

AnRoll: Analytic Unrolling of Painted Pottery Surfaces

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Abstract

Analyzing painted motifs on ancient pottery requires flattened, minimally distorted surface representations. To obtain a flattened and minimally distorted image of the vessel's surface, motifs have traditionally been carefully traced through thin translucent paper. Computational rollout methods, such as projection-based unrolling and Elastic Flattening, reduce distortions but suffer from artifacts, asymmetry, and limited interactivity due to computational requirements. In this work, we present a novel analytic unrolling method based on a low-dimensional mathematical model with four intuitive parameters, derived from observations of Elastic Flattening. Our model produces symmetric rollouts without common artifacts while supporting real-time parameter adjustments. We implement the method in a GPU-accelerated C++ application and also publish an open-source tool, offering an intuitive graphical user interface and supplementary features. Compared to Elastic Flattening, our model achieves similar visual results while drastically reducing execution time, making it well-suited for archaeological research and digital heritage applications.

Keywords

ancient pottery, cultural heritage, analytic unrolling, vessel surface, surface mapping, flattening

1. Introduction

In archaeological research, the study of ancient pottery, specifically the painted figures and ornaments, is of great interest [1]. The analysis of motifs and styles gives us a window into the past. It can inform us about chronological developments, the classification of distinct workshops, and, in some cases, even direct attribution to specific painters [2, 3].

The curvature of the vessels introduces significant distortions to the vase paintings when viewed from a single direction, so methods for creating flattened 2D representations of the 3D surfaces are of great significance in this area of research. These unrollings give a holistic view of the vessel's surface with decreased distortion and without segmentation into multiple images, which is advantageous for analysis. Traditionally, such unrollings have been produced by manually tracing the motifs through translucent tracing paper. This method is not only time-consuming and prone to errors, but also, due to the fragility of the surfaces, often not possible. This is why computer-assisted techniques have increasingly gained traction in the research of ancient pottery over the past decades [4, 5].

Data acquisition for producing these unrollings involves either constructing high-quality 3D scans using a structure from motion photogrammetry technique [6] or taking several photographs and stitching them together to get a high-resolution texture map [7, 8]. The most common rollout creation technique is projection-based, where points on the 3D mesh are projected onto proxy surfaces like cones, cylinders, or spheres. Projection-based methods introduce significant distortions if these surfaces cannot approximate the vessel shape well. A more recent technique, Elastic Flattening [9, 8], improves upon projection-based methods by reducing surface distortions through a global minimization process based on a physical relaxation simulation. Elastic flattening, however, comes with its own set of drawbacks. Depending on the number of vertices used during this process, producing a single result often takes several minutes on standard hardware. The simulated relaxation process can also introduce asymmetry, surface folding, and a characteristic outward bend at the corners of the result.

In this paper, we propose a novel, significantly more efficient approach to creating 2D rollouts using

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an analytic mathematical model with only four intuitive parameters. Our model builds upon Elastic Flattening and, in part, emulates its behavior. It always produces perfectly symmetric results without the aforementioned artifacts, and allows for real-time parameter tuning, even on weaker hardware.

2. Related Work

Developable 3D surfaces, such as cones or cylinders, can be flattened to a 2D representation without introducing distortion. However, the surfaces of interest in archaeological research are typically non-developable and cannot be flattened without introducing some distortion. Related work in this area concerns the holistic or partial unrolling of non-developable rotational bodies with minimal distortion.

The most naive and common approach to this problem is to fit a proxy geometry to the target object and project points from the object’s surface onto the proxy geometry. The GigaMesh Software [10] is a widely used tool to create rollouts using either cones, cylinders, or spheres as the proxy geometry [11, 12, 13]. These rollouts exhibit large distortions for vessels with highly irregular shapes.

Following this principle, Bechtold et al. [14] propose an approach where the target vessel is vertically subdivided into segments based on painted motif and/or surface curvature, and an optimal proxy geometry is used to flatten these parts individually. Rieck et al. [15] demonstrate this approach on high-resolution meshes. It constitutes a significant improvement over the naive method if the vessel can be segmented well without cutting through painted motifs, but the resulting rollouts are non-continuous.

