Well-posedness of boundary-value problems in information security of cyber-physical systems for the bioeconomy*

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Abstract

The article studies a complex cyber-biophysical system and proves the correctness of boundary problems for its information security in bioeconomics (biotechnology as one of the areas of bioeconomics). It is proven that the hardware implementation of biotechnology belongs to one of the types of cyber-biophysical systems, the modeling of which is based on the methods: Formal Concept Analysis (FCA) and Business Process Modeling Notation (BPMN 2.0), which made it possible to substantiate the conditions for the correctness of boundary problems. The proposed algorithm for improving the accuracy and speed of complex cyber-biophysical systems in conditions of increasing detail of the modeled system and taking into account the influence of changing external factors on the system. This algorithm is based on methods for searching and substantiating the conditions for the correctness of boundary problems with differential equations that describe the state of modeled cyber-physical systems. The universal nature of the proposed algorithm allows us to assert the feasibility of applying the results of this article to improve the effectiveness of information security not only for the cyber-physical system proposed in this article, but also for other cyber-physical systems.

Keywords

cyber-biophysical system, information security, bioeconomics, modeling of biotechnological systems, boundary value problems

1. Introduction

As is well known, bioeconomics combines two components: economic and environmental, in terms of the use of biological resources for economic growth, and is based on a bio-approach and sustainability.

Foreign and domestic scientists have paid and continue to pay attention to the study of bioeconomics as a science: H. Daly [1], I. Mateescu, S. Popescu, L. Paun, et al. [2], M.P. Talavira [3], V.V. Bugaychuk, I.F. Grabchuk [4], and others. They use different approaches to defining the concept of «bioeconomy» which allows them to take into account the specifics of areas that are becoming increasingly diverse and narrow. Previously, three areas were distinguished main areas, they are currently narrow in scope and simultaneously penetrate all spheres of the economy. Historically, the main areas have been as follows:

- biotechnology, which means the use of technical and technological innovations in biological processes for the purpose of commercialization and profit, while maintaining the ecological state and without harming the environment;
- the use of biological resources instead of artificially created or fossil resources;
- the formation of bioecological awareness among the population and, in particular, among workers in the manufacturing and processing sectors.

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Highly developed countries have adopted relevant bioeconomy development programs at the national level. For example, in 2011, Germany adopted the National Research Strategy: Our Route Towards a Biobased Economy, the US adopted the National Bioeconomy Blueprint (2012), and Canada adopted The Canadian Blueprint: Beyond Moose and Mountains (2009). Japan has perhaps the largest biotechnology market, while China has seen rapid growth since 2011 (15% per year). The projected total output of China's industrial biotechnology sector for 2022 was estimated at $1.5 trillion in 2019. Ukraine ranks lowest in the Global Biotechnology Innovation Score (53rd out of 54) [5]. The latter is used to assess the level of biotechnology development and consists of a number of indicators, including: indicators of staffing in the biotechnology sector, indicators of research funding by the state and venture companies, the level of administrative support for biotechnology research and socio-economic stability in society, biotechnology productivity indicators, and others. The productivity of biotechnology not only affects the macroeconomic indicators of a country and its ranking, but also demonstrates the value of the end result of biotechnology implementation, which for companies means profit, increased competitiveness, and diversification of production and sales activities.

To increase the efficiency of biotechnological processes, it is necessary to reduce resource consumption and defective output, which depends on the accuracy of project planning and implementation, for example, the accuracy of energy consumption calculations. This is achieved by increasing the accuracy of the construction and implementation of mathematical models that describe the state of the modeled systems. However, increasing the detail of mathematical models and the number of external factors taken into account at the mathematical modeling stage that affect the modeled biotechnological system will increase the complexity of constructing mathematical models. Such factors are, of course, information security factors, both for the project as a whole and for its individual components. And then, to calculate and optimize the objective function and control parameters of the equipment, it is necessary either not to take into account the specifics of the equipment and the structure of the biological object when constructing boundary problems (to neglect external factors affecting the biotechnological system, including the probability of information threats), or propose fundamentally new algorithms that will guarantee the existence and uniqueness of the solution to the mathematical model based on differential equations without taking into account the constraints on the parameters of the biological object (its density, thermal conductivity, and heat capacity).

Such mathematical models are used to solve various non-biotechnological problems. Works [6–17] present examples of mathematical modeling and optimization of control parameters of complex systems. Publication [6] provides mathematical modeling of stochastic signals at random moments in time. The authors obtained characteristic and moment functions for the specified stochastic signals: cardiographic, electricity and gas consumption processes, and radar signals. In [7], a theoretical study of the features of constructing and solving computer-aided geometric design problems for optimizing the parameters of a large number of complex systems is carried out. Methods for cyber protection of information technologies in modern digital society have been developed [8]. Applied problems of optimization of individual hydraulic systems have been solved [9]. With the aim of ensuring the reliability of the formation and restoration of military structures depending on the intensity of losses in military units, an algorithm for the dynamics of random processes has been constructed in article [10]. This allowed its authors to calculate the combat capability of individual military units.

