

Systematic review on quantum algorithms for solving tasks of linear algebra and statistics

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Abstract

Quantum algorithms represent a key direction in the development of modern computing technologies, enabling significant computational advantages over classical methods by offering polynomial or even exponential speedups for certain classes of problems, although their practical implementation remains constrained by current limitations of quantum hardware. However, current research is focused not only on developing new quantum algorithms but also on optimizing existing approaches, creating hybrid quantum-classical methods, and adapting them to real-world application conditions. This paper presents an analysis of quantum algorithms for solving linear algebra problems and provides a multicriteria analysis of these algorithms.

Keywords

quantum algorithms, computing, security, quantum computing, technologies, hybrid quantum-classical algorithms

1. Introduction

Quantum algorithms [1, 2, 3, 4, 5] represent one of the key areas of development in modern computing technologies, based on the principles of quantum mechanics, such as superposition, entanglement, and interference. Unlike classical algorithms, they allow computations to be performed in a multidimensional state space, opening up possibilities for significantly accelerating the solution of certain classes of problems. In particular, for certain problems related to factorization, search, or the processing of large datasets, quantum algorithms demonstrate polynomial or even exponential speedups compared to classical approaches. At the same time, their practical implementation depends on the development of quantum hardware and overcoming limitations related to noise, decoherence, and the limited number of qubits.

Methods focused on solving linear algebra problems occupy a special place among quantum algorithms, as such problems underlie most modern information systems, including machine learning, signal processing, statistics, and optimization. Quantum approaches allow working with vectors and matrices in the form of quantum states, which potentially provides a significant reduction in computational complexity for operations on high-dimensional spaces. Algorithms such as HHL, QSVT, and quantum spectral analysis methods enable the efficient performance of operations for solving systems of equations, calculating eigenvalues, and singular value decompositions, which are critically important for data analysis and model building.

At the same time, the efficiency of quantum algorithms for linear algebra problems depends largely on the method of data representation and the ability to quickly load it into the quantum system. In many cases, the use of specialized structures, such as quantum random-access memory, is envisaged, which is currently technologically challenging to implement. Furthermore, the characteristics of the input data

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play an important role, particularly the sparsity of matrices and their rank, which directly affect the accuracy and speed of the algorithms. Therefore, current research is focused not only on developing new quantum algorithms but also on optimizing existing approaches, creating hybrid quantum-classical methods, and adapting them to real-world application conditions. Let's examine these algorithms:

HHL (Harrow-Hassidim-Lloyd) [6, 7, 8]. The HHL algorithm is a fundamental quantum approach to solving systems of linear algebraic equations and demonstrates potential exponential speedup compared to classical methods, provided the matrix is sparse and has a limited condition number. Its operation is based on the use of quantum phase estimation for the spectral decomposition of the matrix and the inversion of eigenvalues. At the same time, its practical effectiveness is limited by requirements for state preparation, access to data (QRAM), and sensitivity to noise.

QPE (Quantum Phase Estimation) [9, 10, 11]. The quantum phase estimation algorithm is a fundamental tool of quantum computing that allows for the determination of eigenvalues of unitary operators. It underlies many algorithms, including HHL, qPCA, and quantum optimization methods. Its efficiency is determined by the accuracy of the approximation and the number of auxiliary qubits, but it requires deep quantum circuits and is sensitive to decoherence.

QSVT (Quantum Singular Value Transformation) [12, 13, 14]. QSVT is a modern generalization of quantum linear algebra algorithms, allowing arbitrary polynomial functions to be efficiently applied to the singular values of matrices. This approach provides a universal framework for constructing optimal algorithms, including improved versions of HHL, and is characterized by better stability and flexibility, making it one of the most promising areas of development for quantum algorithms.

qPCA (Quantum Principal Component Analysis) [15, 16, 17]. Quantum principal component analysis is designed to identify the main directions of data variation through spectral analysis of density matrices. The algorithm can provide exponential acceleration when processing large datasets; however, its efficiency depends on the ability to effectively encode data into quantum states and access them.

VQLS (Variational Quantum Linear Solver) [18, 19, 20]. The VQLS is a hybrid algorithm tailored for NISQ devices, combining quantum circuits with classical optimization. It does not require deep quantum circuits, like HHL, and is more noise-resistant, but its efficiency depends on the choice of the variational ansatz and the complexity of the optimization.

QAE (Quantum Amplitude Estimation) [21, 22, 23]. The quantum amplitude estimation algorithm provides quadratic acceleration in probability estimation and mathematical expectation problems compared to classical Monte Carlo methods. It is a key component in financial modeling, risk analysis, and statistical problems, although the classical implementation requires complex quantum sub-schemes.

Quantum Monte Carlo [24, 25]. Quantum Monte Carlo methods use QAE to accelerate numerical integration and statistical modeling. They provide a better error-to-iteration ratio, making them promising for high-dimensional problems, particularly in finance, physics, and big data analysis.

Quantum Bayesian Inference [26, 27, 28]. Quantum Bayesian methods aim to accelerate the updating of posterior distributions and the performance of statistical inference. They use quantum representations of probability distributions, which potentially allows for faster processing of complex models, but face challenges in scalability and implementation.

QML (Quantum Machine Learning) [29, 30, 31]. Quantum machine learning combines quantum computing with AI/ML methods to accelerate classification, regression, and clustering tasks. Key approaches include quantum kernels, variational quantum neural networks, and quantum SVMs. The main challenges remain noise, a limited number of qubits, and efficient data encoding.

QRAM (Quantum Random Access Memory) [32, 33, 34]. QRAM is a critical infrastructure component that provides access to large amounts of classical data in quantum algorithms. Many algorithms, including HHL and qPCA, rely on its use; however, the practical implementation of QRAM is complex and remains one of the main barriers to scaling quantum computing.

QAOA (Quantum Approximate Optimization Algorithm) [35, 36, 37]. The QAOA algorithm is designed to solve combinatorial optimization problems through variational minimization of the objective function. It combines quantum evolution with classical parameter optimization and shows promise for problems in statistics, logistics, and network analysis, especially under conditions of limited quantum resources.

The following is a study and analysis of various algorithms.

2. Review of quantum algorithms for linear algebra and statistics

Article [1] presents a new quantum algorithm for signal-to-noise ratio enhancement, which performs threshold filtering in the frequency domain using amplitude amplification and an adaptive threshold based on local averages. The algorithm can be applied to both classical and quantum signals and provides a significant speed-up compared to existing methods. Numerical results demonstrate its effectiveness (Figures 1, 2, 3) in removing both classical and quantum noise, significantly outperforming current approaches to quantum noise cancellation, particularly in high-noise quantum environments.

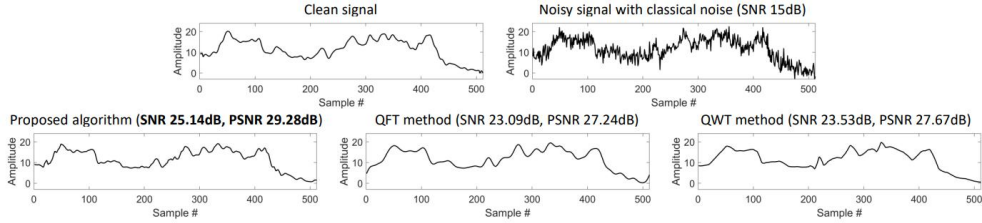


Figure 1: Noise cancellation performance in the presence of standard additive white Gaussian noise.

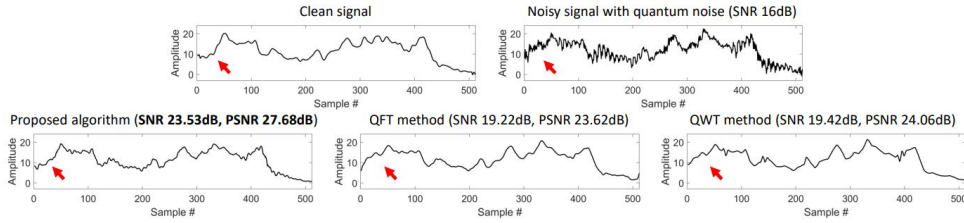


Figure 2: Noise cancellation efficiency in the presence of quantum phase noise arising from random phase angles in the base quantum logic gates.

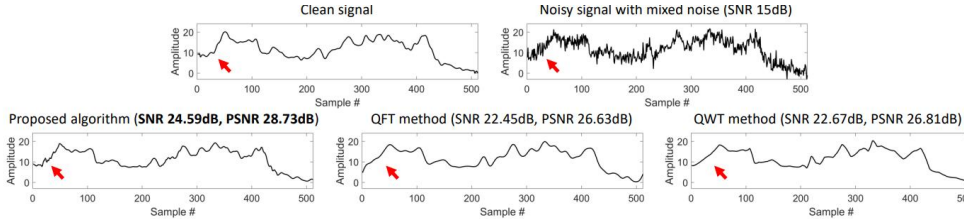


Figure 3: Noise cancellation performance in the presence of mixed noise (classical Gaussian noise + quantum phase noise).

Hybrid quantum-classical algorithms, in particular variational quantum algorithms (VQA), are well suited to devices based on noisy intermediate-scale quantum (NISQ) systems. Work [2] highlights the often underestimated role of classical pre-computations in VQA, where classical methods can simplify and adapt quantum schemes to specific tasks. A key trade-off in VQA exists between solution quality and the complexity of scheme design and optimisation. Exact formulations, such as VQA for MAXCUT, offer optimal solutions but involve NP-hard scheme construction or optimisation, whereas shallow approaches, such as the quantum approximate optimisation algorithm (QAOA), are more accessible but yield weaker approximations. To bridge this gap, Spanning Tree QAOA (Figure 4) is introduced, which utilises classical approximations to determine the ansatz structure, achieving the same performance guarantees as classical methods whilst outperforming standard QAOA at low depths. More broadly, this work demonstrates how integrating classical precomputations into VQA can improve both heuristic and guaranteed performance.

Quantum technologies are developing rapidly, and are expected to have a significant impact on solving

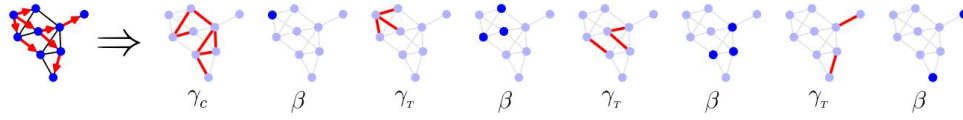


Figure 4: An example of an ST-QAOA round. The classical algorithm computes an approximately optimal solution to the SIGNED MAXCUT problem by finding a suitable spanning tree (left). A QAOA round consists of alternating unitary operators $\sigma_z\sigma_z$ on edges (red) and σ_x on vertices (blue). First, the unitary complement operator is applied, followed by the tree operator, which simulates a directed tree graph through repeated application of $\sigma_z\sigma_z$ and σ_x . This is repeated r times. For $r = 1$, it is guaranteed that this will match the performance of the classical algorithm, and for $r \rightarrow \infty$, the approximation coefficient approaches 1.

complex real-world problems that lie beyond the capabilities of classical algorithms. In parallel, physical-layer security (PLS) has emerged as a promising approach to ensuring secure wireless communication that is resistant to both classical and quantum attacks. Although current PLS methods are based on classical technologies, their effectiveness can potentially be enhanced using quantum algorithms (Figure 5). Article [3] investigates the application of quantum computing methods – including quantum annealing, hybrid quantum-classical algorithms and approaches based on Grover’s algorithm – to PLS problems (Figure 6). An overview of their use in physical-layer techniques is provided, and their potential for improving secure message transmission and secret key generation in wireless communication systems is discussed.

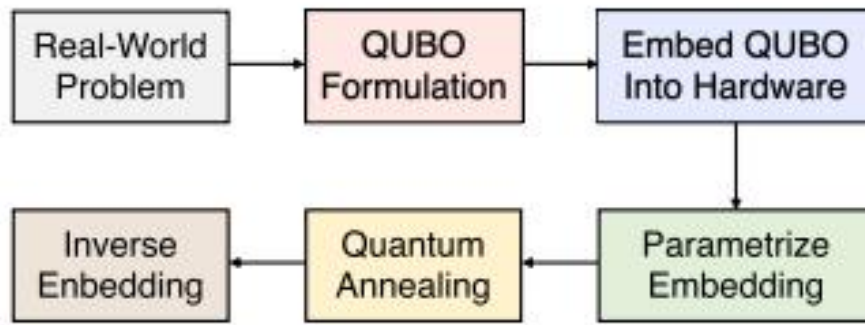


Figure 5: A typical workflow in the quality control department

Quantum computing – an interdisciplinary field based on quantum mechanics and computer science – is attracting considerable attention due to its potential to transform computing capabilities. Article [4] provides a comprehensive introduction to quantum computing, intended for both beginners and experienced researchers. It covers fundamental concepts such as qubits, superposition, entanglement, interference and noise, and provides an overview of quantum hardware, logic gates and circuit design. The study also examines the current state of the field, in particular the era of noisy intermediate-scale quantum (NISQ) systems, and its significance for solving real-world problems. Key quantum algorithms, including Shor’s and Grover’s algorithms, are discussed, along with their practical applications. Furthermore, the paper explores the impact of quantum computing on various fields, such as cryptography, optimisation, machine learning and materials science (Figure 7). Overall, this work serves as a structured guide to the principles, technologies and applications of quantum computing, providing readers with a solid foundation and understanding of future developments in this rapidly evolving field.

