \mathcal{N}} \ell_k$.

The overall objective function can therefore be expressed as:

$$f_{QUBO}(y) = \sum_{t=1}^{H} t y_{n+1,t,1} + \lambda_1 P_1(y) + \lambda_2 P_2(y) + \lambda_3 P_3(y) + \lambda_4 P_4(y) \tag{3}$$

Finally, the overall number of binary variables is $(H+1)(\sum_{i \in \mathcal{A}} |\mathcal{M}_i| + \sum_{k \in \mathcal{R}} \ell_k) + \sum_{k \in \mathcal{N}} \ell_k$.

Example. As an illustrative example[1], Figure 1 depicts a precedence graph in which we have 4 activities $\{1, 2, 3, 4\}$. The instance is characterized by one *renewable* resource with $B_1 = 3$, one *consumable* resource with $B_2 = 7$, and two modes. The related values p_{im} and b_{ikm} are displayed in Table 1. As can be easily observed, the time limit H of the example instance can be computed as $\sum_{i=1}^{n} \max\{p_{i,1}, p_{i,2}\}$, and its value is 13. The number of binary variables $y_{i,t,m}$ is $14 \times (2+2+2+2+1) = 126$, removing the useless variables $y_{0,t,m}$. The number of bits needed to represent B_1 is $\ell_1 = 2$, while B_2 can be represented with $\ell_2 = 3$ bits. Therefore, the number of binary variables $z_{t,k,j}$ is $14 \times 2 = 28$, while the number of variables $\zeta_{k,j}$ is 3. Therefore, the overall number of binary variables necessary to represent the instance is $126 + 28 + 3 = 157$.

[1]The example is taken from [11]

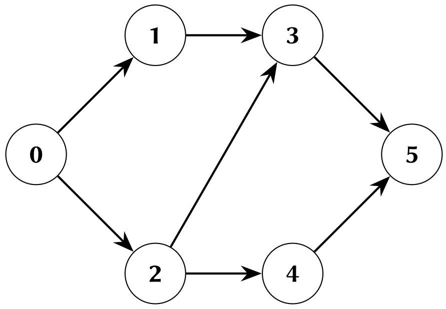

Figure 1: Precedence graph where the nodes represent the problem activities and the edges represent the precedence constraints between activity pairs. Activity 0 and activity 5 are dummy activities.

Table 1
$p_{i,m}$ and $b_{i,k,m}$ values for each activity and mode

	Mode 1			Mode 2		
activity	$p_{i,1}$	$b_{i,1,1}$	$b_{i,2,1}$	$p_{i,2}$	$b_{i,12,}$	$b_{i,2,2}$
1	3	1	2	1	3	2
2	4	1	1	2	2	1
3	2	2	2	3	1	1
4	2	3	2	3	1	1

5. Discussion and future work

This work represents a first step toward the systematic application of quantum annealing to complex project scheduling problems such as MRCPSP, and highlights several challenges toward practical adoption. A point that requires immediate attention in this line of research is the rapid growth in the number of binary variables (and thus qubits) required by QUBO formulations of the MRCPSP. Encoding activity start times, mode selections, precedence constraints, and resource feasibility leads to models whose size scales combinatorially with key parameters such as the number of activities, modes, time horizon, and available resources. In particular, time-indexed formulations yield a number of variables proportional to the product of activities and time periods, quickly becoming impractical for realistic instances. This is further exacerbated by penalty terms enforcing feasibility, which often produce dense quadratic interactions.

When implementing these models on quantum annealers, an additional overhead arises from the minor embedding process, which maps logical variables onto hardware with limited connectivity. Each logical variable may require a chain of physical qubits, significantly increasing the effective qubit count and limiting solvable instance sizes. Consequently, an important research direction is the analysis of qubit growth as a function of problem characteristics, with the aim of identifying more scalable encodings. Promising approaches include alternative formulations (e.g., event-based or hybrid models), constraint reduction, and decomposition strategies.

A second research direction concerns extending the framework to gate-based quantum architectures. Variational algorithms such as the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA) provide alternative solution paradigms. Their application to the MRCPSP requires designing suitable cost Hamiltonians and problem-specific ansätze that capture scheduling constraints. Future work will explore the relative strengths of annealing and gate-based methods, as well as hybrid quantum-classical approaches for improved scalability and robustness. Addressing these challenges will be crucial for enabling the practical use of quantum technologies in project scheduling and bridging the gap between theoretical formulations and real-world applications.

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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