For rational resource allocation under conditions of uncertainty in the initial data in individual technical systems, mathematical models and numerical methods for optimizing such systems are constructed in article [11]. These algorithms consist of well-known and proven numerical methods for searching for optimal control parameters of modeled complex systems. The authors of articles [12, 13] have approximated the solutions of nonlinear control systems and adapted approximate calculation methods for approximating systems on a computer. In article [14], evolutionary pseudo-differential operators are used to search for and justify the conditions for the correctness of boundary value problems in spaces of generalized, bounded functions. The controllability conditions for boundary value problems with systems of heat conduction equations in separate functional spaces are determined [15, 16]. In article [17], conditions for decomposing functions of several variables into power series are obtained.2. Formulating the problem in general terms.

Increasing the detail of modeled biotechnological systems and the number of external factors taken into account in the modeled biotechnological system increases the complexity of constructing and implementing boundary value problems with differential equations. The internal structure of a biological object and the presence of microscopic irregularities on its outer shell do not allow us to guarantee the correctness of boundary value problems describing the state of a biological object. The existence and uniqueness of the solution to boundary value problems with differential equations can only be guaranteed in the case of a simple (segment, sphere, ball) spatial form of the initial domain of the biological object under study. In this case, the correctness of the boundary value problem is justified by methods from the traditional theory of existence and uniqueness of solutions (correctness) from the course of differential equations. In the case of a complex spatial form of the initial domain of the bio-object, due to the consideration of its structure, technical features of the sources of influence, and external factors affecting the specified biotechnological system when constructing boundary problems, it is not possible to guarantee the existence and uniqueness of the solution for boundary problems with differential equations.

This means that increasing the number of features of the modeled system and factors affecting the modeled system when constructing boundary value problems will increase the complexity of differential equations and boundary value problems. And without averaging the parameters of the modeled systems, it is impossible to guarantee the correctness of such boundary value problems. To prove the correctness of computational and optimization mathematical models, the authors of the article suggest using methods from the theory of pseudodifferential operators over the space of generalized functions of slow power growth.

It is fair to say that if the mathematical model is not correct, then the class of solutions can be narrowed down to compact sets, in which case the problem becomes stable to small changes in the initial data and the quasi-solution method or quasi-inverse method can be used. A quasi-solution of a non-homogeneous equation is understood to be a function from a compact subset of the class of functions (in which we are looking for a solution) that minimizes the deviation of the image of this function from the given right-hand side. The quasi-inverse method consists in replacing the incorrect problem with a close perturbed problem that is correct, and then letting the small parameter tend to zero.

If the compactness condition is not satisfied, then one can use the regularization method, which allows obtaining an approximate solution that is stable to small changes in the initial data.

To implement and study the computational mathematical model describing the state of a biological object under the influence of a laser beam, the Fourier method of separated variables and variational methods are used. The essence of the Fourier method is that the solution to the boundary value problem is sought in the form of a series of products of functions depending on time and functions of spatial variables. Moreover, the functions depending on spatial variables are the eigenfunctions of the boundary value problem for the Helmholtz equation. To justify the Fourier method and establish the correctness of the boundary value problem, the maximum principle is used, the essence of which is that the solution of a homogeneous boundary value problem reaches its maximum at the boundary.

If an exact analytical solution to the boundary value problem cannot be found, approximate numerical methods are used to find solutions. Among the approximate methods, the most widely used are difference methods. The essence of difference methods is that the initial domain of independent variables is replaced by a discrete set of points — a grid — and the derivatives in the equation are approximated by difference relations. As a result of this procedure, the initial problem is matched with a system of a finite number of algebraic equations, called a difference scheme. The solution of the difference scheme is used as an approximate solution to the boundary value problem. The accuracy of the approximation depends on the approximation method and the frequency (step) of the grid. When using difference methods to solve the boundary value problem of the heat conduction equation, first-order accuracy is ensured for the time coordinate and second-order accuracy for the heat penetration depth coordinate in the biomaterial.

When constructing a difference scheme, an important issue is the correctness of difference schemes. It should be noted that a difference scheme is called correct if, for any boundary conditions, there is a unique solution that is stable to small changes in the initial data.

There are direct and iterative methods for solving difference equations. Direct methods provide an exact solution to a difference equation for a finite number of arithmetic operations. Direct methods include the run method, the fast Fourier transform method, the summation method, and other methods. The most effective of these methods is the fast Fourier transform method. Iterative methods include the Richardson method, the triangular method, and the iterative method of variable directions. Iterative methods are more versatile and easier to implement, and therefore have become widely used in solving difference equations.