Elastic Flattening [9] implements a global elastic relaxation process, where the vertices of an initially naively flattened 3D mesh represent nodes connected to their neighbors by springs. The simulation of this mass-spring system attempts to globally minimize the length distortion of individual edges in relation to the length of edges on the original model. Results of this approach are continuous, but can exhibit surface folding artifacts and are usually not symmetric around the vertical center axis, a property that is generally desirable. Houska et al. [8] partially solve these problems by using a uniform triangle mesh defined on a cylindrically unrolled image texture, but asymmetry remains in cases of irregular distance maps due to bad input geometry. Elastic Flattening results also display an outward bend in the corner regions, an artifact of the physical simulation process, which may be undesirable.

To address these limitations, we propose an analytic unrolling approach that produces continuous, symmetric rollouts while substantially reducing computational demands, enabling an interactive workflow with responsive parameter control for domain users.

3. Approach

We use results from the Elastic Flattening approach as a reference to construct our analytic flattening model. To that end, 3D models of different widths and profile shapes were unrolled using elastic relaxation. We then measured the lengths of vertical and horizontal arcs and segments in both the

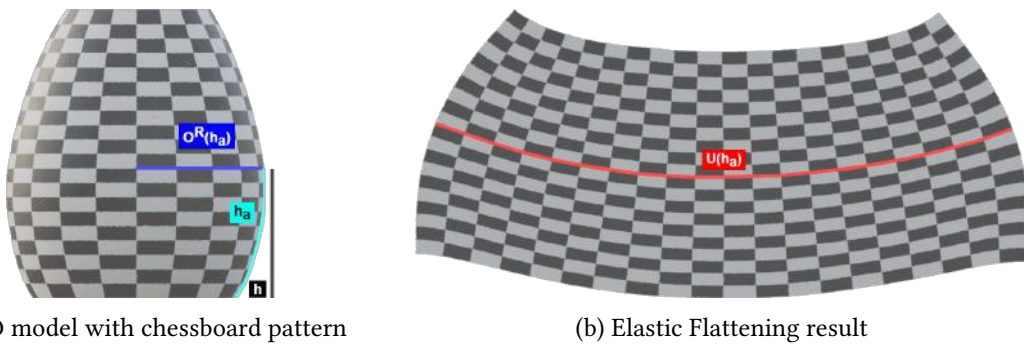


Figure 1: Sample model and Elastic Flattening result used for our segment length measurements (alabastron shape), including illustrations of relevant length measures h , h_a , $O^R(h_a)$, and $U(h_a)$.

original model and the unrolled result to examine the resulting length distortions. To obtain consistent reference arcs and segments for measurement, we applied a 20×20 chessboard pattern to each of the input 3D models before unrolling (Figure 1). The inspection of the resulting mapping leads to two fundamental observations.

Observation 1. Let $h_a := h_a(h)$ denote the absolute arc length along a vertical contour on the vessel surface from its bottom boundary up to a height h , $O^C(h_a)$ the circumference, and $O^R(h_a)$ the corresponding radius of the original vessel at this height (Figure 1a). Then by $U(h_a)$ we denote the corresponding horizontal arc length of this contour in the unrolled result (Figure 1b).

We found that $U(h_a)$ can be well approximated by a linear interpolation between the circumference function $O^C(h_a)$ and the linear least-squares regression line $L(h_a)$ of $O^C(h_a)$ using an appropriate interpolation factor α_{IF} . Figure 2 illustrates this observation for our measurements on the alabastron-shaped vessel shown in Figure 1a.

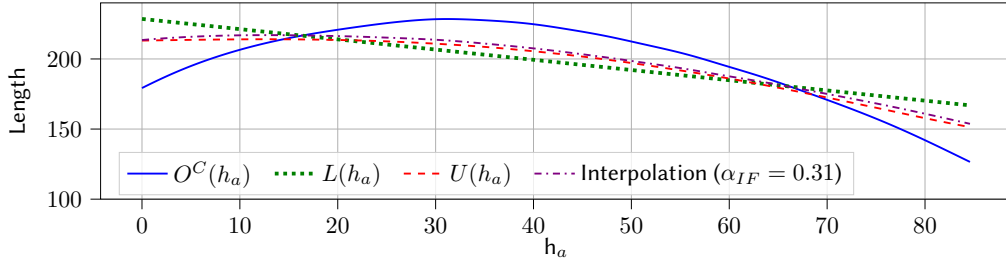


Figure 2: Length approximation of Elastic Flattening arc lengths $U(h_a)$ by linear interpolation between the circumferences $O^C(h_a)$ and its linear fit $L(h_a)$ with an interpolation factor α_{IF} .