Calculating the fidelity $F(\rho, \sigma)$ between two unknown mixed quantum states in an N -dimensional Hilbert space is a fundamental problem in quantum computing, with applications in the verification of results and the analysis of algorithms. In [5], a quantum algorithm is proposed that estimates fidelity in polynomial time of the order of $\log N$, r and $1/\varepsilon$, where r is the lower rank of the states and ε is the desired accuracy, provided access to their purification via quantum oracles. The proposed method

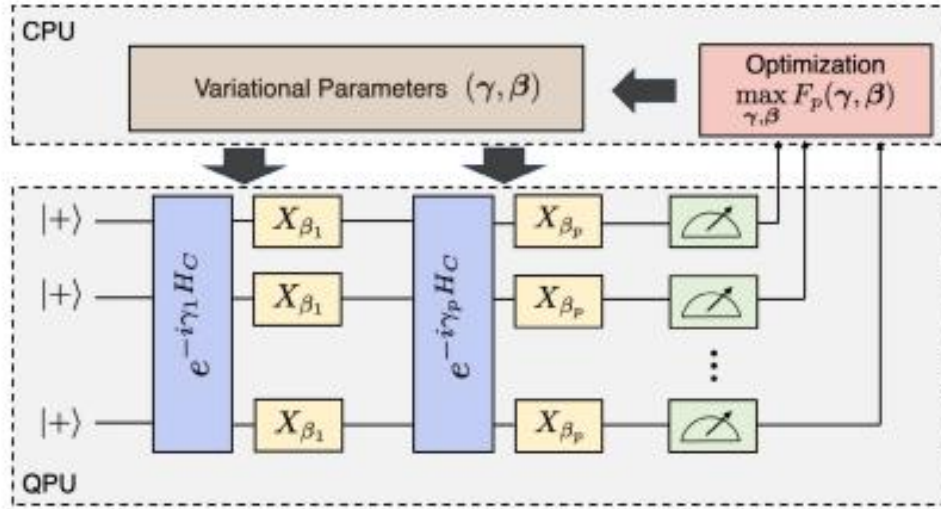


Figure 6: The QAOA procedure

Task	Algorithm/ Model	Study	Used for
Secure Key Distribution	Quantum channels	Rana et al. [81]	Secure communication and detection of eavesdropping.
Classification	Quantum Variational Classifier	Adhikary et al. [82]	Fisher's Iris, Sonar, and Wisconsin's Breast Cancer (WBC) datasets.
	Quantum inspired Nearest Mean Classifier	Sergio et al. [83]	Idiopathic pulmonary fibrosis (IPF) dataset.
	Quantum Neural Network	Hellstem et al. [84]	Image classification using finance and MNIST datasets.
Regression	Quantum Neural Network	Ngo et al. [85]	Li-ion battery degradation dataset.
	Continuous Variable - Quantum Neural Network (CV-QNN)	Kanimozhi et al. [86]	Surface plasmon resonance sensor dataset.
Reinforcement Learning	Hybrid-Quantum Proximal Policy Optimization	Rainjonneau et al. [87]	Satellite path planning and task decision using multiple satellites dataset.
	Hybrid Quantum Deep Q Network	Bar et al. [88]	Synthetic robot navigation dataset.
Dimension Reduction	Quantum PCA	Dri et al. [89]	Synthetic financial interest rate dataset.
Image Generation	Quantum-Classical GAN (QGANs)	Tsang et al. [90]	High-resolution image generation using MNIST digits and Fashion datasets.
Contrastive Learning	Quantum SimCLR Model	Jaderberg et al. [91]	Image processing using CIFAR-10 dataset.

Figure 7: A brief overview of quantum applications, indicating the algorithms/models used.

provides exponential acceleration compared to classical approaches based on quantum state tomography, the complexity of which grows polynomially with N (Figure 8).

2.1. HHL (Harrow–Hassidim–Lloyd)

Article [6] investigates the processes of error generation and propagation in the Harrow–Hassidim–Lloyd (HHL) quantum algorithm, executed on IBM Quantum hardware, using MATLAB-based simulation (Figure 9). Although the HHL algorithm provides exponential speed-up compared to classical methods for solving systems of linear equations, its performance in practical implementations is highly sensitive to noise. An HHL scheme for two variables has been implemented, and errors are analysed at each stage and compared with the simulation results. Three main sources of error have been identified: the flipping of a single qubit, the inaccuracy of logic gates, and cumulative error propagation. The study shows that errors increase significantly during the auxiliary qubit rotation stage, although the encoded solution retains high accuracy at this stage. However, accuracy deteriorates after the inverse quantum phase estimation stage, leading to the loss of the final solution. These findings highlight the need to prioritise error correction resources in later stages of the circuit to improve overall performance.

Article [7] presents a quantum-optimised Newton-Raphson method for power flow analysis using the Harrow-Hassidim-Lloyd (HHL) algorithm (Figure 10). The proposed approach converges in four iterations, as opposed to the five iterations required by the classical Newton-Raphson method. The

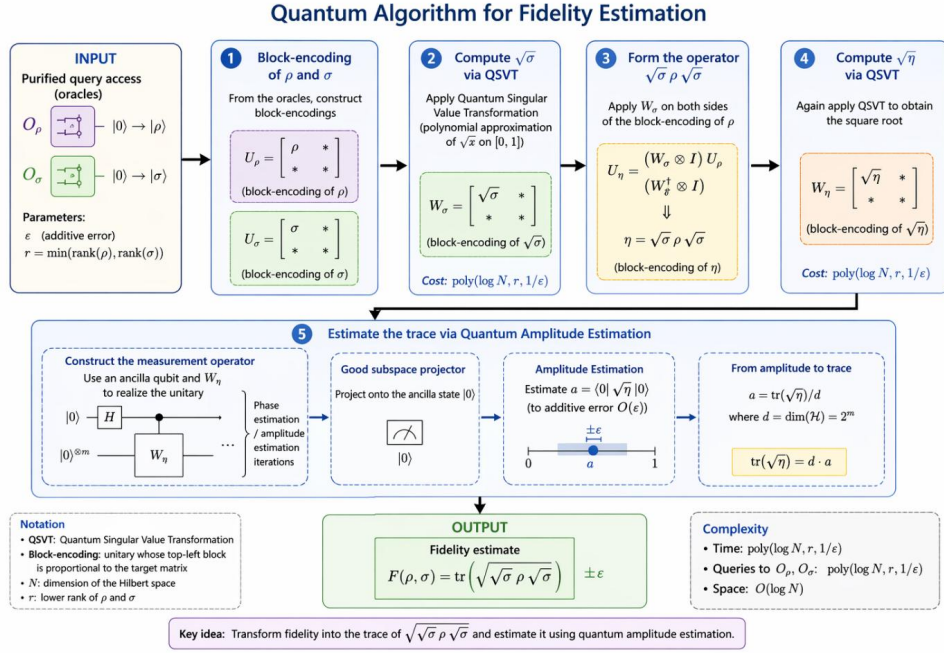


Figure 8: Block diagram of a quantum algorithm for accuracy estimation (*generated by ChatGPT 5.0*)

quantum method yields comparable results: voltage $V_2 = 0.98173$ pu, angle $\delta_2 = -0.05018$ rads, angle $\delta_3 = 0.04021$ rads, which almost coincide with the classical solutions. Implementation on a 3-node system demonstrates a convergence time of 0.228 s with an accuracy of 0.9814. The quantum scheme uses 4 qubits and has a depth of 126 gates. These results indicate the potential of quantum algorithms for improving computations in power systems.

In [8], the Harrow–Hassidim–Lloyd (HHL) quantum algorithm is used as a teaching tool to reinforce the basic concepts of quantum computing. The HHL algorithm, known for its exponential speedup in solving linear systems, is examined through an analytical explanation and a detailed numerical example for a 4-qubit system in bracket notation. Supplementary materials include MATLAB code for matrix-based understanding and a Qiskit implementation for execution on IBM quantum hardware (Figure 11). This approach gives students a practical understanding of key concepts such as superposition, entanglement, basis transformations, the quantum Fourier transform and quantum phase estimation. Additional explanations are provided to aid understanding of the concepts.

2.2. QPE (quantum phase estimation)

Quantum computing offers significant advantages over classical methods due to parallelism and entanglement, particularly in terms of reducing computational complexity. Quantum phase estimation (QPE) is a fundamental algorithm used for estimating eigenvalues and is a key component of many quantum applications. Article [9] discusses three QPE-based quantum machine learning algorithms: quantum principal component analysis (PCA), the Harrow–Hassidim–Lloyd (HHL) algorithm for solving linear systems, and quantum singular value thresholding (SVT) for matrix completion (Figure 12). The study analyses their advantages and limitations compared to classical approaches, discusses potential improvements, and concludes with recommendations for enhancing their performance and applicability.

Quantum phase estimation (QPE) is a fundamental algorithm for solving electronic structure problems in quantum computing, enabling the efficient estimation of the eigenvalues of molecular Hamiltonians. A common approach to implementing QPE is the Trotter approximation (TA), which decomposes a complex Hamiltonian into a sequence of simpler, easily diagonalised fragments. However, this approximation introduces errors into the unitary evolution operator, and the magnitude of these errors depends heavily on both the chosen decomposition of the Hamiltonian and the order of its constituent

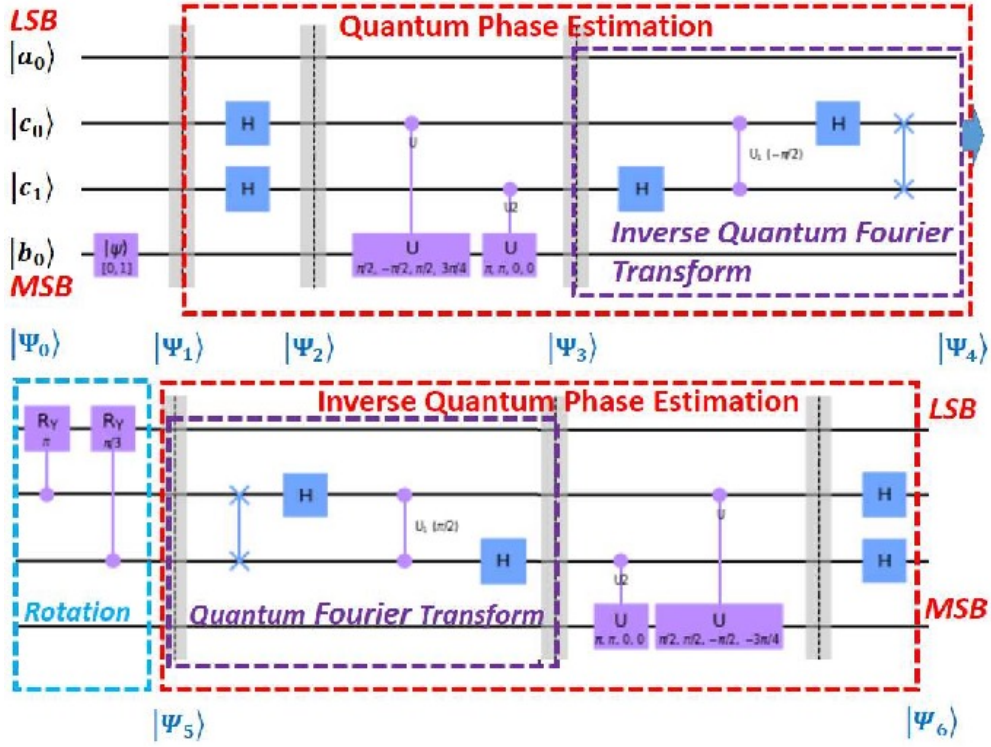


Figure 9: The HHL quantum circuit implemented in this study. The circuit is divided into two parts (upper and lower). The 4-qubit input signal, $|\Psi\rangle$, flows from left to right. The output of the upper part, $|\Psi_4\rangle$, becomes the input to the lower part. The output of the HHL circuit is $|\Psi_6\rangle$. Note that the most significant bit (MSB) is at the bottom.

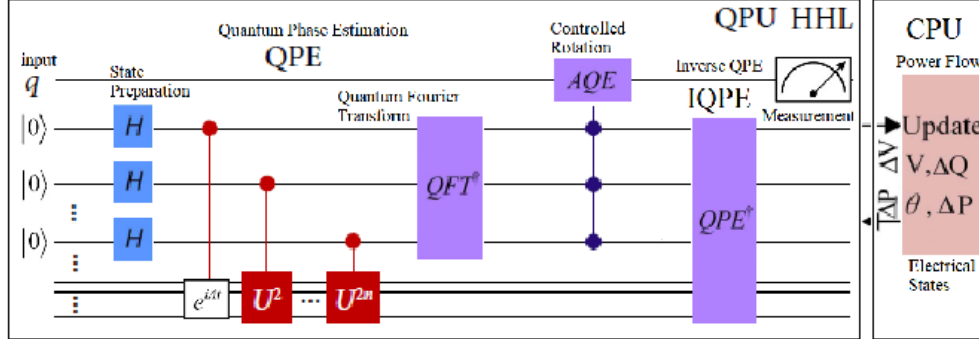


Figure 10: Quantum circuit for implementing the HHL algorithm

terms. In [10], a new two-stage algorithm is proposed, designed to determine the near-optimal order of the Hamiltonian fragments for a given decomposition, with the aim of minimising the Trotter error whilst maintaining computational efficiency. This method systematically reduces the search space compared to exhaustive approaches, allowing the optimisation to be accelerated without a significant loss of accuracy (Figure 13). The proposed algorithm is evaluated on the electronic Hamiltonian of the H_2 molecule using a decomposition scheme with qubit commutation. The results demonstrate that our approach achieves accuracy comparable to exhaustive ordering strategies, whilst reducing computational time by approximately a factor of six. These findings highlight the effectiveness of the proposed method in improving the practical implementation of QPE and point to its potential applicability to larger and more complex quantum systems.