The research conducted in this article will improve the accuracy of calculating the energy costs incurred during the biotechnological process of exposing a biological object to a radiation source. This means an increase in the quality of the process due to the increased detail of the modeled systems and the consideration of external factors affecting these systems while ensuring, directly, the search procedure and step-by-step traversal of local extrema of the objective function. This increases the accuracy of the calculated global extremum of the objective function in terms of technical parameters. According to the authors of this article, the presented research results should be applied to solve applied biotechnological problems of improving the efficiency of methods for forecasting and controlling biomaterial losses and saving energy costs with technical devices. However, when the object of study and the sources of load on the biotechnological system change, the boundary problems with systems of differential equations will change, and, consequently, the methods for solving boundary problems will also change. Nevertheless, the algorithm proposed in this article can be left without significant changes or additions.

2. Biotechnology as a cyber-biophysical system

The hardware implementation of biotechnology belongs to one of the types of cyber-physical systems (CPS), which we propose to clarify as a cyber-biophysical system. We believe that this term best reflects its characteristics, especially since we have already dealt with certain issues [18–20]. We propose a definition of a cyber-biophysical system (CBPS) based on the definition of an earlier term, taking into account its limitations and specifics. Thus, a cyber-biophysical system is a complex system consisting of computer technology or other computing devices; network sensors; physical and biological objects used for data processing and analysis, decision-making, and other actions in real time in a physical and biological environment. In such a system, computers typically control processes and interact with physical and biological processes, providing integrated communication between the cybernetic and real environments. As Ed. A. Lee notes, such control and management of processes occurs through feedback loops, when physical processes (and, we add, biological processes as well) influence the calculation, processing, and analysis of data [21]. On the other hand, using the wording [22], a cyber-biophysical system can be considered an information and technological system or concept that involves the integration of computer technology into physical, biological, and human processes. Its distinctive feature is the distribution of the information system across the physical system and the synergistic connection with its constituent elements. However, the use of different terms in scientific and research publications does not exclude the interaction of system components, technical mutual adjustment, and joint adaptation to changes in physical and biological processes. Artificial intelligence systems, quantum computing, nanotechnology, autonomous robots, and information security systems are considered to be striking examples of cyber-physical systems with a biological object [23]. Cyber-physical systems affect not only individual types of work, but also economic sectors and countries. The relevance of information security issues in cyber-physical systems requires the search for non-standard solutions that can be implemented at the theoretical and applied levels.

2.1. Description and characteristics of the biotechnology under study

The biotechnology under study is a CBF system consisting of:

- a biological object (cattle embryo);
- a laser;
- an electron-optical system for focusing the laser beam on the biological object;
- an information unit for determining the maximum control parameters;
- an information unit for comparing the obtained parameter values with their permissible values;
- an information unit for determining the permissible values of control parameters;

• software for determining laser parameters;
• a system of executive mechanisms;
• a grid model system with variable structure and parameters;
• system for setting grid model parameters;
• information and communication registration system;
• system for entering output information [24].

The described biotechnology uses a continuous-wave argon laser with a radiation wavelength of 515 nm or 488 nm and adjustable power in the range of 0–2 W. The electron-optical system is equipped with an acousto-optic modulator that forms a laser beam in the form of pulses with a duration of 1 ms or less; the pulse energy is 0.002 J; the beam divergence is 0.9 mrad. The system of executive mechanisms based on the AVR ATmegal6(L) microcontroller and the corresponding assembler program provides laser power in the range of 0.5–2 W. The use of a bovine embryo placed in a cannula with a nutrient solution, where a stable temperature of 37^{0}C is maintained, is proposed as a bio-object. A cattle embryo is used as a bio-object, which is placed in a cannula with a nutrient solution, where a stable temperature of 37^{0}C is maintained. The size of the embryo corresponds to the number of elastomers 18, 36, or 64. In the process of implementing this biotechnology, the embryo is divided by laser using a focused laser beam into two or more viable parts for transplantation to recipient cows. More detailed characteristics and results of the implementation of this biotechnology are presented in the authors' previous scientific works [18–20, 24].

For practical use of this system in production conditions of breeding farms or specialized veterinary clinics for the creation of a bank of embryos of cattle with high productivity (live weight, average annual milk yield), it is necessary to determine the parameters (laser energy, permissible temperature of embryo cell heating) of the CBF system, or more precisely, its physical processes included in the information block for determining the maximum control parameters of the cyber-physical system.

2.2. Modeling of a cyber-biophysical system based on Formal Concept Analysis

One such method of finding solutions is to model biotechnology as a cyber-biophysical system and perform the appropriate mathematical calculations to find acceptable values and parametric maxima for physical and biological processes. The selection of optimal methods for modeling systems through their analysis and application restrictions has settled on the following two, which, in our opinion, are optimal. This method turned out to be the modeling of a cyber-biophysical system in the form of templates based on Formal Concept Analysis (FCA) [25]. This model usually consists of several systems interconnected by information transmission channels. In the context of the previous definition of a cyber-biophysical system, its subsystems (physical and biological) are capable of functioning independently and providing a synergistic result. We propose further development of the mathematical model of such a system [26] in terms of supplementing it with elements of the biological system and corresponding connections with other subsystems.