Observation 2. We also observed that uniform segment lengths along a horizontal arc on the original surface are not mapped uniformly onto the flattened image. The mapping near the center in the output image fits more closely with a conical unrolling. Figure 3 shows that the measured horizontal arcs $U^I(h_a)$, representing the inner 50% of the unrolled image normalized to the full length, closely resemble the regression line $L(h_a)$. Contrarily, the normalized arc lengths when measuring only the outer 50% of segments $U^O(h_a)$ compensate in the opposite direction towards the real circumferences $O^C(h_a)$.

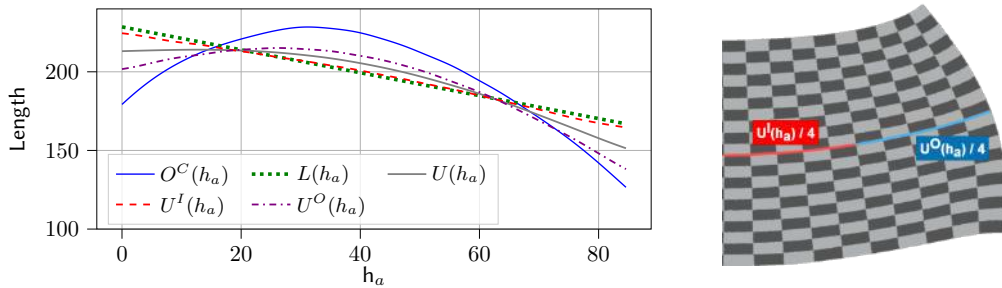


Figure 3: Comparison between the inner 50% ($U^I(h_a)$) and the outer 50% ($U^O(h_a)$) of measured horizontal segments' arc lengths of the Elastic Flattening result, indicating a non-uniform length distribution along horizontal arcs. The image to the right illustrates how we measured and calculated these lengths.

In the following, we incorporate these observations for the design of an efficient, low-dimensional mapping model that still provides a large space of result mappings.

3.1. Model Design

Based on the observations in the reference mappings above, this section describes our derived analytic mapping model and its four main parameters, the interpolation factor α_{IF} , the radius factor λ_{RF} , the

uniformity β_U , and Enforce Isotropy δ_{EI} . Together, they parameterize a coordinate mapping from a 2D input texture, which represents an initial cylindrical unrolling of the target vessel, to a vertically symmetric curvilinear grid. Our model design is based on the fundamental assumption that the target vessels are rotationally symmetric. Due to the nature of pottery manufacturing using a potter’s wheel, this assumption holds true in most practical cases.

3.1.1. Image Mapping

Let $x \in [-1, 1]$ and $y \in [0, 1]$ be normalized horizontal and vertical image coordinates in the input domain, where $x = 0$ represents the horizontal center, $|x| = 1$ the seam, $y = 0$ and $y = 1$ the bottom and top, respectively. Further, let $h_a := h_a(yH)$ be the absolute vertical arc length parameterized by y , where H denotes the absolute height of the vessel.

Our analytic model maps a point $P_{x,y}$ from the input domain to a point $\hat{P}_{x,y}$ in the unrolled image. This mapping follows a circular arc in the horizontal direction, defined by three quantities: The center point $C(h_a)$, its radius $R(h_a)$, i.e., the distance of the center points to the center of a horizontal arc, and the displacement along the contour $D(x, h_a)$, which determines the opening angle $\phi(x, h_a)$ (illustrated in Figure 4). The mapping is thus defined as

$$\hat{P}_{x,y} = (R(h_a) \sin \phi(x, h_a), C(h_a)_{(y)} - R(h_a) \cos \phi(x, h_a)). \quad (1)$$

Below, we describe the elements of this mapping function in detail.