Quantum phase estimation (QPE) is a key component of many quantum algorithms, but its integration with variational methods often requires resource-intensive parameter optimisation. Article [11] presents

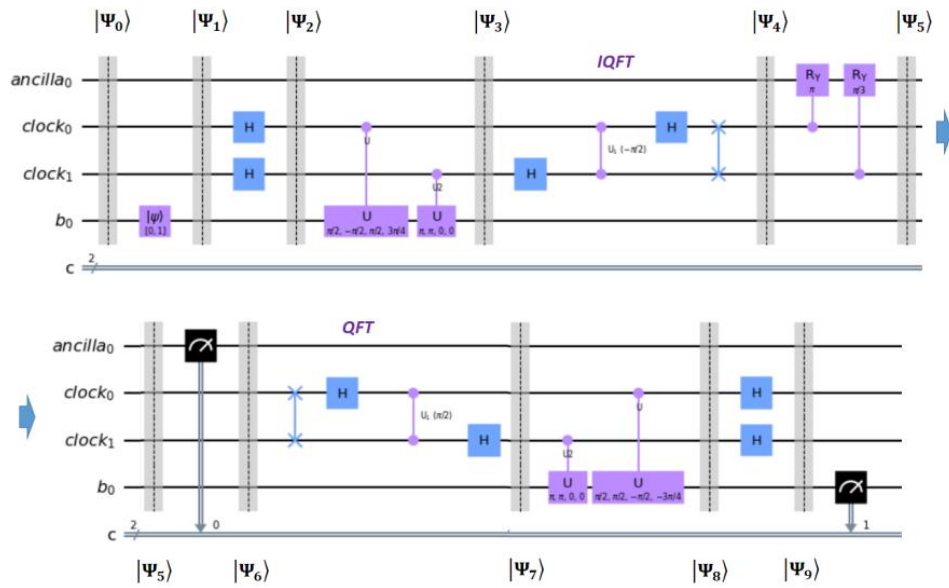


Figure 11: An HHL diagram corresponding to the numerical example, created for execution on IBM-Q

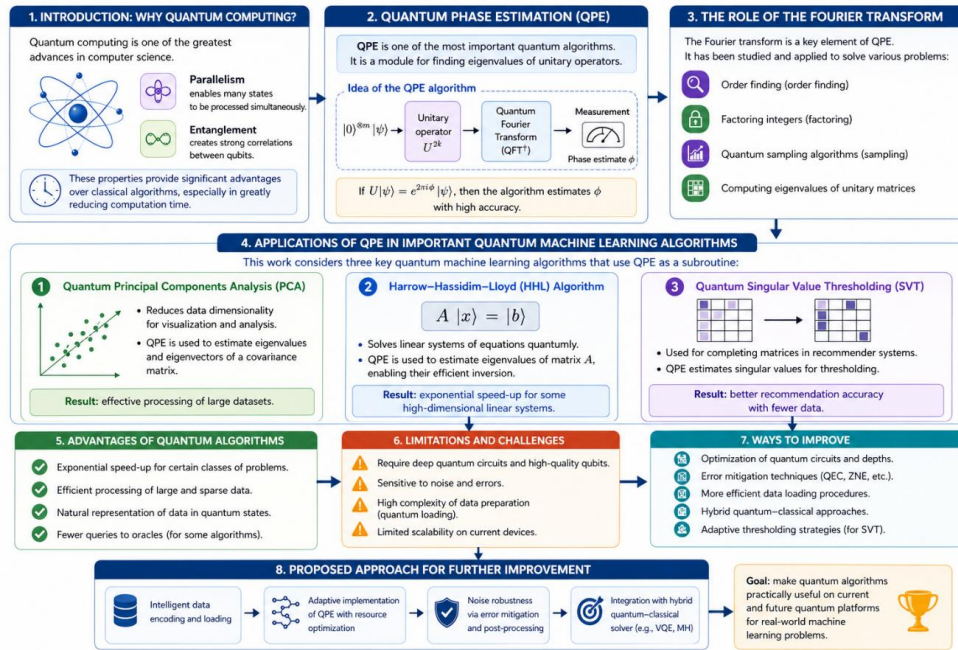


Figure 12: Research framework for the article (*generated by ChatGPT 5.0*)

a QPE-based quantum evolution in imaginary time (QPE-QITE) – a post-variational approach to ground state estimation on fault-tolerant quantum computers (Figure 14). By using auxiliary qubits for projection onto low-energy states, QPE-QITE eliminates the need for parameter tuning. The method is demonstrated on the example of the low-autocorrelation binary sequences (LABS) problem, showing its effectiveness for complex optimisation tasks. Scaling estimates for magic state requirements and potential implementations on near-future hardware are also discussed, highlighting its relevance for future quantum supremacy.

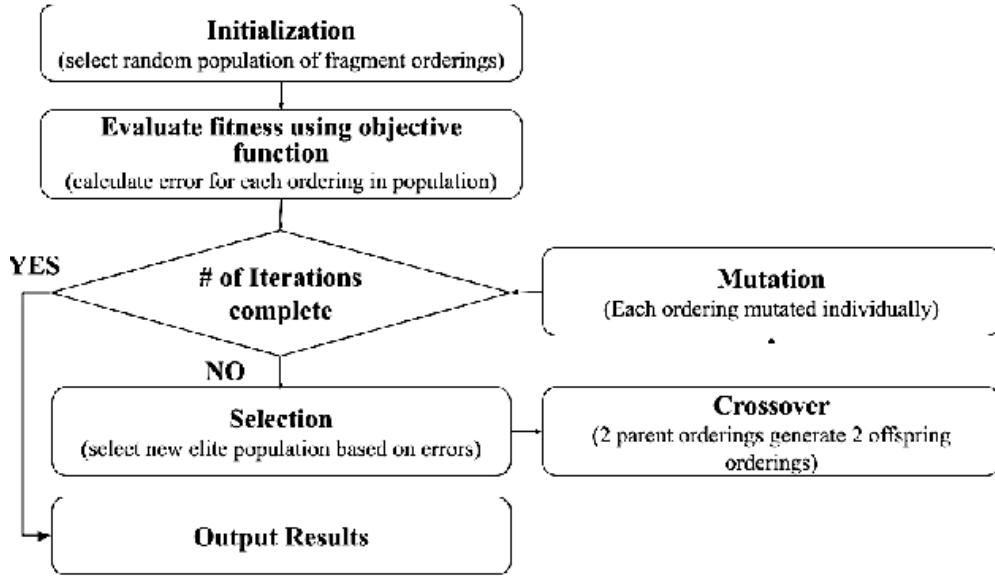


Figure 13: Flowchart illustrating the stages of a genetic algorithm

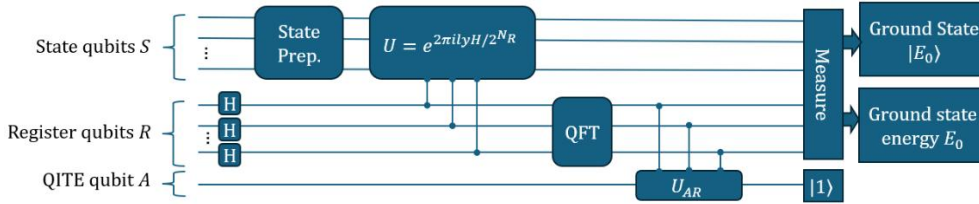


Figure 14: A representation of the QPE-QITE algorithm as a quantum circuit.

2.3. QSVT (quantum singular value transformation)

Quantum Singular Value Transformation (QSVT) is a universal meta-algorithm that enables a wide range of quantum applications through polynomial transformations encoded as quantum gate operations. Paper [12] investigates potential security risks associated with information leakage at the gate level, specifically regarding rotation angles and phase parameters (Figure 15). The paper analyses how various QSVT functions map to specific polynomials and corresponding phase angles, focusing on correlations between the values of the angles and their number. The results show that access to information about phase angles – even partially via side channels – may allow an attacker to infer the implemented basic functionality, highlighting the potential vulnerability of QSVT-based quantum computations.

Quantum Singular Value Transformation (QSVT) is a powerful quantum algorithm capable of performing polynomial singular value transformations of a linear operator embedded in a unitary matrix. In [13], QSVT is extended to enable polynomial transformations of quantum state amplitudes. In particular, an algorithm is proposed that maps the quantum state of the form $\sum_{i=0}^{N-1} x_i |i\rangle$ onto a normalised state proportional to $\sum_{i=0}^{N-1} f(x_i) |i\rangle$, where $f(\bullet)$ is a polynomial function (Figure 16). The practical utility of the proposed method has been demonstrated through its application to the extraction of small amplitudes of a quantum state. This approach extends the scope of application of QSVT and provides new opportunities for the controlled manipulation of quantum state amplitudes in quantum information processing tasks.

In [14], a quantum algorithm is proposed for solving the control problem of a discrete-time, finite-horizon linear-quadratic-Gaussian (LQG) model, which combines optimal control and state estimation in the presence of stochastic disturbances and noise (Figure 17). Conventional solutions are based on backward Ricatti recursion and forward Kalman filtering, both of which involve computationally

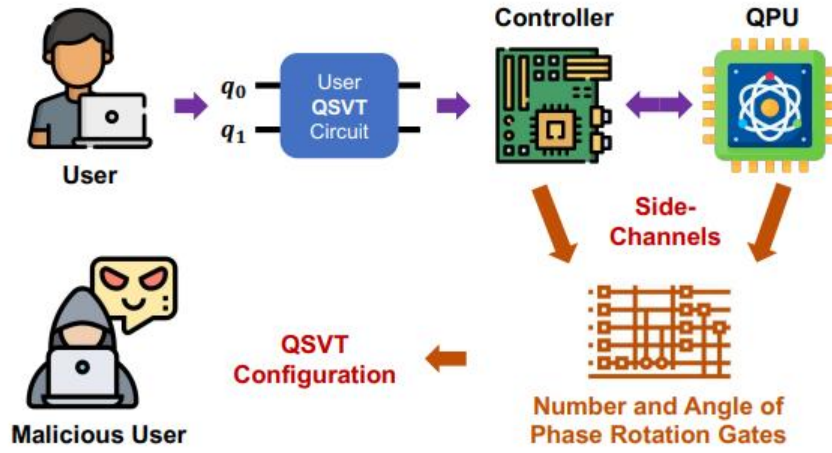


Figure 15: Overview of the threat model and attack on the QSVT algorithm: the user sends a QSVT circuit to a quantum computer, whilst an attacker may use side channels in the controller or QPU to attempt to discover the logical gates being executed, and subsequently information regarding the QSVT configuration

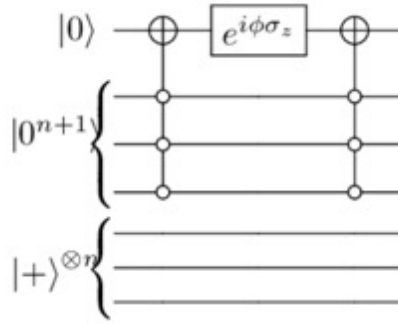


Figure 16: Quantum circuit for a matrix

intensive matrix operations with a total time complexity of $O(Tn^3)$, where n is the system dimension and T is the time horizon. Although effective classical methods exist for small- and medium-scale systems, their computational cost increases rapidly as the system size grows. To overcome this limitation, the LQG pipeline is reformulated using primitives of quantum linear algebra, in particular block-coded matrix representations and the quantum singular value transform (QSVT) for matrix inversion and multiplication. A formal complexity analysis of each component is provided. Under standard assumptions regarding conditional numbers and coding precision, the proposed quantum algorithm achieves a runtime scaling of poly-logarithmic in n and linear in T , providing asymptotic quantum speedup compared to classical approaches.

2.4. qPCA (quantum principal component analysis)

In [15], a low-complexity quantum principal component analysis (qPCA) algorithm is proposed, which reduces dimensionality by directly extracting the principal components of the data matrix; this, in turn, reduces the number of quantum registers and measurements required. Similar to existing approaches, including Lin's qPCA, the method is based on quantum singular value transformation (QSVT). Unlike Lin's approach, which combines QSVT with modified QSVT (MQSVT) to generate superpositions of principal components, this method simplifies the scheme by replacing the rotation-controlled operation in QSVT with a controlled NOT operation. This modification significantly reduces the complexity of the scheme. The proposed qPCA (Figure 18) requires only three phase estimation steps, compared to five in state-of-the-art methods, resulting in a runtime of approximately three-fifths that of existing approaches. Simulations on IBM's quantum computing platform confirm that the algorithm generates

Input:

- 1: Time horizon T
- 2: Block encodings $U_A, U_B, U_C, U_M, U_N, U_S, U_\Sigma, U_\Gamma, U_Y$.
- 3: Initial estimate U_{μ_0} , initial covariance U_{R_0}
- 4: Block encodings of measurement stream $\{U_{y_1}, \dots, U_{y_T}\}$

Phase I: Backward Pass – LQR Gain Computation

- 5: Compute block-encoded feedback gains $\{U_{K_0}, \dots, U_{K_{T-1}}\}$ using **Algorithm 2**.

Phase II: Forward Pass – Kalman Filtering

- 6: Run quantum Kalman filter using **Algorithm 3**, with inputs $U_{\mu_0}, U_{R_0}, \{U_{y_k}\}, \{U_{K_k}\}$
- 7: Obtain block-encoded state estimates $\{U_{\mu_1}, \dots, U_{\mu_T}\}$

Phase III: Control Computation

- 8: **for** $k = 0$ to $T - 1$ **do**
- 9: Compute classical control input: $u_k \leftarrow (U_{K_k} \cdot U_{\mu_k})$
- 10: **end for**

Output:

- 11: Classical control inputs $\{u_0, \dots, u_{T-1}\}$
- 12: Block-encoded conditional means $\{U_{\mu_1}, \dots, U_{\mu_T}\}$

Figure 17: A quantum algorithm for controlling an LQG with output feedback over a finite horizon

the expected quantum states.

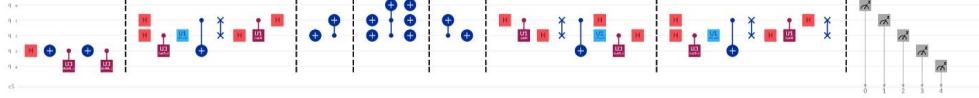


Figure 18: Block diagram of the proposed qPCA for a 2×2 matrix A_0 on IBM QX

Quantum Principal Component Analysis (QPCA) extends the classical PCA framework, enabling an exponential acceleration of computations through the use of quantum computing (Figure 19). As an unsupervised dimensionality reduction method, it maps data of exponentially high dimensionality into a lower-dimensional space, whilst preserving the features with the highest variance. Quantum computing, characterised by superposition and implicit parallelism, allows for the efficient extraction of principal components, offering advantages over classical approaches. In particular, QPCA utilises quantum superposition to perform an eigenvalue decomposition of the density matrix – which is equivalent to the covariance matrix – using fundamentally different computational mechanisms. In [16], a general methodological framework for the development of QPCA algorithms is presented, covering quantum data encoding, quantum memory processing and the implementation of quantum circuits. Existing approaches to QPCA are systematically classified taking into account recent developments, and potential improvements are outlined, which will serve as a guide for the development of more efficient quantum algorithms for principal component analysis. Finally, the proposed concepts are compared with current methods in terms of computational complexity and related factors, highlighting their significance for future research directions.