Therefore, the mathematical expression (formula) of such a model should be as follows:

$$\{P, B, C, CBPS, R^P, R^C, R^B\} \tag{1}$$

where P is a set of physical components (subsystems) that are related by a relationship R^P; C is a set of cyber components (subsystems) that are related by a relationship R^C; B is a set of biological components (subsystems) that are related by R^B; $CBPS$ is a cyber-biophysical system consisting of cybernetic, biological, and physical subsystems.

2.3. Modeling of a cyber-biophysical system based on Business Process Modeling Notation (BPMN 2.0)

In our opinion, the Business Process Modeling Notation (BPMN 2.0) method, which is used to describe and model cyber-physical systems taking into account their specifics, is interesting [27]. The basis of such a model is the internal processes of a cyber system based on logical and causal relationships, despite the simplicity and banality of its interpretation. As noted by the authors of this fundamental scientific work [27], the BPMN 2.0 modeling method is considered the standard for illustrating any business processes, but it has a number of shortcomings when it comes to technological or biotechnological processes. Its use in biotechnological processes requires different approaches, which is explained by the participation and interrelationships of diverse subsystems: information or cyber systems, biological systems involving humans and biological objects, and physical systems, which is typical for cyber-biophysical systems. The authors emphasize the special importance of BPMN 2.0, provided that the roles of each subsystem are clearly defined and modeled, which allows for model variability and application to solve numerous tasks in systems with complex and sensitive processes. The development of such a model allows solving the problem of ensuring the information security of a cyber-biophysical system and finding the only correct solution to boundary problems for this purpose.

To this end, we have constructed a meta-model of the biotechnology under study as a cyber-biophysical system (Figure 1), which consists of five components: a biological system (bio-object), a physical system, a quality control system, a data storage and processing system, and a database of CBFS performance results.

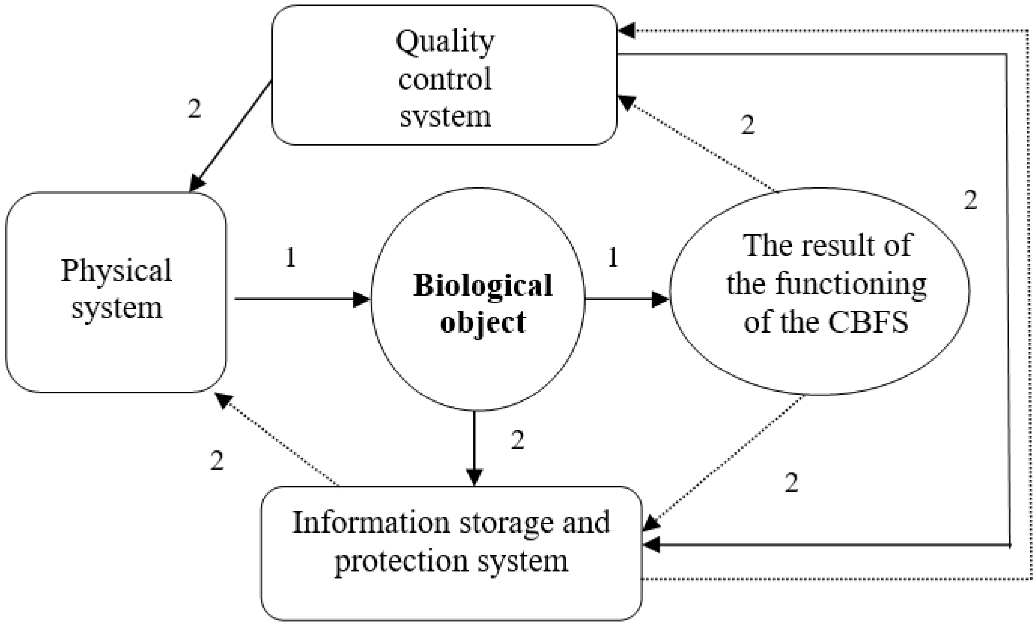

Figure 1: Meta-model of a cyber-biophysical system (developed by the authors)

Figure 1 also shows the intercomponent interaction of the CBFS or the interrelationships between the elements of the system. Solid lines indicate the directions of direct links from the control systems to the controlled ones, while dotted lines indicate the directions of feedback links for adjusting the received information and technical and biological parameters, the viability of the biological object and its structure. Number 1 shows physical and biological connections, and number 2 shows cyber connections.

Figure 2 shows the implementation of the model of the studied biotechnology as a cyber-biophysical system in BPMN4 terminology.