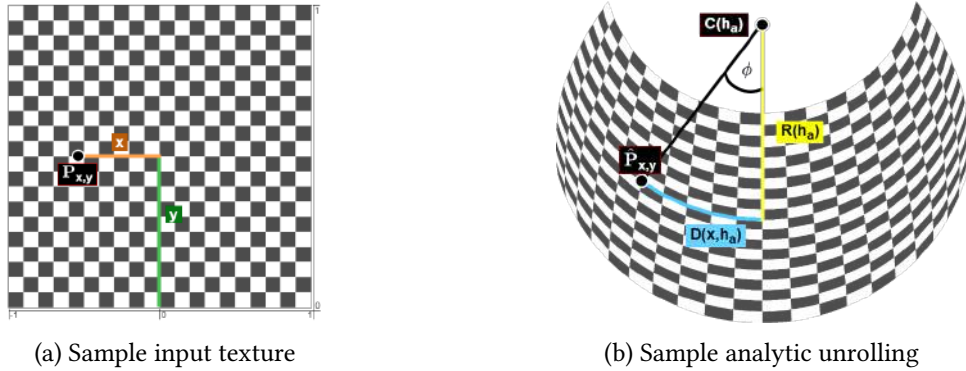


Figure 4: Mapping from $P_{x,y}$ to $\hat{P}_{x,y}$ for $x = -0.5$ and $y = 0.5$, showing the center point $C(h_a)$ of a circular grid line $U(h_a)$, its radius $R(h_a)$, horizontal displacement $D(x, h_a)$ along this line, and displacement angle $\phi(x, h_a)$.

3.1.2. Interpolation

Based on observation 1, the interpolation factor $\alpha_{IF} \in [0, 1]$ globally defines the total absolute length of horizontal arcs as

$$U(h_a) = L(h_a) + \alpha_{IF} \cdot (O^C(h_a) - L(h_a)). \quad (2)$$

At $\alpha_{IF} = 0$, the result resembles a conical unrolling, whereas at $\alpha_{IF} = 1$, the horizontal arc lengths equal the circumferences of the object (see Figure 5).

3.1.3. Curvature

A perfect flattening of a cone results in a mapping where horizontal lines of the input grid map onto circular contours. Elastic flattenings of approximately cone-like objects exhibit a similar overall curvature, as seen in the upward-curved mapping in Figure 1b.



Figure 5: Shape change introduced by $\alpha_{IF} \in \{0, 1\}$ when unrolling the model shown in Figure 1a.

In our model, we therefore also map points along circular contours. The radius of the respective circle is defined as a multiple of the given vertical arc length $U(h_a)$, expressed by

$$R(h_a) = \frac{U(h_a)}{2\pi} \lambda_{RF}. \quad (3)$$

The radius factor λ_{RF} used in this expression provides an intuitive and consistent radius parameterization. A radius factor of $\lambda_{RF} = 1$ in conjunction with the division by 2π describes a particular type of unrolling, namely a “rolled up” appearance, as shown in the second column of Table 1. An additional parameterization of the radius is necessary because the exact radius required for an optimally unrolled result is dependent on the shape of the target object. A sensible radius factor generally satisfies $\lambda_{RF} \in \mathbb{R} \setminus (-1, 1)$, because a value $\lambda_{RF} \in (-1, 1)$ leads to an overlap of the mapped horizontal arcs. Conversely, if λ_{RF} diverges towards $\pm\infty$, the arcs become lines (see Table 1, right column).

To ensure symmetry, the corresponding center point, defined as

$$C(h_a) = (0, R(h_a) + h_a), \quad (4)$$

lies on the central vertical axis and is offset by h_a in addition to the circle’s radius. This leads to a vertically distortion-free mapping at $x = 0$, where the distance between two mapped points $\hat{P}_{0,0}$ and $\hat{P}_{0,1}$ is equal to the absolute vertical arc length of the vessel.