Quantum Principal Component Analysis (QPCA) theoretically offers exponential speed-up compared to classical PCA; however, its practical effectiveness under real-world conditions remains poorly understood. The study [17], currently under development, examines the practical implementation of QPCA by creating a scheme based on quantum phase estimation (QPE) using Qiskit and testing it on ten well-known classical datasets within the IBM Quantum simulator environment. Performance is evaluated in terms of the accuracy of reproducing eigenvalues and eigenvectors, as well as the subsequent impact on regression and classification tasks. The results show that QPCA can reproduce classical

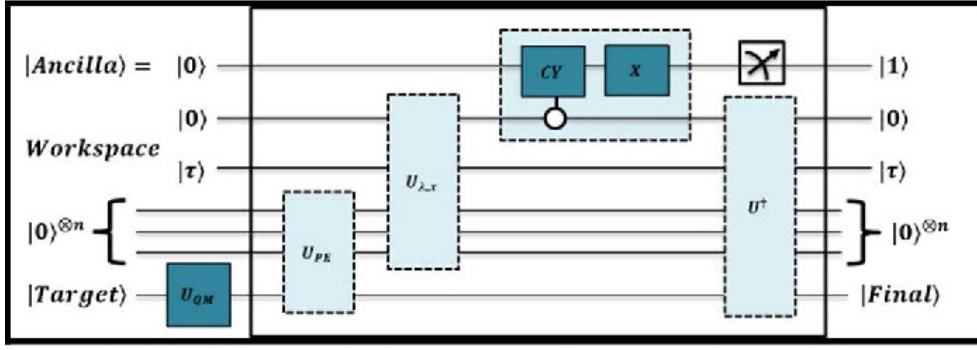


Figure 19: Quantum circuit for QPCA based on an extended approach

eigenvalues with high accuracy (error $< 10^{-4}$), although the accuracy of eigenvectors deteriorates in the presence of nearly degenerate spectra (Figure 20). Furthermore, classification tasks benefit more consistently from PCA than regression tasks, regardless of the implementation approach. These findings provide an initial empirical basis for understanding the strengths and limitations of QPCA and establish baseline metrics for future research focused on hardware noise, scalable circuit design and hybrid quantum-classical methods.

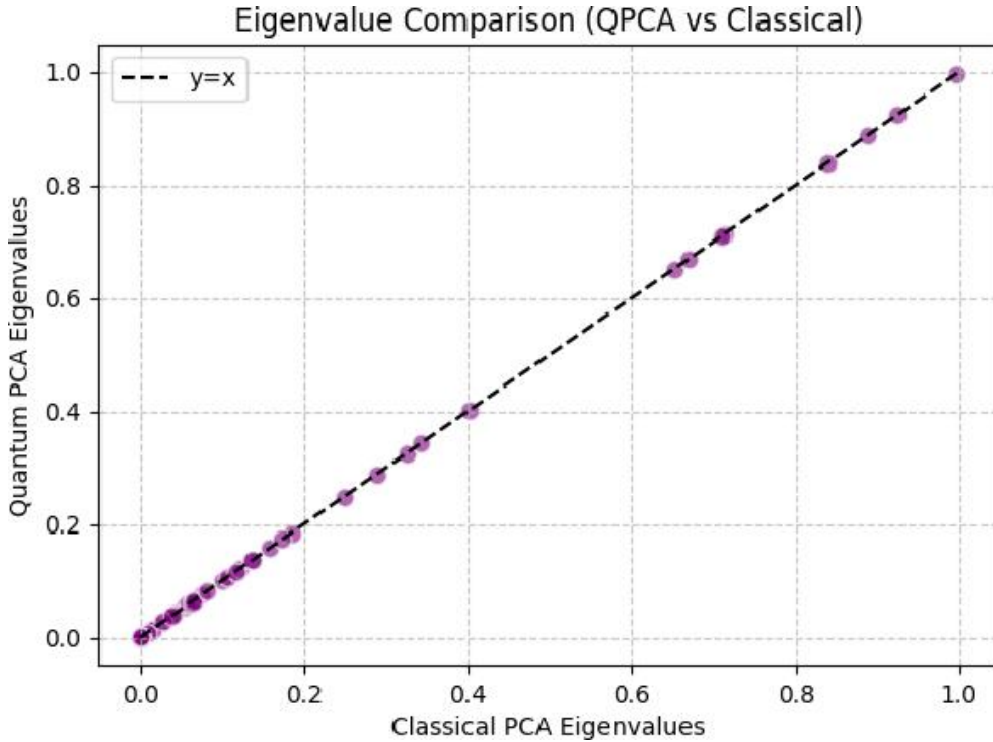


Figure 20: Maximum absolute error in eigenvalues between QPCA and CPCA for each dataset. The average error across all datasets was 1.38×10^{-4}

2.5. VQLS (variational quantum linear solver)

Electromagnetic modelling plays a crucial role in radar technology, but it is computationally intensive, particularly when using finite element methods (FEM), which rely on linear algebraic procedures with polynomial complexity that depends on the mesh size. The Variational Quantum Linear Solver (VQLS) is a quantum algorithm capable of providing exponential acceleration for certain classes of problems. Although VQLS is not universal, it represents a promising approach to improving the computational

efficiency of radar cross-section calculations (Figure 21). Work [18] describes the application of VQLS to electromagnetic problems related to radar, with a particular focus on the statistical estimation component of the algorithm. The discussion highlights how quantum methods can be adapted to this field and assesses their potential for reducing computational costs in practical scenarios.

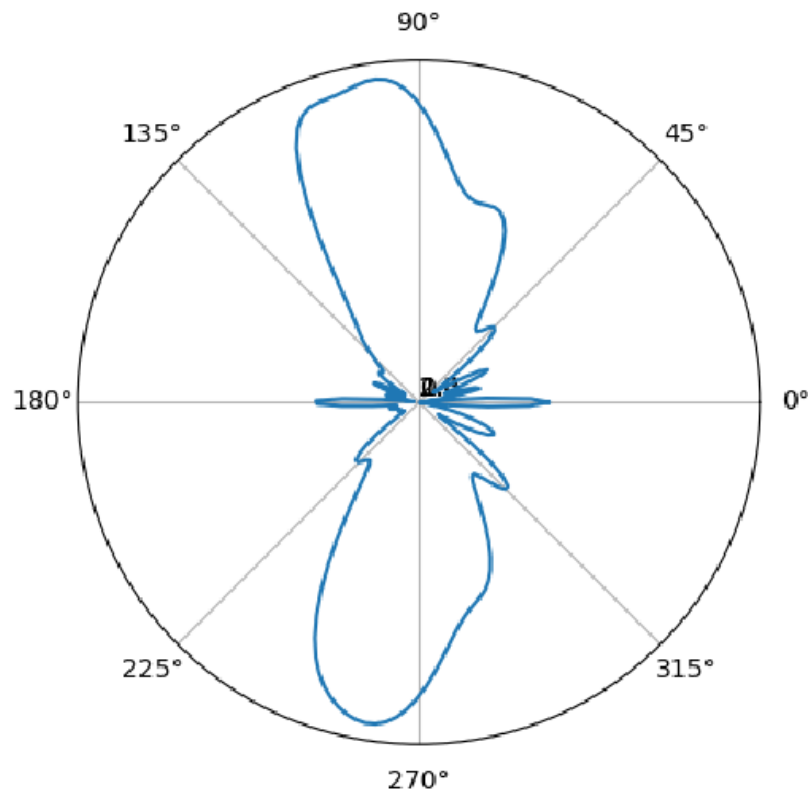


Figure 21: Radar cross-section associated with the object. For each possible angle of incidence, the intensity of the far-field signal reflected in that direction is calculated.

In [19], a new approach was developed to improve the efficiency of the Variational Quantum Linear Solver (VQLS) for structured sparse matrices, which are frequently encountered in the numerical solution of partial differential equations in science and engineering. Traditional implementations are based on Pauli basis expansions within the framework of the linear combination of unitary operators (LCU) method for computing global and local cost functions. However, in the worst case, such expansions require a number of terms that grows quadratically with the matrix size. To overcome this limitation, an alternative basis has been introduced that better exploits the sparsity and structure of the matrix, reducing the number of terms in the tensor product to logarithmic scaling with respect to the matrix size. Since this basis consists of non-unitary operators, the concept of unitary complementation is used to construct efficient quantum schemes for estimating VQLS cost functions. The proposed method is compared with similar techniques, including unitary expansion and Bell-based measurement, highlighting the respective advantages and limitations (Figure 22). Its effectiveness is demonstrated through application to the heat conduction equation, illustrating potential benefits in terms of efficiency and scalability.

Recent advances in the field of quantum linear solvers open up promising avenues for accelerating computationally intensive scientific applications, such as fluid dynamics. However, the depth and complexity of many quantum schemes limit their practical applicability on current hardware. The Variational Quantum Linear Solver (VQLS) offers a more realistic alternative for near-future noisy intermediate-scale quantum (NISQ) devices, and previous studies demonstrate its applicability to specific hydrodynamic problems. In [20], the use of the VQLS for canonical hydrodynamic problems is investigated with the aim of identifying strategies to generalise its application to a wider range of systems (Figure 23). The influence of various circuit design approaches and classical optimisation methods on

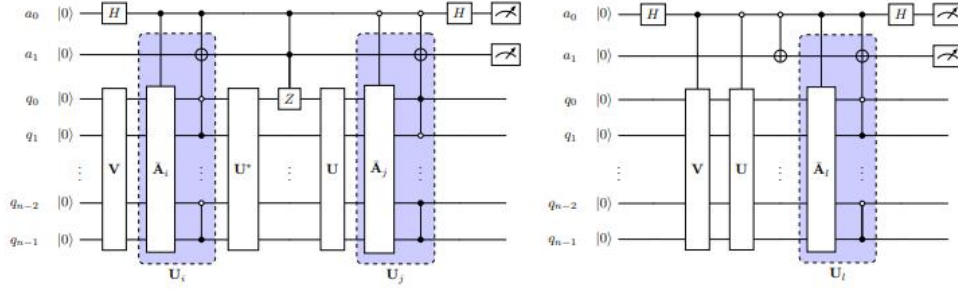


Figure 22: The Hadamard test for computing the terms of the global and local value functions.

the accuracy and convergence of solutions is systematically analysed. Furthermore, the possibility of integrating VQLS into a hybrid structure of quantum computing and high-performance computing (HPC) is assessed by implementing it in QFw, a modern software stack for quantum computing and HPC. The results provide an insight into the practical aspects of implementation and highlight opportunities for the development of quantum simulations of hydrodynamics (Figure 24).

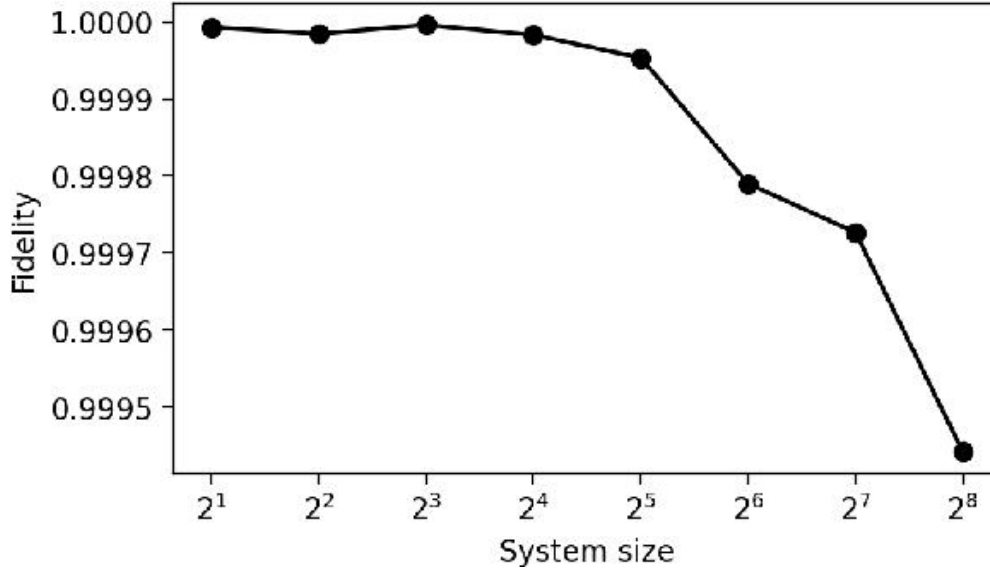


Figure 23: The effect of problem size on the accuracy of the VQLS method's results (compared with the classical method) is demonstrated for an LCU-based system.

2.6. QAE (quantum amplitude estimation)

Article [21] presents a Gaussian noise model for quantum amplitude estimation, derived from a series of simple and physically motivated assumptions. The model is designed to capture the dominant noise characteristics that influence the practical implementation of amplitude estimation on modern quantum hardware. To verify its accuracy, experimental results obtained on various platforms are presented, in particular on IBM superconducting quantum computers and the H1 trapped-ion system from Quantinuum (Figure 25). Empirical data demonstrate that the proposed Gaussian noise model corresponds well with real-world observations across different hardware technologies. Furthermore, it is shown that the Gaussian noise model can be flexibly combined with other noise models, allowing it to capture more complex noise characteristics that deviate from purely Gaussian behaviour. Based on this, a generalised framework has been developed for incorporating the noise model into quantum amplitude estimation algorithms without phase estimation, enabling the design of 'noise-sensitive' quantum algorithms that explicitly account for hardware imperfections. Finally, experimental results are presented

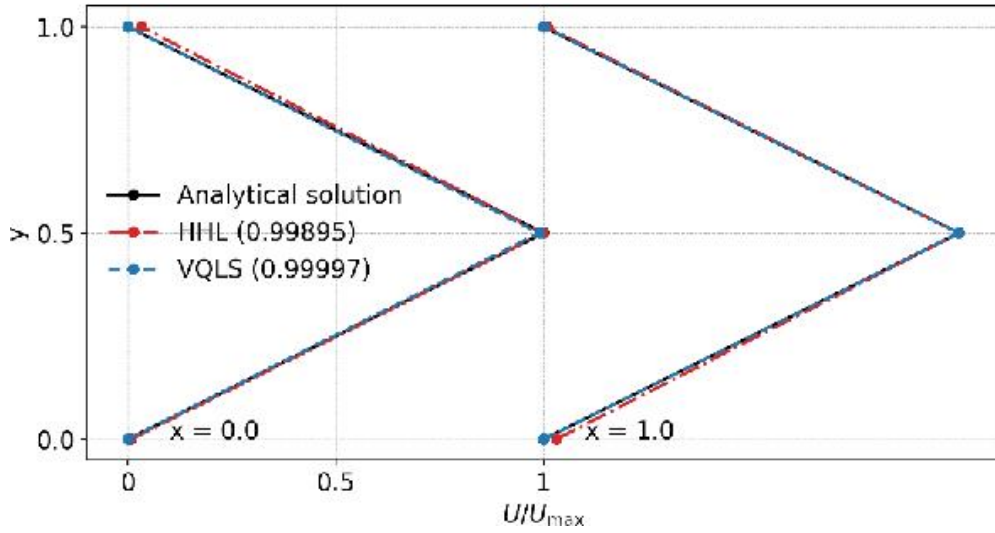


Figure 24: Comparison of the horizontal velocity (normalised by the maximum velocity, U_{max}) in the vertical direction, y , at different horizontal points, x , for a Hele-Shaw flow with a six-point grid, using the HHL and VQLS algorithms.

for the implementation of noise-sensitive quantum amplitude estimation on IBM superconducting devices. The results show that accounting for noise significantly improves estimation accuracy and acts as an effective form of quantum error reduction. Overall, this work highlights the importance of realistic noise modelling for improving the reliability and performance of quantum algorithms in the near future.