The BPMN2.0 model consists of four parts, between which physical and cyber processes occur that require information security within the processes and when transitioning from one to another. Information security points are represented by a dotted line with two dots. Please note that the main problem of such a cyber-biophysical system is ensuring the interaction of the quality control system and the information storage and protection system with the systems of physical and biological processes. Of course, information systems are constantly becoming more complex, and even the smallest information leak has negative consequences for the cyber-biophysical system and each of its components [28]. Therefore, further research by the authors of the article will focus on systems aimed at ensuring information balance within the CBPF and protection against external information security risks.

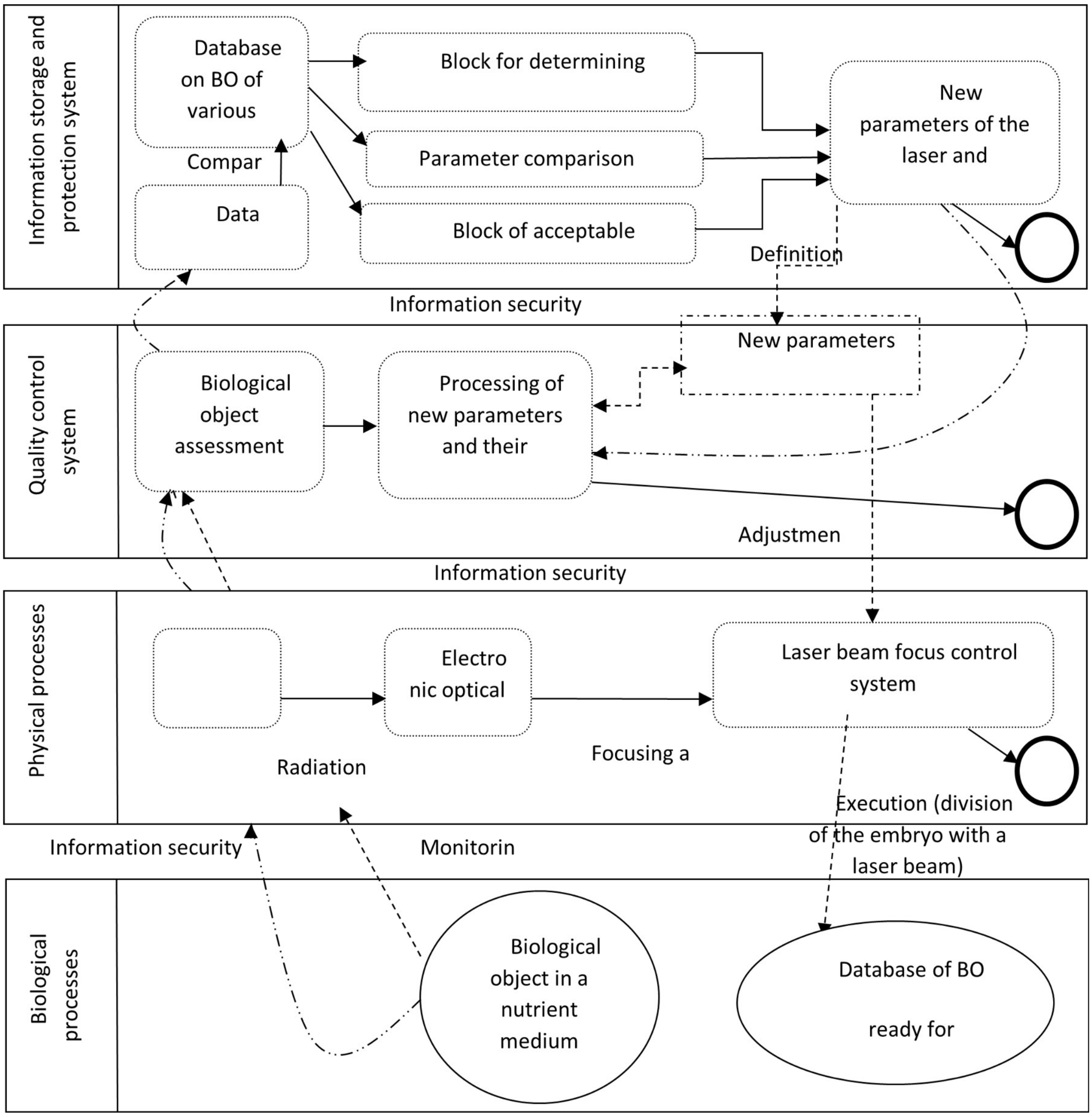

Figure 2: BPMN4 notation model of a cyber-biophysical system (developed by the authors)

Any changes in the external environment in relation to the CBFS: temperature conditions for biological processes, personnel errors in setting up the system for operation, software and device configuration failures, basic personnel negligence, etc. Therefore, when in an active-passive or active state, the cyber-biophysical system controls its environment. In the first case, changes in the behavior of the system are planned in order to adjust it to predefined parameters. In the active state, other systems can be connected that will adapt to the existing conditions and launch the appropriate physical processes to, for example, reduce the temperature of the environment with a bio-object to a reasonable level.