3.1.4. Non-Uniform Mapping

To model observation 2, it is necessary to vary the interpolation factor along the horizontal contours of the resulting mapping. To this end, we define a local interpolation factor $A_{IF}(x)$, which varies between different normalized horizontal image positions based on a quadratic function

$$A_{IF}(x) = ax^2 + \beta_U. \quad (5)$$

Note that the integral over the local interpolation factors has to match the global interpolation factor α_{IF} in order to produce the same total arc length $U(h_a)$, i.e.,

$$\int_0^1 A_{IF}(x) dx = \frac{a}{3} + \beta_U = \alpha_{IF}. \quad (6)$$

We choose β_U as the model parameter to control the uniformity of the mapping, as it intrinsically describes the local interpolation factor at $x = 0$ and intuitively defines the curvature of $A_{IF}(x)$ to be convex ($\beta_U < \alpha_{IF}$), concave ($\beta_U > \alpha_{IF}$) or zero, resulting in a uniform mapping ($\beta_U = \alpha_{IF}$), see Figure 6. The missing parameter a can then be derived from Equation (6) as $a = 3 \cdot (\alpha_{IF} - \beta_U)$ to find a suitable quadratic function. At given height h_a , the local arc-length at x is given by

$$U(x, h_a) = L(h_a) + A_{IF}(x) \cdot (O^C(h_a) - L(h_a)). \quad (7)$$

Note that in order to obtain a continuous mapping without singularities or overlaps, we have to ensure that all local arc lengths $U(x, h_a) > 0$. We derive suitable boundaries for the free parameter β_U that meet this constraint as shown in Appendix A.

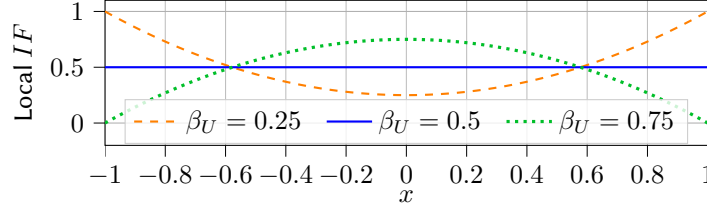


Figure 6: $A_{IF}(x)$ for $\alpha_{IF} = 0.5$ and different values of β_U

Given $U(x, h_a)$ allows us to express the ratio between the local and global arc-lengths by

$$\rho(x, h_a) = \frac{U(x, h_a)}{U(h_a)}. \quad (8)$$

This even function with the property $\int_0^1 \rho(x, h_a) dx = 1$ can then be used to calculate the horizontal displacement along the circular contour,

$$D(x, h_a) = \text{sgn}(x) \frac{U(h_a)}{2} \int_0^x \rho(u, h_a) du, \quad (9)$$

shown in Figure 4b. Uniformly sampled points in the input domain then lead to a non-uniform mapping based on the local interpolation factors. Finally, given this displacement $D(x, h_a)$ and the radius $R(h_a)$, the opening angle $\phi(x, h_a)$ in radians is expressed by

$$\phi(x, h_a) = \frac{D(x, h_a)}{R(h_a)}. \quad (10)$$

3.1.5. Enforcing Central Isotropy

The previously described mapping approximates Elastic Flattening and displays no vertical distortion $K_V(x, h_a)$ at $x = 0$ due to the parameterization by h_a . However, unless $\alpha_{IF} = 1$ and $\beta_U = 1$, a distortion along horizontal arcs will be introduced. This discrepancy introduces anisotropic distortion as illustrated in Figure 7, where equal-sized circles on an input model (Figure 7a) become elliptical (Figure 7b) while the vertical size remains constant.

We define the horizontal distortion factor K_H as the ratio between the local horizontal arc length and the corresponding circumference on the original vessel, given by

$$K_H(x, h_a) = \frac{U(x, h_a)}{OC(h_a)}. \quad (11)$$

We extend our model by a fourth parameter δ_{EI} , with the aim of minimizing anisotropic distortion toward the x-center of the unrolling. This is done by introducing the constraint $K_V(0, h_a) = K_H(0, h_a)$, i.e., ensuring the distortion of the vertical mapping at the central axis is the same as the horizontal distortion. This is achieved by summing the differential horizontal distortion factors up to h_a at the center axis and using the result to calculate the center points in place of h_a . This extends the definition of the center point as follows:

$$C(h_a) = \left(0, R(h_a) + \begin{cases} h_a & \text{if } \delta_{EI} = 0, \\ \int_0^{h_a} K_H(0, u) du & \text{if } \delta_{EI} = 1 \end{cases} \right). \quad (12)$$

If $\delta_{EI} = 1$, this extension changes the central vertical mapping, leading to no anisotropic distortion at $x = 0$. Figure 7c shows that the unrolled circles now preserve conformality along the central axis, with the disadvantage that they are no longer of uniform size. Nevertheless, this extension can be useful in circumstances where the proportions of motifs, especially near the x-center, are important.