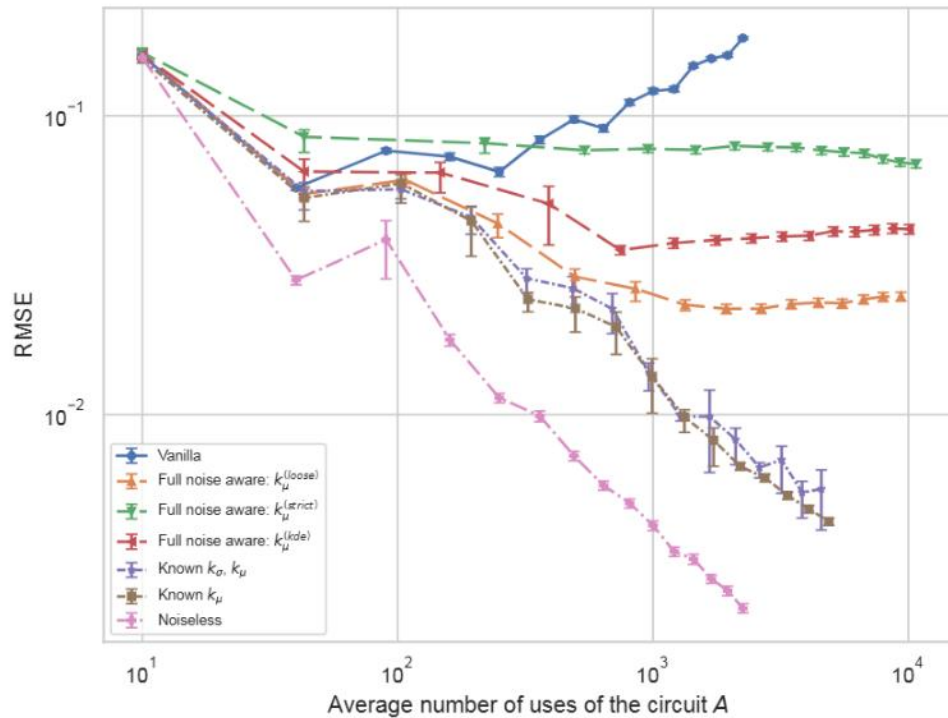


Figure 25: QAE for various experimental configurations using the IBM five-qubit machine in Perth. To determine the 68% confidence intervals, on the basis of which the errors of the data points are calculated, a bootstrapping method is used, implemented using a bootstrapping interval with bias correction and acceleration

The emergence of practical quantum computing systems has made it possible to implement quantum algorithms in real-world conditions; however, their performance remains highly sensitive to noise.

Article [22] investigates the effect of noise on quantum amplitude estimation using the maximum likelihood method (MLQAE) by analysing and modelling circuit depth under three conditions: ideal implementation, a theoretical noise model, and a real hardware noise model (Figures 26, 27). The results show significant differences in performance across these scenarios, with the error rate increasing substantially in deeper circuits and with a larger number of qubits. Even under low-noise conditions, the performance degradation becomes noticeable as the circuit complexity increases. These findings highlight the important role of noise modelling in the evaluation of quantum algorithms and the need for effective error reduction strategies in practical quantum computing environments.

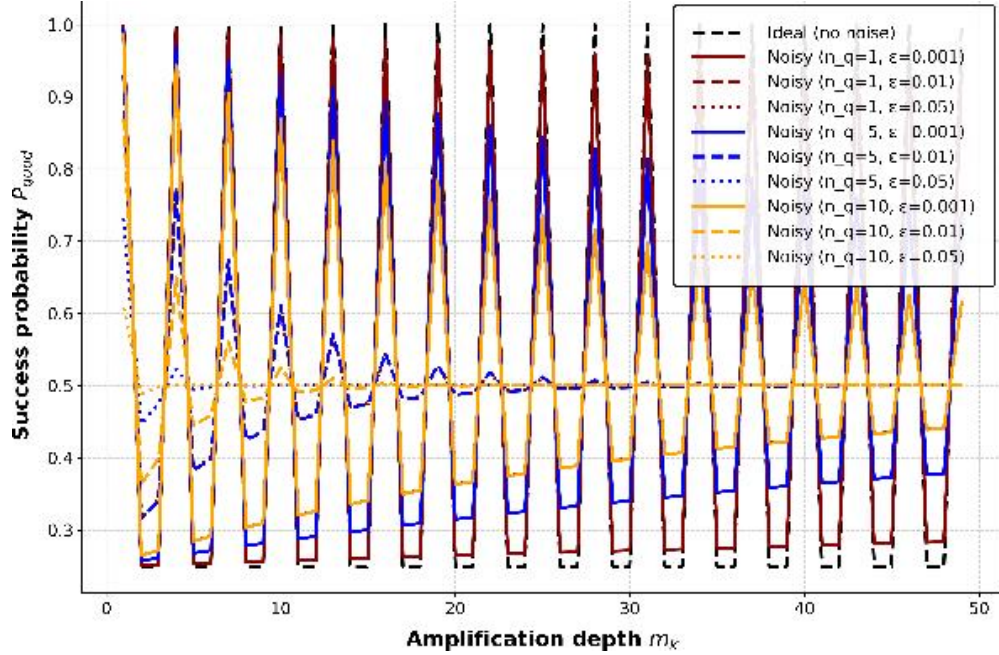


Figure 26: Probability of success for MLQAE gain depths at different noise levels (0, 0.001, 0.01 and 0.05) and numbers of qubits (1, 5 and 10).

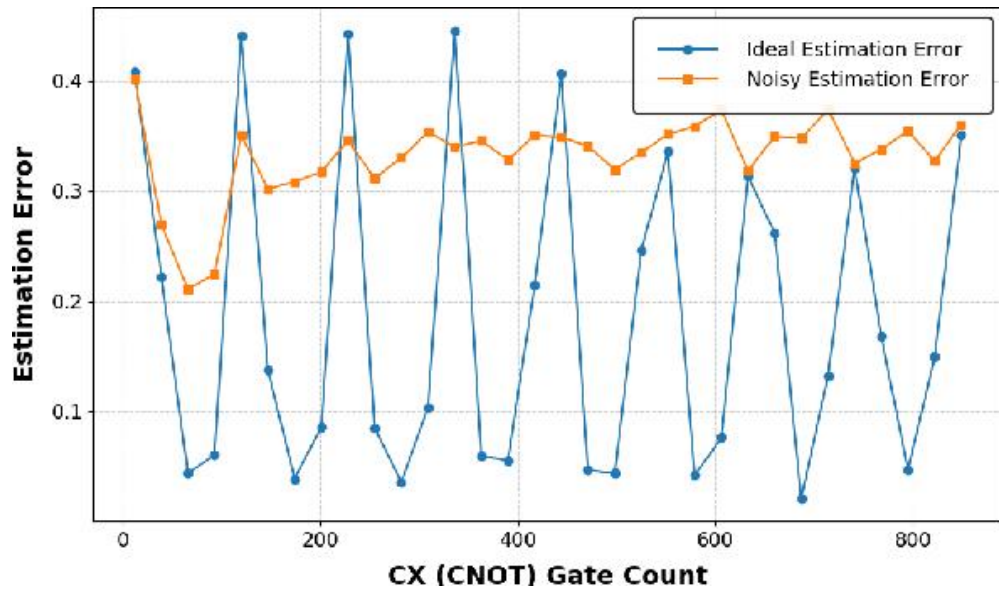


Figure 27: Simulated estimation error for different numbers of CX in the MLQAE scheme used to estimate (3) in ideal and noisy environments.

Monte Carlo integration is a widely used method for approximating integrals, but it often requires significant computational resources. Quantum computing offers the potential for quadratic acceleration,

which is driving the development of integration methods utilising quantum technologies. Article [23] proposes a method for quantum iterative Fourier amplitude estimation (QFIAE) – a new algorithm that combines quantum machine learning and amplitude amplification for efficient Monte Carlo integration (Figure 28). This method decomposes the objective function into a Fourier series using a parameterised quantum scheme, namely a quantum neural network, and computes each component using iterative quantum amplitude estimation (IQAE). Unlike existing approaches to Fourier-based quantum integration, QFIAE avoids explicit numerical evaluation of the Fourier coefficients, thereby reducing computational costs whilst maintaining the quadratic acceleration of IQAE. We test this approach on scattering functions in particle physics and compare its performance with other quantum integration methods and analytical results. The results show that QFIAE achieves comparable accuracy whilst remaining suitable for implementation on modern quantum hardware. Furthermore, it is shown that accuracy improves with an increase in the number of Fourier terms, highlighting the scalability and efficiency of the proposed method.

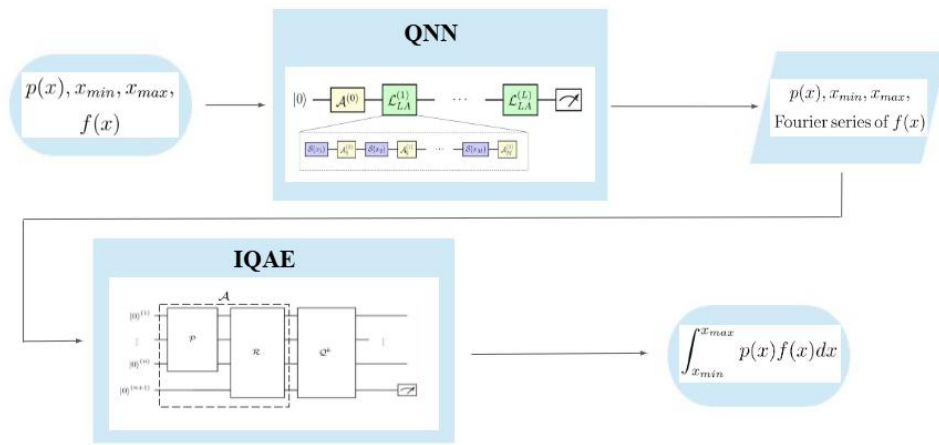


Figure 28: The operating principle of QFIAE. Firstly, the input parameters are entered, comprising the function $f(x)$, the probability distribution $p(x)$ and the integration interval. Secondly, a quantum neural network (QNN) is used to approximate $f(x)$, and a Fourier series is obtained based on the quantum scheme. Thirdly, $p(x)$, the integration interval and the Fourier series of the function $f(x)$ are entered as input parameters. Fourthly, IQAE computes the integral for each trigonometric term of the Fourier series within the specified interval. Fifthly, the integrals are summed with the corresponding coefficients to obtain the final integration result.

2.7. Quantum Monte Carlo

Quantum Monte Carlo integration (QMCI), based on quantum amplitude estimation (QAE), offers quadratic acceleration compared to classical methods and is attracting interest in applications involving a large number of measurements, such as finance. However, traditional implementations of QMCI require a large number of qubits, as the probability distributions for multiple random variables must be encoded in separate quantum registers. In [24], a more resource-efficient QMCI approach is proposed, which sequentially generates pseudorandom numbers in a single register, significantly reducing the number of qubits required. Furthermore, for integral expressions where the contributions of different variables are separable, nested QAE is combined with pseudorandom number generation to reduce the time complexity relative to the dimension. This approach enables more scalable quantum integration for high-dimensional problems, whilst retaining computational advantages (Figure 29).

In [25], the Zero-Temperature Quantum Monte Carlo Algorithm is investigated; this technique enables the determination of key properties of the ground state of a quantum Hamiltonian whilst simultaneously controlling quantum fluctuations to guide the system towards optimised solutions to computationally complex problems. By carefully controlling these fluctuations, the method facilitates an efficient exploration of the solution space and helps to avoid local minima, which typically hinder

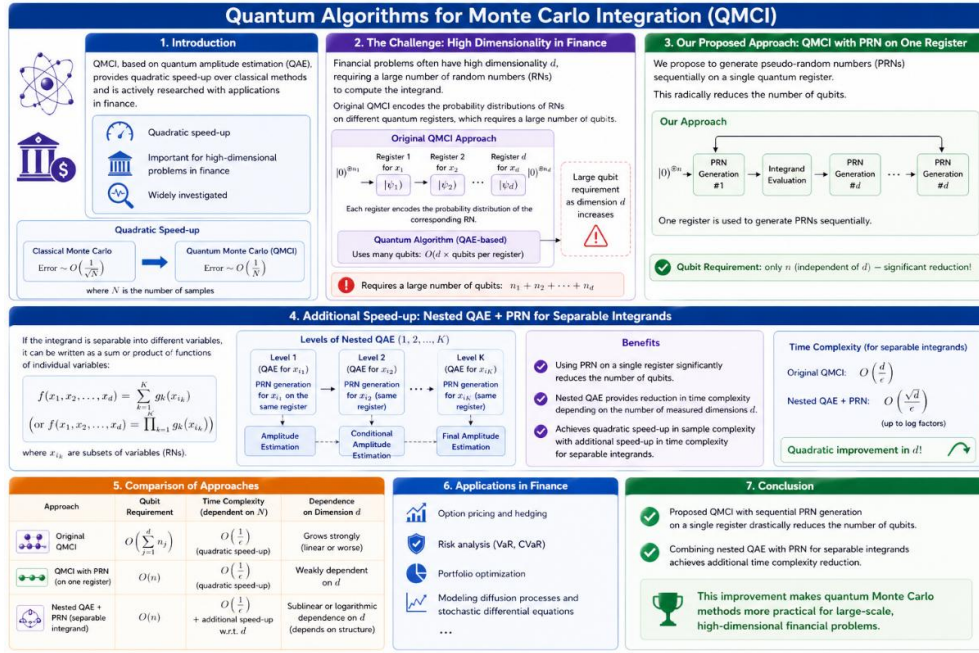


Figure 29: Algorithm structure for Monte Carlo integration (*generated by ChatGPT 5.0*)

classical approaches to optimisation. Consequently, this improves convergence to high-quality solutions in complex optimisation problems (Figure 30). Furthermore, this approach is particularly relevant for quantum algorithms, such as quantum annealing and adiabatic quantum computing, where the evolution of the system is driven by gradual changes in the Hamiltonian. Its applicability spans a wide range of fields, including combinatorial optimisation, machine learning and materials science. Overall, this technique highlights the potential of quantum methods to extend problem-solving capabilities beyond classical limitations, whilst contributing to the development of more robust and efficient quantum optimisation systems.