The types of information security threats to the CBFS we are studying may include attempts to destroy, gain unauthorized access to, and steal information from the data processing system [28]. In our opinion, information security threats are most likely to arise in the form of false obstacles and barriers to information flow. We have previously studied the impact of information security on management projects and demonstrated the universality of approaches to ensuring it for any system, including cyber-biophysical ones [30–33]. For the system we are studying, it is particularly important to ensure information security in all its components, from the assessment of the bio-object to the determination of the parameters of the CBFS and the laser beam control system. A direct attack on the system with the aim of gaining commercial access to its database is also likely. That is why optimizing the parameters of the physical CBFS system by substantiating the correctness of boundary tasks, i.e., determining the factors influencing the system, including information security, is a logical continuation of our research for the future.

3. Justification of the conditions for the correctness of boundary problems for a cyber-biophysical system

3.1. Description of initial conditions

Thus, the biological object (embryo) is immersed in a cannula with a nutrient medium at a temperature of 37 °C. At the beginning of the laser action, the temperature of the laser beam is 100 °C, and at the end, it is 37 °C. The permissible (maximum) values of the controlled technical parameters of the cyber-physical system (embryo under laser exposure) are taken from an expert assessment of the permissible parameters of laser emitters. We also note that a thermophysical analysis of this cyber-biophysical system leads to the conclusion that in order to increase the accuracy of calculating the percentage of embryo damage and optimize the energy consumption of laser emitters, it is necessary to increase the number of external factors affecting the cyber-physical system taken into account when constructing boundary value problems with differential equations, as well as to increase the level of detail of the system. Of course, this will complicate the boundary value problems, as the number of differential equations will increase and the right-hand side of the differential equations will become more complex. In addition, to control the temperature distribution in the embryo layers, it is advisable, along with the boundary conditions at the beginning at the end of the laser action and the boundary conditions of the specific heat flux at the boundary of the outer shell (pelucid zone) of the embryo, it is advisable to introduce the conditions of thermal contact in the layers of the embryo and the equality of continuity in the layers of the embryo into the boundary problems [34–36].

That is, an algorithm is needed, the implementation of which will ensure the correctness of such already complicated boundary value problems with differential equations. According to the authors of this article, this algorithm should be based on well-known and proven computational methods, the combination of which will create a computational structure [37–39].

By applying methods from distribution theory and evaluation to the fundamental function of solutions in this article, conditions for the correctness of boundary value problems for differential equations with perturbations in their right-hand side are obtained. These perturbations correspond to external factors affecting the modeled cyber-physical system that are taken into account when constructing boundary value problems. It is shown that the condition of parabolicity of the resolving function of solutions is a necessary and sufficient condition for the correctness of the boundary value problems studied in this article. The results obtained are useful for calculating and optimizing the energy consumption of other biotechnological and technical systems, and the algorithm proposed in this article can be used to check whether there will be a unique solution in boundary value problems with differential equations that take into account external factors affecting the modeled systems, the internal structure of the studied bio-object, and the technical characteristics of the emitters. In other words, this will increase the accuracy of calculation and optimization of parameters not only for the cyber-physical system studied in this article, but also for other complex systems [41].

3.2. Solving the task at hand

Let us consider homogeneous and inhomogeneous boundary value problems, respectively:

$$\begin{gathered}
\frac{\partial u(x,t)}{\partial t}=A_1\left(\frac{\partial}{i\partial x}\right)u(x,t), t\in[0;t_1], x\in R^m;\\
\frac{\partial u(x,t)}{\partial t}=A_2\left(\frac{\partial}{i\partial x}\right)u(x,t), t\in[t_1;t_2], x\in R^m;\\
\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\
\frac{\partial u(x,t)}{\partial t}=A_n\left(\frac{\partial}{i\partial x}\right)u(x,t), t\in[t_{N-1};T], x\in R^m.
\end{gathered} \tag{2}$$

With boundary conditions

$$B_0\left(\frac{\partial}{i\partial x}\right)u(x,0)+...+B_n\left(\frac{\partial}{i\partial x}\right)u(x,T)=\varphi(x) \tag{3}$$

and

$$\begin{aligned}
&\frac{\partial u(x,t)}{\partial t}=A_1\left(\frac{\partial}{i\partial x}\right)u(x,t)+f(x,t), t\in[0;t_1];\\
&\frac{\partial u(x,t)}{\partial t}=A_2\left(\frac{\partial}{i\partial x}\right)u(x,t)+f(x,t), t\in[t_1;t_2];\\
&\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\
&\frac{\partial u(x,t)}{\partial t}=A_n\left(\frac{\partial}{i\partial x}\right)u(x,t)+f(x,t), t\in[t_{N-1};T].
\end{aligned} \tag{4}$$

with boundary conditions

$$B_0\left(\frac{\partial}{i\partial x}\right)u(x,0)+...+B_n\left(\frac{\partial}{i\partial x}\right)u(x,T)=0, \tag{5}$$

where $A_k\left(\frac{\partial}{i\partial x}\right)$ and $B_k\left(\frac{\partial}{i\partial x}\right)$ are pseudodifferential operators with symbols from the space of infinitely differentiable functions of power growth $C_{-\infty}^{\infty}$; $u(x,t)$ is a solution function, continuous in t throughout the layer and differentiable at $t \neq t_k$.