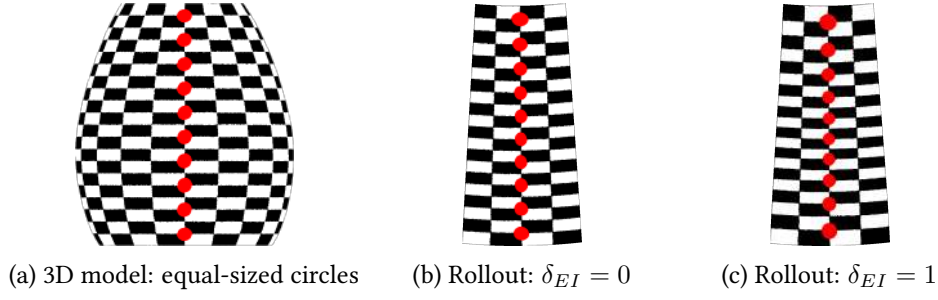


Figure 7: Rollout comparison of a model with filled red circles along the center axis. For $\delta_{EI} = 0$, the circles become ellipses, whereas for $\delta_{EI} = 1$, the circles retain their shape but not relative sizes.

4. Implementation

We implement our model in C++, available either as a command-line tool or as a backend server application. The model takes as input a 2D texture (cylindrical rollout) as well as a binary profile mask of the target vessel. For our analytic rollout, the point mapping and rendering are performed on the GPU, and the necessary preparation steps are done on the CPU. Our implementation also supports supplementary features like cropping the unrolling based on the area of interest, rotating the texture horizontally to center different parts of the vessel, and performing a weighted minimization of the four distortion measures described in Section 5.1.

We also created a user-friendly, interactive frontend application implemented in TypeScript using the ReactJS [16] library and available both as a browser version (frontend) and as an Electron [17] app (frontend + backend) for Debian and Windows systems. The tool can be accessed online¹ and is published under an open-source license.

Generation of the required input data based on either a 3D mesh or a single orthogonal photograph is possible using this tool. All features of the command-line tool are accessible through the user interface, together with further features, such as visual region of interest selection, distortion map overlays, creating parameter transition animations, and more.

5. Results and Discussion

In the following, we evaluate our model by performing several parameter studies, considering different distortion measures. We compare results and execution times to those of Elastic Flattening [9].

5.1. Distortion Measures

Throughout the evaluation, we will refer to four distinct measures of distortion, which are all defined as ratios between related quantities from the target and the source domain, such that a measure of 1 represents a distortionless mapping.

Horizontal Distortion. As introduced in Section 3.1.5, the horizontal distortion factor $K_H(x, h_a)$ describes the length conformity of local arc lengths $U(x, h_a)$ to the corresponding circumferences of the original vessel $O^C(h_a)$.

Vertical Distortion. Similarly, the vertical distortion factor measures the length conformity in the vertical direction. Let $\hat{P}_{1,x,y}$ and $\hat{P}_{2,x,y+dy}$ be two points in the mapped image that differ by a differential step dy in the input domain. Then the vertical distortion factor is calculated as

$$K_V(x, h_a) = \frac{\|\overrightarrow{\hat{P}_1 \hat{P}_2}\|}{h_a((y + dy)H) - h_a(yH)}. \quad (13)$$

¹Available for download at <https://github.com/aaronginer/anroll>

Anisotropic Distortion. This measure describes the ratio between horizontal and vertical distortion:

$$K_R(x, h_a) = \frac{K_H(x, h_a)}{K_V(x, h_a)}. \quad (14)$$

Angular Distortion. Let $\hat{P}_{3+x+dx,y}$ be a third point in the unrolled image obtained by a differential step outward from the central symmetry axis in the source domain (increasing $|x|$). Considering two orthogonal differential segments represented by vectors $\overrightarrow{\hat{P}_1\hat{P}_2}$ and $\overrightarrow{\hat{P}_1\hat{P}_3}$ in the source domain, the angle between their mapped segments is given by

$$\gamma = \arccos \left(\frac{\overrightarrow{\hat{P}_1\hat{P}_2} \cdot \overrightarrow{\hat{P}_1\hat{P}_3}}{\|\overrightarrow{\hat{P}_1\hat{P}_2}\| \|\overrightarrow{\hat{P}_1\hat{P}_3}\|} \right). \quad (15)$$