2.8. Quantum Bayesian inference

Bayesian networks (BNs) are widely used to model complex infrastructure systems under conditions of uncertainty, as they enable probabilistic reasoning and decision-making. The study [26] examines the application of quantum Bayesian networks (QBNs) for predicting infrastructure wear and tear in railway systems. QBNs extend the capabilities of classical BNs by utilising quantum schemes to represent prior knowledge and conditional dependencies between system variables. In the example presented, a hierarchical model of the railway track condition was developed, which includes elements of the lower and upper parts, such as ballast, subgrade, rails and sleepers, as well as environmental factors, in particular temperature and humidity. Conditional relationships were initially defined using classical conditional probability tables and then converted into a parameterised quantum circuit using quantum gate operations (Figure 31). The QBN was implemented on an IBM quantum simulator, where inference was performed via repeated quantum sampling. Its performance was evaluated by comparing estimates of marginal, joint and conditional probabilities with those obtained from an equivalent classical BN using exact inference methods. The results show significant convergence between the quantum and classical models, with probability distributions and bit-string statistics exhibiting only minor deviations during repeated runs. These results confirm the suitability of using quantum networks (QBNs) for interpretable infrastructure condition assessment and highlight their potential as a scalable tool for future railway system monitoring and preventive maintenance under conditions of uncertainty.

The detection of defects and anomalies in 3D objects is of paramount importance for ensuring reliability and proper functionality in industrial applications. In [27], a hybrid approach is proposed

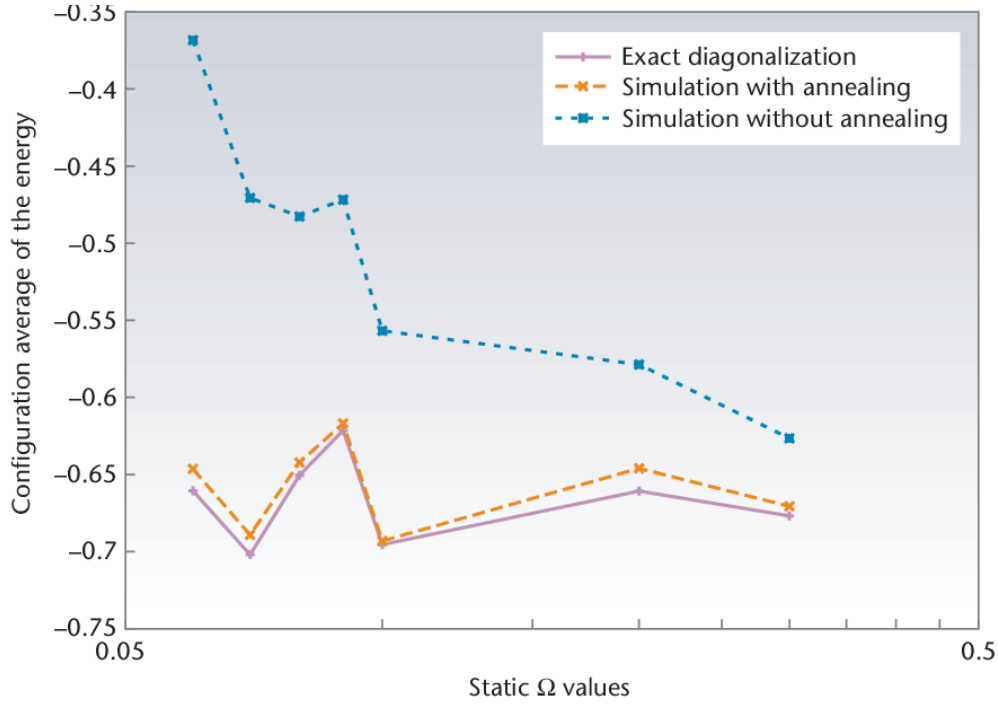


Figure 30: Simulation results (with and without pre-annealing) of the ground state of the Hamiltonian for various static values of Ω , compared with the corresponding exact results obtained via exact numerical diagonalisation for the same set of samples

that combines Bayesian learning with quantum and quantum-inspired machine learning methods to address this problem (Figure 32). Bayesian learning provides a reliable quantitative assessment of uncertainty, whilst orthogonal neural network architectures ensure stable and efficient learning thanks to constrained weight matrices. Orthogonal (quantum) variants of 3D convolutional neural networks have been developed, tailored for detecting anomalies in volumetric data. These models are evaluated on an industrially relevant dataset and demonstrate high performance in detecting structural anomalies. To further explore the practical integration of quantum computing, parts of the proposed approach have been implemented on the 127-qubit IBM Brisbane quantum device. Hardware experiments evaluate the impact of noise and a limited number of measurement snapshots on the model's performance. Our results show that orthogonal models inspired by quantum technologies can effectively detect anomalies in 3D objects while maintaining training stability. Furthermore, the hardware experiments highlight both the current limitations and the future potential of integrating quantum resources into anomaly detection pipelines, paving the way for scalable industrial inspection systems enhanced by quantum technologies.

Laser phase noise is a key factor limiting the performance and transmission range of coherent quantum communication systems. In [28], the application of Bayesian inference methods to mitigate the effects of phase noise was experimentally investigated. By utilising probabilistic estimates, the proposed approach improves phase noise compensation and enhances signal quality (Figure 33). The results demonstrate a reduction in excess noise of approximately 15%, leading to an increase in communication range and an improvement in the speed of secure key generation. These findings highlight the potential of Bayesian methods as an effective tool for enhancing the reliability and efficiency of practical quantum communication systems.

2.9. QML (quantum machine learning)

Article [29] proposes a quantum machine learning (QML) algorithm, based on the compact amplitude encoding method, for efficient demapping in modern communication systems. This approach utilises a vacuum quantum state to estimate the measurement probabilities of qubits corresponding to the received

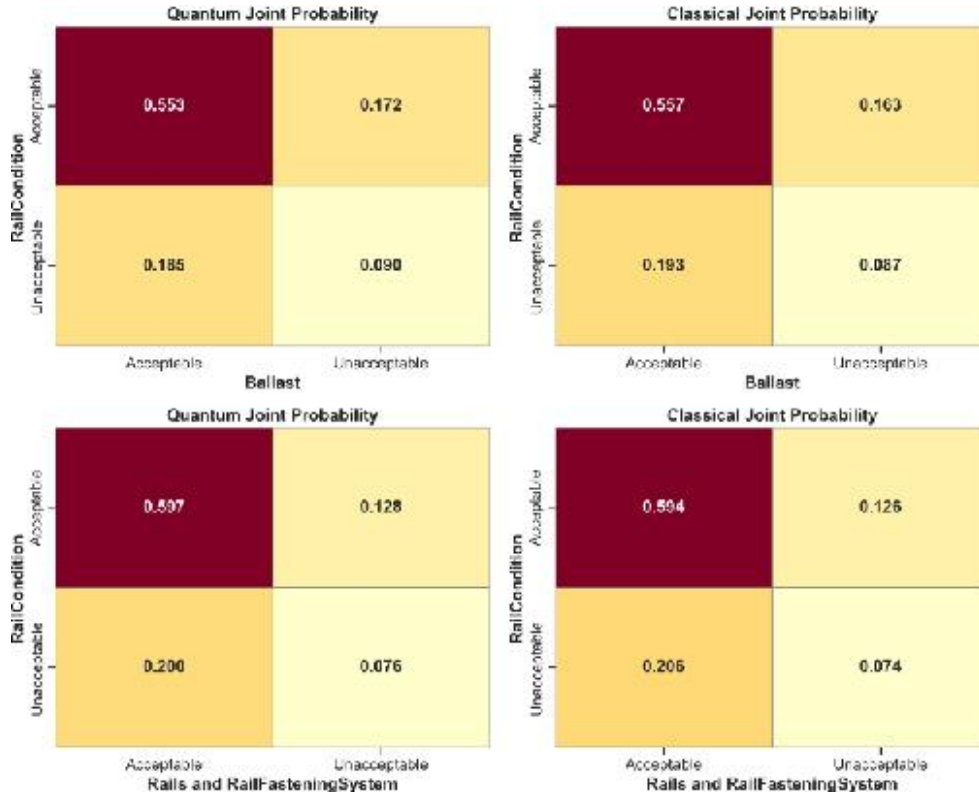


Figure 31: Cumulative probability distributions: quantum (left) versus classical (right) for various RailCondition output variables

signal, enabling the detection of bits by selecting the lowest probability derived from the amplitudes of the quantum state. The proposed QML-based demapping method significantly reduces the number of qubits required compared to existing quantum approaches and demonstrates shorter computational processing times than classical demapping methods, particularly for higher-order constellations. Experimental verification demonstrates successful training and testing for complex binary phase-shift keying (CBPSK) using only three qubits, with minimal additional qubit costs required for larger constellation sizes. Furthermore, the method effectively supports irregular modulation schemes, such as 8-quadrature amplitude modulation (8-QAM) (Figure 34), highlighting its flexibility and scalability. Overall, the results indicate that the proposed approach is a promising solution for efficient signal processing in next-generation communication systems and may contribute to the development of sixth-generation (6G) wireless technologies.

The quasi-maximum likelihood (QML) algorithm is recognised as a highly accurate estimator for polynomial phase signals (PPS) (Figure 35). To enhance its practical applicability, this article [30] presents an efficient hardware implementation of the QML algorithm. The proposed design is implemented on a field-programmable gate array (FPGA) and is built using an architecture that utilises multiple clock cycles to optimise computational efficiency and resource utilisation. The implementation has been tested on arbitrary PPS signals corrupted by white Gaussian noise, demonstrating reliable estimation results in noisy conditions. The results confirm that the proposed FPGA-based solution provides high estimation accuracy whilst enabling faster and more efficient real-time processing. This work highlights the potential of hardware-accelerated QML algorithms for signal processing applications requiring high accuracy and low latency.

Anomaly detection is a critical component of cybersecurity, enabling organisations to counter threats such as network intrusions, financial fraud, identity theft and unauthorised access to systems. Due to the rapid development of cyber-physical systems and the constant emergence of new attack patterns, traditional approaches to machine learning often prove incapable of providing comprehensive protection. Quantum machine learning (QML) has recently emerged as a promising paradigm capable of improving

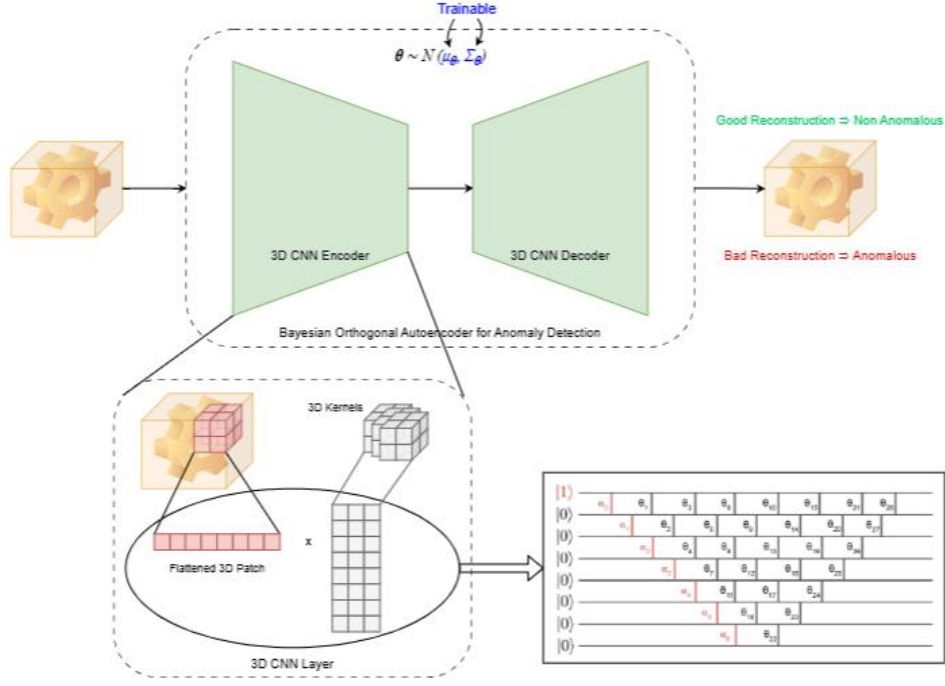


Figure 32: Overview of the proposed anomaly detection algorithm using orthogonal neural networks.

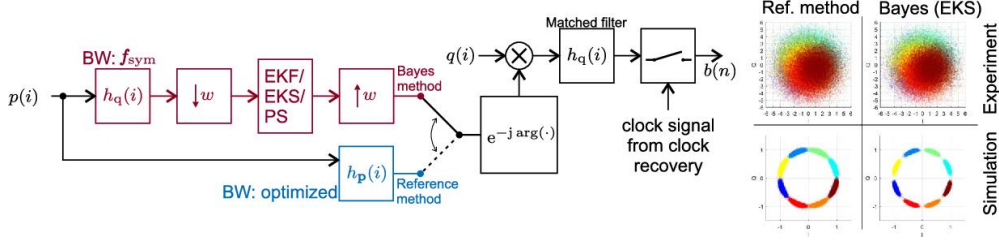


Figure 33: Estimation of the carrier phase used in the experiments and simulations. The constellation diagrams were plotted for a carrier signal-to-noise ratio (SNR) of 11 dB, whereas in the experiment the SNR of the quantum channel was -3 dB. In the simulation, the quantum channel was noise-free, as $\Delta\hat{P}_Q$ depends only on phase noise. EKF/EKS: extended Kalman filter/smoothen; PS: particle smoother; BW: bandwidth.

anomaly detection through enhanced computational efficiency and improved pattern recognition capabilities. Article [31] provides an overview of QML and its application for anomaly detection in consumer electronics. A general framework for integrating QML algorithms into anomaly detection pipelines is proposed, and key approaches within supervised, unsupervised and reinforcement learning paradigms are discussed (Figure 36). Furthermore, five recent case studies are examined to illustrate the practical implementation and performance benefits of QML methods in real-world scenarios. The results obtained highlight the potential of QML to address increasingly complex challenges in the field of cybersecurity and demonstrate that quantum methods can play a significant role in the future of anomaly detection systems.