We will conduct our analysis in Sobolev-Slobodetsky spaces H_l^s, as well as in their projective limit $S=\bigcap_{s,l} H_l^s$ the space of generalized functions.

To determine the conditions for the correctness of boundary problems (2)–(3) and (4)–(5), we need to define correctly solvable homogeneous and inhomogeneous boundary problems.

Let us apply the Fourier transform (with respect to spatial variables) to the equations from the homogeneous boundary value problem (2)–(3). We obtain the dual boundary value problem

$$\begin{cases}
\frac{\partial \widetilde{u}(s,t)}{\partial t}=A_1(s)\widetilde{u}(s,t),\ \ t\in[0;t_1],\ \ s\in R^m;\\
\frac{\partial \widetilde{u}(s,t)}{\partial t}=A_2(s)\widetilde{u}(s,t),\ \ t\in[t_1;t_2],\ \ s\in R^m;\\
\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\
\frac{\partial \widetilde{u}(s,t)}{\partial t}=A_n(s)\widetilde{u}(s,t),\ \ t\in[t_{N-1};T],\ \ s\in R^m
\end{cases} \tag{6}$$

with boundary conditions

$$B_0(s)\widetilde{u}(s,0)+...+B_n(s)\widetilde{u}(s,T)=\varphi(s) \tag{7}$$

Let us apply the Fourier transform to the equations from the inhomogeneous boundary value problem (4)–(5):

$$\begin{cases}
\frac{\partial \widetilde{u}(s,t)}{\partial t}=A_1(s)\widetilde{u}(s,t)+f(s,t),\ \ s\in R^m;\\
\frac{\partial \widetilde{u}(s,t)}{\partial t}=A_2(s)\widetilde{u}(s,t)+f(s,t),\ \ s\in R^m;\\
\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\
\frac{\partial \widetilde{u}(s,t)}{\partial t}=A_n(s)\widetilde{u}(s,t)+f(s,t),\ \ s\in R^m
\end{cases} \tag{8}$$

with boundary conditions

$$B_0(s)\tilde{u}(s,0)+...+B_n(s)\tilde{u}(s,T)=0, \quad (9)$$

where $A_k(s)$, $B_k(s)$ are symbols of corresponding pseudo-differential operators.

We will seek a solution to the homogeneous boundary value problem (5)–(6) in the form:

$$Q(s,t)=\begin{cases} \exp t\cdot A_1(s)\cdot\varphi_1(s), & t\in[0;t_1]; \\ \exp t\cdot A_2(s)\cdot\varphi_2(s), & t\in[t_1;t_2]; \\ \cdots & \cdots \\ \exp(t-t_{n-1})\cdot A_n(s)\cdot\tilde{\varphi}_n(s), & t\in[t_{n-1};T]. \end{cases} \quad (10)$$

From the continuity condition, we obtain the solution:

$$\varphi_k(s)=\exp((t_1,A_1,(s))+...+(t_{k-1}-t_{k-2})A_{k-1}(s))\varphi(s). \quad (11)$$

If we substitute solution (11) into condition (7), we obtain

$$B_0(s)\varphi_1+...+\varphi_1 B_n(s)\exp(t_1 A_1(s))+...+(T-t_{n-1})A_n(s)=\varphi. \quad (12)$$

If the function

$$\Delta(s)=B_0(s)\varphi_1 B_n(s)\exp(t_1 A_1(s))+...+(T-t_{n-1})A_n(s)\neq 0 \quad (13)$$

Then $\varphi_1(s)=\varphi(s)/\Delta(s)$, and the solution to the boundary value problem (6)–(7) is as follows:

$$\tilde{u}(s,t)=Q(s,t)\cdot\varphi(s,t) \quad (14)$$

where the resolving function for the homogeneous boundary value problem (1)–(2) is:

$$Q(s,t)=\begin{cases} \exp t A_1(s)/\Delta(s); \\ \exp t A_2(s)/\Delta(s); \\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots \\ \exp((t-t_{n-1})A_n(s)+\ldots+t_1 A_1(s))/\Delta(s). \end{cases} \quad (15)$$

Function (15) is called the resolving function of problem (6)–(7).

Thus, the following statement has been proven.