We express angular distortion at $\hat{P}_{1,x,y}$ in the unrolled image by the deviation from orthogonality,

$$K_A(x, h_a) = 1 + \frac{\gamma - \frac{\pi}{2}}{\frac{\pi}{2}}. \quad (16)$$

5.2. Parameter Studies

With four parameters, our model can produce a large variety of different results. In the following, we analyze interesting subsets of parameter combinations, using the Graz G 28 (Typhon) vessel [18].

Interpolation and Curvature. Table 1 shows the resulting unrollings of varying α_{IF} and λ_{RF} parameters. As shown in the first column, the model can not only produce an upward curvature that most complete unrollings would require, but can also curve downward. A downward curvature is usually required when the radius for the linear regression line (Figure 2) decreases towards the bottom of the vessel. The second column shows the interesting effect at $\lambda_{RF} = 1$, where the mapping appears “rolled up”. Together with an interpolation factor $\alpha_{IF} = 1$, it almost represents a tilted view of the original vessel, only stretched due to our parameterization by h_a instead of h . The first row with $\alpha_{IF} = 0$ shows different possible conical unrollings. With increasing α_{IF} we can observe an increasing bulge towards the left and right boundaries, as arcs along this direction increasingly preserve length, while angular distortion increases. The last column shows the results of a flat unrolling.

Non-Uniformity. In our second study, we investigate the impact of the uniformity parameter β_U by interpolating $\beta_U \in [0, 0.35]$. Table 2 shows the resulting images partially overlaid by a 10×20 grid to emphasize the changes, along with a color map encoding angular distortion. At $\beta_U = 0$, we can observe that the vertical lines towards the center are straighter, which more closely resembles a conical unrolling. With increasing β_U , all the vertical lines become more and more curved toward the sides. The color maps show that this also increases angular distortion closer to the center than at lower β_U .

Enforcing Isotropy. Table 3 examines the effect of δ_{EI} . With $\delta_{EI} = 0$, the vertical distortion for a conical unrolling ($\alpha_{IF} = 0$) is minimal. For this configuration, the anisotropic distortion closely matches the horizontal distortion. If $\delta_{EI} = 1$, our model deliberately distorts the vertical mapping at $x = 0$, which results in a negligible anisotropic distortion, but also in a large vertical distortion that closely resembles the horizontal one. Using a higher interpolation factor of 0.5, the vertical distortion noticeably deviates from the horizontal one due to larger changes in the curvature of vertical arcs, and thus also the anisotropic distortion increases. In general, if the model enforces the central isotropy, low anisotropic distortions are obtained at low interpolation factors, as vertical and horizontal distortions remain more similar. One clear advantage of isotropy enforcement can be seen in the flower ornament above the Typhon’s head. Without enforcement, the ornament, which exhibits a nearly circular shape on the original vase, appears more elliptical, while active isotropy enforcement remedies this effect, maintaining the proportions of the ornament.

Table 1Study 1: Varying α_{IF} and λ_{RF} , $\beta_U = \alpha_{IF}$ (uniform horizontal distortion K_H), $\delta_{EI} = 0$

$\lambda_{RF} \backslash \alpha_{IF}$	-5	1	5	500
0				
0.5				
1				

Table 2Study 2: Varying β_U , $\alpha_{IF} = 0.25$, $\lambda_{RF} = 10$, $\delta_{EI} = 0$

β_U	0	0.175	0.35
Image+Grid			
Angular Dist.			