2.10. QRAM (quantum random access memory)

Quantum computing is a new paradigm that utilises the principles of quantum mechanics to process information at the atomic and subatomic levels, offering significant advantages over classical computing. A key component of quantum systems is quantum memory, which acts as the quantum analogue of classical memory and plays a crucial role in the processing of quantum information. Quantum memory (Figure 37) ensures synchronisation between computational processes, stores quantum states in quantum

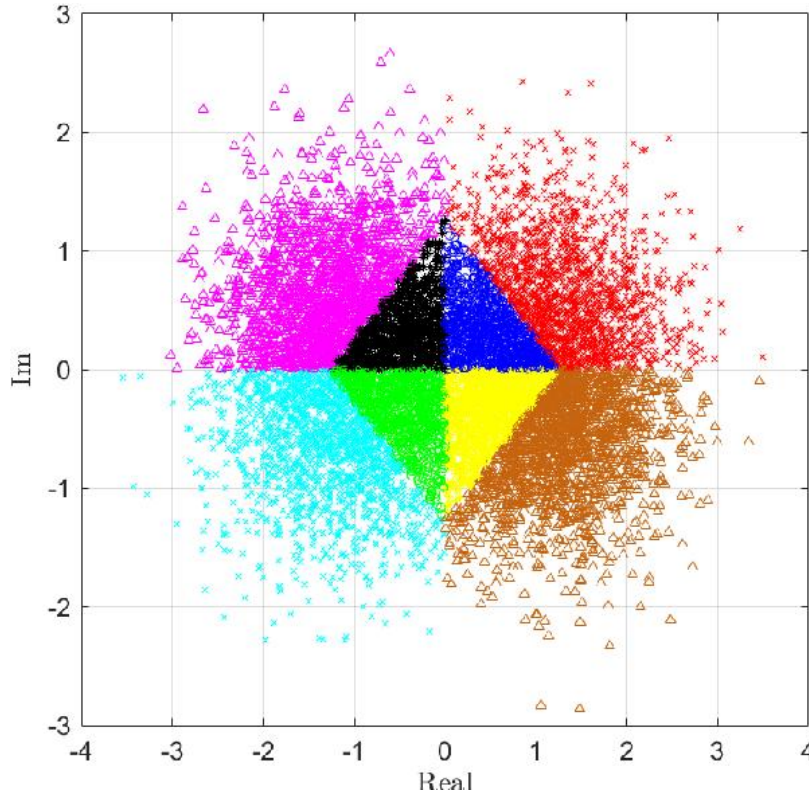


Figure 34: The proposed QML effectively demodulates 8-QAM, as evidenced by the virtual line separating the received symbol

logic gates, and supports applications such as controlled photon generation. In [32], a random-access quantum memory (QRAM) architecture is proposed, designed to exploit the parallelism inherent in quantum computing to ensure high-speed data access and processing. The proposed design is based on the fundamental operations of quantum gates, in particular CNOT, V and V+ gates, which are used to construct basic logical functions such as AND, OR, NOT and NOR. This gate-level implementation demonstrates the possibility of constructing scalable quantum memory structures capable of supporting complex quantum computations. The proposed QRAM architecture highlights the potential for efficient data processing in quantum systems and contributes to the development of high-performance quantum information processing technologies.

Convolutional neural networks (CNNs) have achieved significant success in the fields of computer vision and deep learning; however, when processing large volumes of data, they often require substantial computational resources and training time. Quantum convolutional neural networks (QCNNs) have emerged as a promising alternative, utilising quantum computing to enhance efficiency and scalability. However, current limitations of quantum hardware, particularly the limited number of qubits, necessitate resource-efficient approaches to processing high-dimensional data. In [33], a QCNN model enhanced with QRAM is proposed, which enables efficient processing of large volumes of input data. By utilising quantum random-access memory (QRAM), the model encodes data logarithmically using quantum superposition and entanglement, which significantly reduces the requirements for the number of qubits. The proposed architecture focuses on improving resource utilisation and chain depth, whilst supporting larger input sizes and an increased number of output channels. Experimental analysis demonstrates that the proposed model provides more efficient feature extraction with lower computational costs compared to traditional approaches. These results highlight the potential of integrating QRAM with QCNN (Figure 38) to create scalable and practical quantum machine learning solutions for processing large volumes of data.

Quantum machine learning (QML) combines machine learning (ML) with quantum computing (QC) by using variational quantum circuits (VQC) to solve complex learning problems. Classical data is

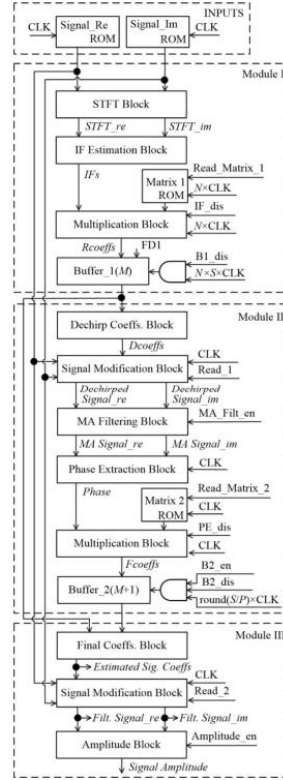


Figure 35: Hardware implementation of the estimator core based on the QML algorithm for PPS. M , N , S and P represent, respectively, the order of the observed signal, the width of the STFT delay window, the length of the observed signal and the length of the moving average filter

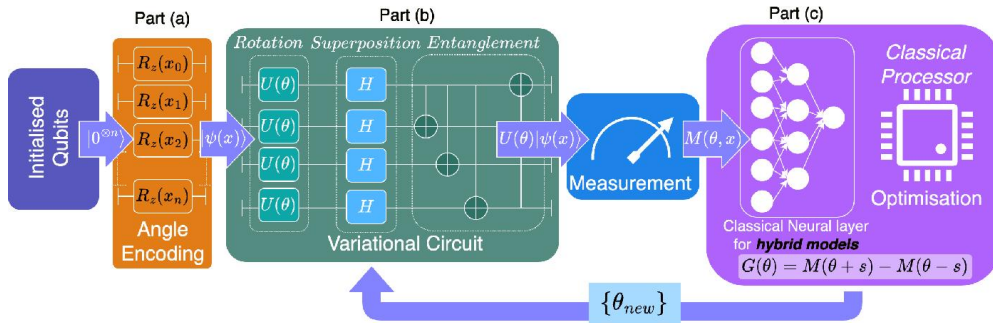


Figure 36: A variational model for QML; Part (a): Quantum encoding, Part (b): Parameterised quantum scheme, Part (c): Classical optimisation

mapped onto the quantum Hilbert space using embedding methods, which allows quantum operations to be performed using parameterised logic gates. In [34], a new embedding scheme based on random-access quantum memory (QRAM) is proposed, where address input allows data to be retrieved in a manner analogous to classical random-access memory (Figure 39). A QRAM architecture based on these schemes has been developed, and we demonstrate its application for both machine learning and binary data storage. For ML tasks, datasets of digit images are stored in QRAM and classified using a quantum neural network (QNN). The proposed QRAM-based implementation achieves faster convergence (by the 6th epoch) with 100% accuracy, outperforming standard QNNs and classical fully connected neural networks (FCNNs), which converge at later epochs. Experiments on IBM quantum hardware yield approximately 63% accuracy. For binary data storage, performance is evaluated using the Hamming distance (HD) and prediction accuracy. The results show a decline in performance as the address size increases. To address this issue, two optimisation techniques are proposed: data clustering using multiple QRAMs and bit-level partitioning for processing large data volumes. Clustering significantly improves

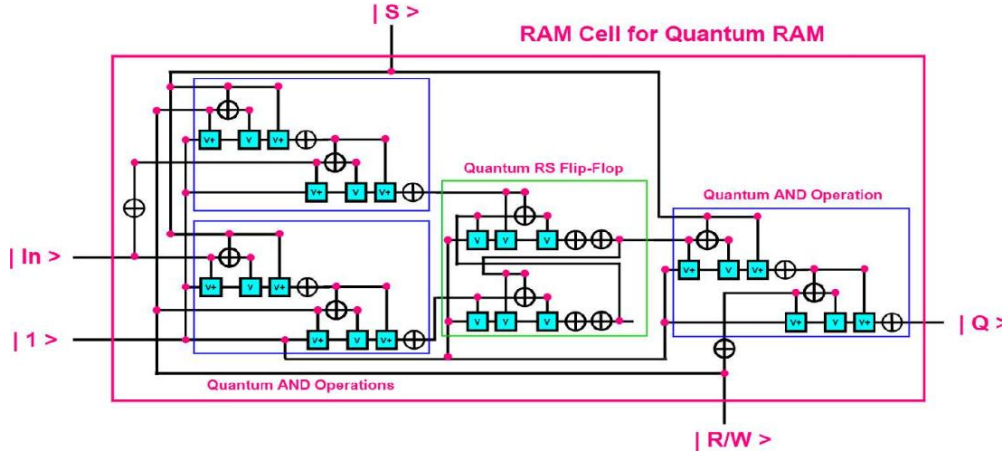


Figure 37: Quantum cell of random-access memory

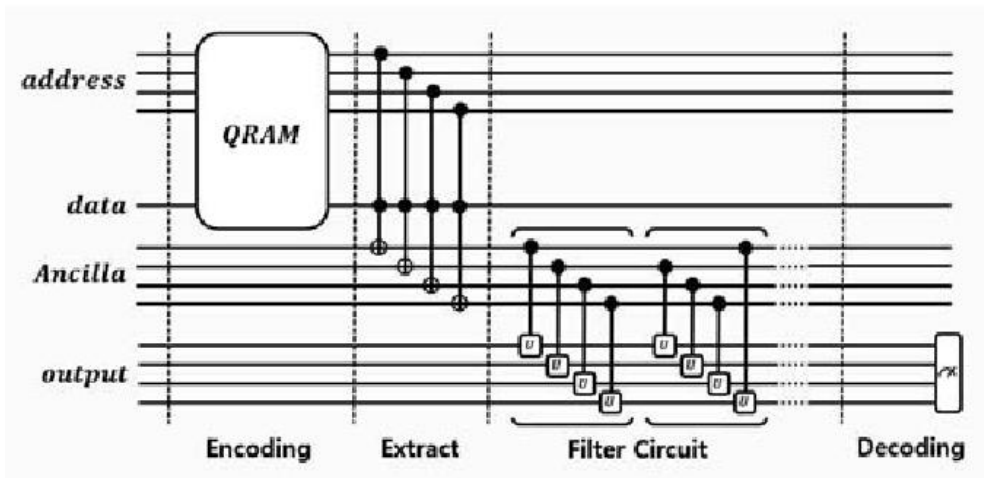


Figure 38: A quantum diagram illustrating an example of a QCNN using QRAM. The feature extraction circuit extracts the required portion of the feature maps, enabling them to be processed efficiently in the filtering circuit thanks to the parallelism of quantum computing.

performance, achieving a $1.95\times$ improvement compared to standard QRAM and outperforming bit-level partitioning-based approaches. Overall, this work demonstrates that QRAM-based embedding enhances both training efficiency and data storage performance, offering a scalable approach for quantum machine learning applications.

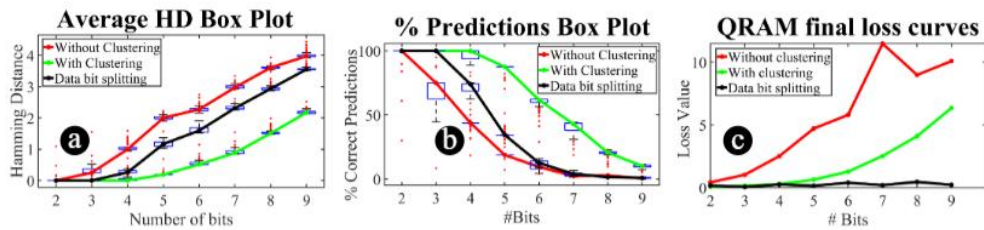


Figure 39: Results for binary QRAM without clustering, with clustering, and with data-bit partitioning: (a) Hamming distance (HD), (b) percentage of correct predictions, and (c) cumulative loss curve for different numbers of address lines.

2.11. QAOA (quantum approximate optimization algorithm)

In recent years, research into quantum algorithms has advanced rapidly, and cloud-based quantum computing platforms have become a vital environment for developing algorithms and conducting experiments. Among these algorithms, the quantum approximate optimisation algorithm (QAOA) has demonstrated significant potential for solving combinatorial problems, such as the ‘maximum cut’ problem. However, the performance of QAOA is significantly influenced by the choice of classical optimisers and the presence of noise in the quantum hardware. In the study [35], the HiQ quantum computing simulation platform was used to experimentally evaluate the performance of QAOA under various conditions. In particular, the influence of different optimisation strategies, noise models and noise parameter settings on the quality of solutions, measured by the maximum number of cut edges, was analysed. The results demonstrate that both the choice of optimiser and the noise characteristics play a decisive role in determining the algorithm’s effectiveness (Figures 40, 41). Furthermore, the broader implications of these findings for the future development of quantum algorithms are discussed, emphasising the need for robust optimisation methods and noise reduction strategies. This work contributes to a deeper understanding of the practical challenges in implementing quantum algorithms and identifies avenues for improving performance on quantum systems in the near future.

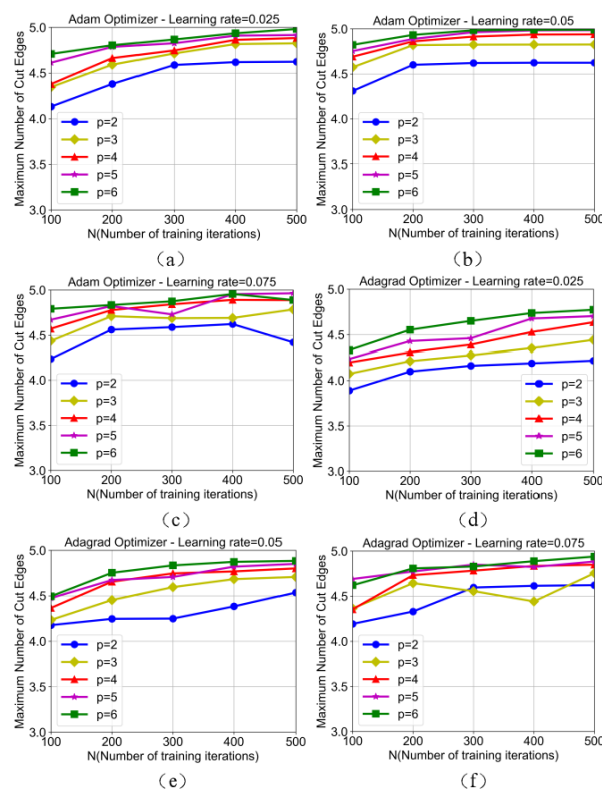


Figure 40: Conduct an experiment with QAOA using the Adam optimiser (a)(b)(c) and the Adagrad optimiser (d)(e)(f).