Statement 1. Problem (2)–(3) is correctly solvable from space S to space $C([0,T],S)$ if and only if the solving function, i. e., $Q(s,t)\in C_{-\infty}^{\infty}$, i.e., when for any k, there exist $p_k, C_k>0$ such that $\left|D^k Q(s,t)\right|\le C_k\left(1+|s|\right)^{p_k}$.

To justify the correct solvability of homogeneous (2)–(3) and inhomogeneous (4)–(5) boundary value problems, we use the definition of a homogeneous parabolic boundary value problem:

Problem (2)–(3) is called parabolic if the solution function satisfies the following estimate:

$$|Q(s,t)|\leqslant C\ \exp\left(-bp(t)\cdot|s|^h\right) \quad (16)$$

with some $b>0, h>0$. Here $\rho(t)=\min\limits_{0\le k\le n}|t-t_k|$.

Let us now consider the boundary value problem for the inhomogeneous equation (4)–(5). To do this, we need to define the Green's function for the dual boundary value problem (8)–(9).

Note that if there exists a resolving function in problem (6)–(7) $Q(s;t)$, then in the task (8)–(9) there exists a Green's function, and for this Green's function the following system holds:

$$G(t,\tau,s)=\begin{cases} \sum\limits_{k,t_k>\tau} B_k(s)Q(s,t-\tau+t_k), & t\le\tau; \\ \sum\limits_{k,t_k\le\tau} B_k(s)Q(s,t-\tau+t_k), & t>\tau. \end{cases} \quad (17)$$

Next, we will find out how the correct solvability of the homogeneous boundary value problem

(2)–(3) from the space *S* of generalized functions to the space $C([0,T],S)$ with correct solvability of the inhomogeneous boundary value problem (4)–(5) in space $C([0,T],S)$. As shown in [20] earlier, the correct solvability of the homogeneous boundary value problem (2)–(3) from space to space *S* $C([0,T],S)$ determines the correct solvability of the inhomogeneous boundary value problem (4)–(5) in space $C([0,T],S)$.

The condition of parabolicity of the homogeneous boundary value problem (2)–(3) is necessary for correct solvability from space $C([0;T],H^{s})$ into space $C([0;T],H^{s-q})$ perturbed equation with homogeneous boundary conditions for sufficiently small perturbations.

Thus, the results obtained above determine the conditions for correct solvability for pseudodifferential equations with symbols varying in *t*. We proved the correctness of boundary value problems with differential equations that take into account external factors affecting the system.

4. Conclusions

The article studies a complex cyber-biophysical system, which is generally represented by an embryo under laser exposure, and proves the correctness of boundary problems for its information security in bioeconomics (biotechnology as one of the areas of bioeconomics). It has been proven that the hardware implementation of biotechnology belongs to one of the types of cyber-biophysical systems, the modeling of which is based on the methods: Formal Concept Analysis (FCA) and Business Process Modeling Notation (BPMN 2.0). The authors show that to increase the level of information security of such a system, it is not enough to take into account only the modes of laser action and the technical characteristics of the hardware of thermal action sources, but it is also advisable to take into account the characteristics of the biological object (three-layer internal structure, micro-irregularities on the outer shell, laser beam movement prohibition routes on the outer shell of the embryo).

The cyber-physical system studied by the authors belongs to systems with distributed parameters. From a computational point of view, the authors had to solve a special type of nonlinear mathematical programming problem. Such characteristics of the modeled system affect the methods used to ensure the information security of cyber-physical processes in the specified system. It is noted that the structure of the embryo, the technical characteristics of laser emitters, and the changing conditions of external influence on the system, including information security factors, increase the complexity of boundary problems with differential equations describing their state. That is, the complication of boundary problems affects the possibility of solutions in general or the existence of an infinite number of solutions. Therefore, we consider the proposed list of conditions for the correctness of the constructed boundary problems to be justified for the mathematical models we have developed with an unambiguous description of the processes in the modeled systems.

The article proposes an algorithm for improving the accuracy and performance of complex cyber-biophysical systems in conditions of increasing detail of the modeled system and taking into account the influence of changing external factors on the system. This algorithm is based on methods for searching and substantiating the conditions for the correctness of boundary value problems with differential equations that describe the state of modeled cyber-physical systems. The selected methods from the theory of pseudo-differential operators over the space of generalized functions made it possible to study the solutions of homogeneous and inhomogeneous boundary value problems for systems of differential equations and to prove that the parabolicity condition of the allowing function of solutions is a necessary and sufficient condition that guarantees the existence of a unique solution for cyber-physical systems from this article. The proposed algorithm is universal in nature, which allows us to assert the feasibility of applying the results of this article to improve the effectiveness of information security not only for the proposed cyber-physical system in bioeconomics, but also for other cyber-physical systems.

Declaration on Generative AI

While preparing this work, the authors used the AI programs Grammarly Pro to correct text grammar and Strike Plagiarism to search for possible plagiarism. After using this tool, the authors reviewed and edited the content as needed and took full responsibility for the publication's content.

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