The Quantum Approximate Optimisation Algorithm (QAOA) has emerged as a promising approach to solving optimisation problems in the era of noisy intermediate-scale quantum (NISQ) systems, particularly those formulated as quadratic unbounded binary optimisation (QUBO) problems. Due to existing hardware limitations, classical quantum simulators play a crucial role in evaluating the performance of QAOA. However, the performance and accuracy of QAOA can vary significantly depending on the modelling method employed. The study [36] analyses the performance of QAOA for various quantum modelling methods, including state vector, density matrix and matrix product state methods. It also investigates the benefits of integrating high-performance computing (HPC) with quantum computing (QC) resources within an active learning workflow. Experiments are conducted

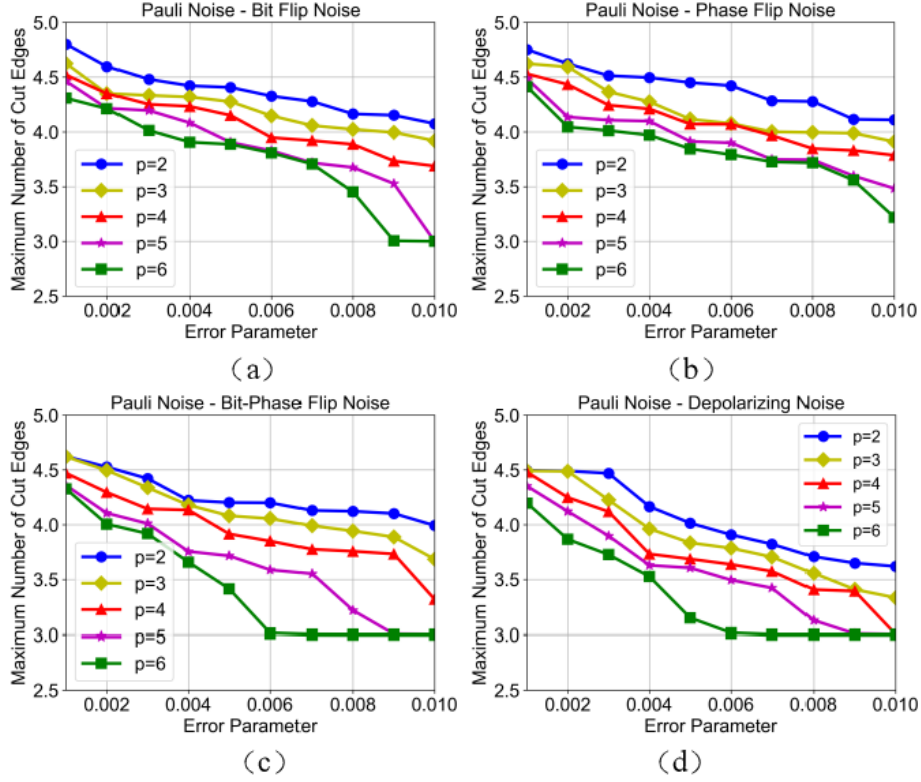


Figure 41: Experiment with QAOA under Pauli noise conditions for the following scenarios: bit flipping (a), phase flipping (b), bit and phase flipping (c), and depolarisation (d).

on large-scale, dense QUBO instances to evaluate solution quality and computation time (Figure 42). Furthermore, the performance of QAOA is compared in local computing environments and integrated HPC-QC systems, utilising resources such as the Frontier supercomputer at the Oak Ridge Leadership Computing Centre, as well as local Qiskit Aer simulators and remote IBM Quantum simulators. The results show that HPC-QC integration significantly improves scalability and performance, enabling more efficient exploration of complex optimisation problems.

The multi-angle quantum approximate optimisation algorithm (ma-QAOA) has been proposed as an alternative to standard QAOA for solving combinatorial optimisation problems on noisy intermediate-scale quantum (NISQ) devices. In [37], QAOA and ma-QAOA are implemented and compared (Figure 43) for solving the MaxCut problem on both weighted and unweighted Erdős–Rényi graphs and random cubic graphs with 6 and 10 vertices. The circuit parameters are iteratively optimised using a classical feedback loop, whilst the forward pass is simulated using Qiskit without noise. The optimised circuits are then evaluated under three settings: noise-free simulation, simulation with experimentally calibrated noise models, and execution on real quantum hardware (the 127-qubit IBM ibm_brisbane processor). The results show that ma-QAOA consistently outperforms standard QAOA in terms of approximation factor, whilst requiring fewer circuit layers under all evaluation conditions. We also extend the analysis to random 14-node cubic graphs under noise-free simulation conditions, where ma-QAOA again demonstrates competitive performance with reduced circuit depth, highlighting its efficiency and suitability for quantum devices of the near future.

3. Multi-criteria analysis of the algorithms

Table 1, based on previous research, systematizes and presents a detailed analysis of approaches to ensuring and assessing sustainability according to the following 10 criteria (proposed by the author).

1. Quantum speedup (QS) – The degree to which a quantum algorithm asymptotically outperforms

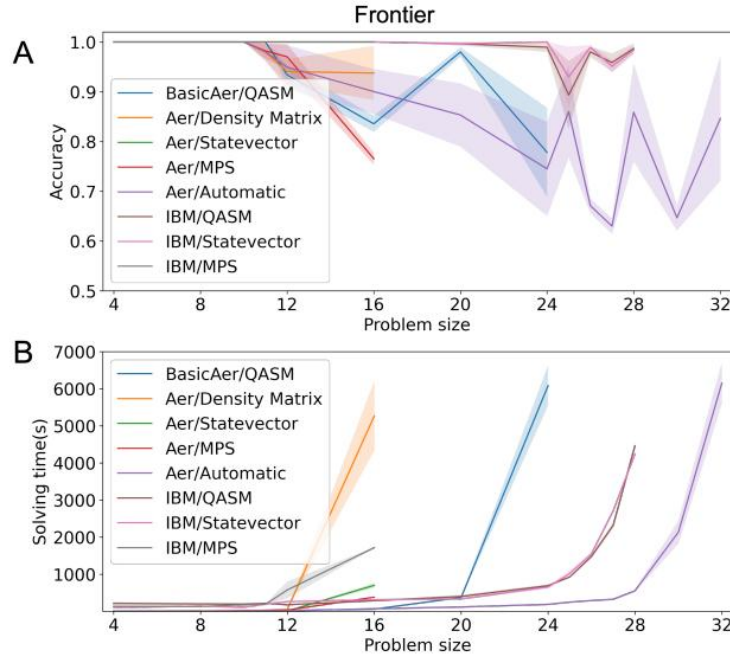


Figure 42: (A) Accuracy and (B) solution times for QAOA problems on QUBO, executed or submitted on the HPC-QC integrated system (Frontier).

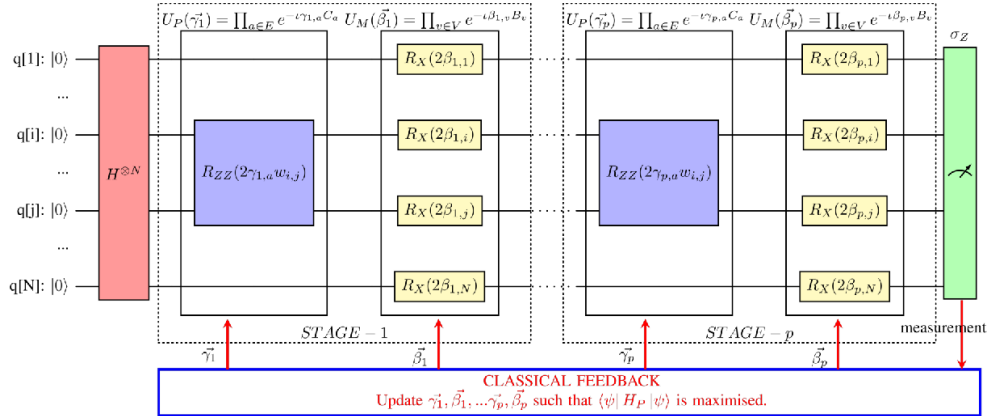


Figure 43: A ma-QAOA scheme for solving the MaxCut problem using a quantum forward-passing circuit consisting of single-qubit RX and multi-qubit RZZ logic gates, as well as gate parameters updated via classical feedback (for further details on the parameters, see Section IIC of the text).

the best known classical algorithm for the same problem, typically measured in terms of gate or time complexity (e.g., exponential vs polynomial scaling).

2. Resource complexity (RC) – The total “quantum cost” of the algorithm, usually expressed via the scaling of qubit count, two-qubit gate count, and circuit depth with input size; often analyzed in big- O notation.
3. Noise resilience (NR) – The robustness of an algorithm to decoherence, gate errors, and readout noise; algorithms with high noise resilience degrade gracefully and maintain acceptable success probability even on imperfect hardware.
4. State preparation complexity (SPC) – The number and type of gates (or other resources) required to prepare the initial quantum state or input states for the algorithm from a simple reference state such as $|0\rangle \oplus^n$.
5. QRAM dependence (QD) – The extent to which the algorithm relies on a quantum random-access memory model: strong QRAM dependence implies the algorithm is only efficient if rapid, coherent

access to large classical data sets is available, which is not realistic on near-term hardware.

6. Scalability (S) – How well the algorithm’s performance and resource requirements scale with problem size; a scalable algorithm maintains reasonable qubit count, circuit depth, and fidelity as the input grows.
7. NISQ compatibility (NC) – The degree to which the algorithm can be implemented on noisy intermediate-scale quantum devices, i.e., with shallow depth, moderate qubit count, and tolerance to low-fidelity gates and connectivity constraints.
8. Practical applicability (PA) – The feasibility of actually deploying the algorithm in real-world settings, considering not only theoretical complexity but also hardware limitations, data encoding, calibration overhead, and expected runtime or error budget.
9. Generality/framework capability (GC) – The breadth of problem classes the algorithm can address, or how well it fits into a broader framework (e.g., variational ansatzes, amplitude amplification, or quantum walks) that can be adapted to many tasks without re-deriving the core scheme.
10. Future optimization potential (FOP) – The scope for improving the algorithm using techniques such as better ansätze, error mitigation, compiler optimization, or tighter complexity bounds, so that it can gain more advantage as hardware and theory evolve.

Table 1
Multi Criteria Analysis of the Algorithms

No.	Algorithm	Criteria									
		QS	RC	NR	SPC	QD	S	NC	PA	GC	FOP
1	HHL	+	-	-	-	-	+-	-	+-	+	+
2	QPE	+	-	-	+-	+-	+-	-	+-	+	+
3	QSVT	+	+-	+-	+-	+-	+	+-	+-	+	+
4	qPCA	+	+-	+-	-	-	+	+-	+-	+-	+
5	VQLS	+-	+	+	+-	+	+-	+	+	+-	+
6	QAE	+-	+-	+-	+-	+-	+	+-	+	+	+
7	Quantum Monte Carlo	+-	-	+-	+-	+-	+	+-	+	+	+
8	Quantum Bayesian Inference	+-	-	+-	-	+-	-	+-	+-	+	+
9	QML	+-	+-	+-	-	+-	+	+-	+	+	+
10	QRAM	-	-	+-	-	-	-	-	-	+-	+
11	QAOA	+-	+	+-	+-	+	+	+	+	+-	+

Table 1 shows that none of the analyzed algorithms meets all criteria simultaneously. Most often, high performance is achieved for the criteria Quantum speedup (QS), Scalability (S), and Future optimization potential (FOP), while Resource complexity (RC) and State preparation complexity (SPC) remain insufficiently addressed for most methods. Algorithms focused on full fault tolerance (HHL, QPE) are more centered on theoretical performance, while variational approaches (VQLS, QAOA) focus on noise resilience and compatibility with NISQ devices. Noise resilience (NR) and QRAM dependence (QD) are underrepresented in a number of promising algorithms, indicating the presence of a scientific and technological gap. This underscores the need to develop an integrated evaluation methodology that combines theoretical complexity analysis with the real hardware limitations of modern quantum processors, while also taking into account all of the listed functional features and advantages.

4. Conclusions

Quantum algorithms for linear algebra problems, such as HHL, QPE, and QSVT, form the foundation for processing large datasets, as they allow vectors and matrices to be represented as quantum states. The HHL algorithm provides a theoretical exponential speedup when solving systems of linear equations; however, its practical accuracy depends significantly on the noise level during the rotation of auxiliary qubits and the inverse phase estimation stages. Despite these limitations, HHL is already being used

to analyze power flows in power systems, where it demonstrates faster convergence compared to the classical Newton-Raphson method. A more general approach is QSVT, which serves as a meta-algorithm for performing polynomial singular value transformations of matrices. This allows for the construction of optimal solutions for complex control problems, particularly within the framework of linear-quadratic Gaussian (LQG) models, where asymptotic acceleration is achieved compared to classical Ricatti recursions.

For modern NISQ-era devices, characterized by high noise levels, hybrid quantum-classical methods such as VQLS and QAOA are the most suitable. The VQLS variational linear solver uses shallower schemes, making it more resistant to decoherence when solving radar and hydrodynamics problems. In turn, the QAOA algorithm and its improved multi-angle version, ma-QAOA, effectively solve combinatorial optimization problems, ensuring high-quality approximations even with limited hardware resources. An important aspect of these algorithms is the ability to integrate classical precomputations to simplify quantum schemes, which helps balance solution accuracy and computational complexity.

In the fields of statistics and machine learning, quantum methods, particularly the quantum amplitude estimation (QAE) algorithm, provide quadratic acceleration for Monte Carlo computations, which is critical for financial modeling and risk assessment. New approaches, such as the quantum iterative Fourier amplitude estimation (QFIAE), allow for numerical integration without the need for explicit evaluation of coefficients, reducing overall computational costs. In addition, quantum Bayesian networks (QBN) are successfully applied to predict the condition of railway infrastructure, demonstrating high convergence with classical probabilistic models. Quantum machine learning (QML) is also finding applications in anomaly detection in consumer electronics and 3D objects, where the use of orthogonal neural networks ensures training stability and effective recognition of structural defects.

Despite its significant potential, the main barrier to scaling remains the lack of practical implementation of quantum random-access memory (QRAM), which is necessary for logarithmic loading of classical data into most linear algebra algorithms. Although the use of QRAM in convolutional networks (QCNN) allows for a significant reduction in the number of qubits required for image processing, the complexity of building such systems remains high. Multi-criteria analysis confirms the existence of a scientific and technological gap: algorithms with the highest theoretical speedup (HHL, QPE) have the lowest noise robustness, while robust variational methods require complex classical optimization procedures. Thus, the development of the field requires the creation of integrated methodologies that combine theoretical complexity analysis with the real-world constraints of modern quantum processors.

Declaration on Generative AI

The authors have not employed any Generative AI tools.

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