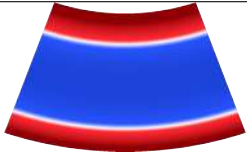
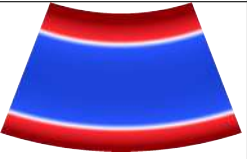
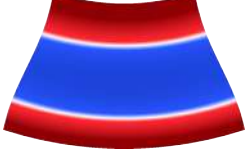
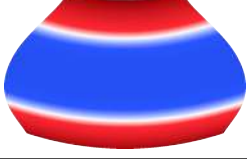
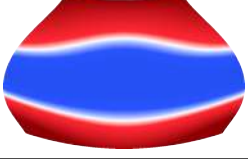
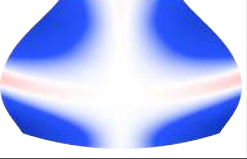
5.3. Execution Time

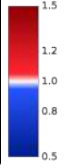
A drawback of the iterative Elastic Flattening approach is its relatively long execution time of up to several minutes for a single rollout [8] and its resulting inflexibility. Table 4 shows the execution times of our approach on two different hardware setups, with and without isotropy enforcement, for various input image resolutions and a fixed output resolution of about 1000×1000 pixels. The results indicate that enforcing isotropy has a minimal impact on execution time due to the additional integral calculations. We can also observe that quadrupling the number of input pixels results in roughly four times the execution time. Lastly, the results show a stark difference between the higher-end hardware setup S1 and the lower-end setup S2, but even on weaker hardware, our results can be produced within a few seconds. This way, our method can provide interactive feedback on parameter changes in the provided UI tool, enabling an intuitive and responsive mapping and analysis workflow for domain users.

5.4. Results

For the mapping results shown in Figure 8, we used stitched photographs and silhouette masks acquired by Houska et al. [8] as input. The original photographs were taken from select volumes of the Corpus

Table 3Study 3: Results with $\alpha_{IF} = \{0, 0.5\}$, $\delta_{EI} \in \{0, 1\}$, $\beta_U = \alpha_{IF}$, and $\lambda_{RF} = 10$

$\delta_{EI} / \alpha_{IF}$	Result	Horizontal Dist.	Vertical Dist.	Anisotropic Dist.
0 / 0				
1 / 0				
1 / 0.5				

**Table 4**Average execution times in seconds ($N = 50$ samples) of unrolling an object (pixel mapping + rendering) using our proposed model with input images of different sizes.

Input Resolution	1000 × 1000		2000 × 2000		4000 × 4000	
δ_{EI}	0	1	0	1	0	1
S1: 3.8GHz i7-10700k, 32GB RAM, NVIDIA 3060ti	0.078	0.087	0.286	0.296	0.829	0.866
S2: 1.6GHz i5-8250U, 8GB RAM, Intel UHD 620	0.358	0.415	1.482	1.812	5.879	5.975

Vasorum Antiquorum [19]. We also use their Elastic Flattening results as a reference for this comparison.

For the results without isotropy enforcement, we chose parameters for which the result most closely resembles that of Elastic Flattening. Figure 8a shows a large deviation between the arc lengths towards the upper boundary. This is a distinct side effect when using a fixed global interpolation factor, and is especially apparent if the vessel has a large curvature towards the top or bottom of the unrolled region. In Figure 8b, Elastic Flattening produces a large vertical distortion of Herakles' head, an artifact that is not present in our result. This result also shows that the motif located on the steep incline toward the neck of the vessel looks noticeably less vertically compressed in our result, which is attributed to the strict vertical unrolling constraints at the central axis, where curvature is flattened without distortion.

Enforcing isotropy, all three results show significant differences in shape and relative size. In Figure 8b, the top and bottom motifs are now almost equal in size, whereas the top motif is originally significantly smaller. However, the shape proportions on the top motif now appear to more closely match the original. Figure 8a and 8c demonstrate that the top part of the unrolling is disproportionately enlarged to balance the vertical and horizontal distortions. Since there is no motif on that part of the vessels, this effect may not be desirable. A slight vertical stretch or compression is generally visible in all the results.

Figure 9 shows the result of sectioning the Graz G 53 vessel based on different motifs and unrolling these parts individually. The model uses an updated $L(h_a)$ calculated from the contour within the selected range of y . The radius modifier λ_{RF} is chosen based on a global minimization of the vertical distortion $K_V(x, h_a)$. We observe a visible improvement in the partial rollouts compared to the combined result. Our model can perform such updates with minimal performance overhead.

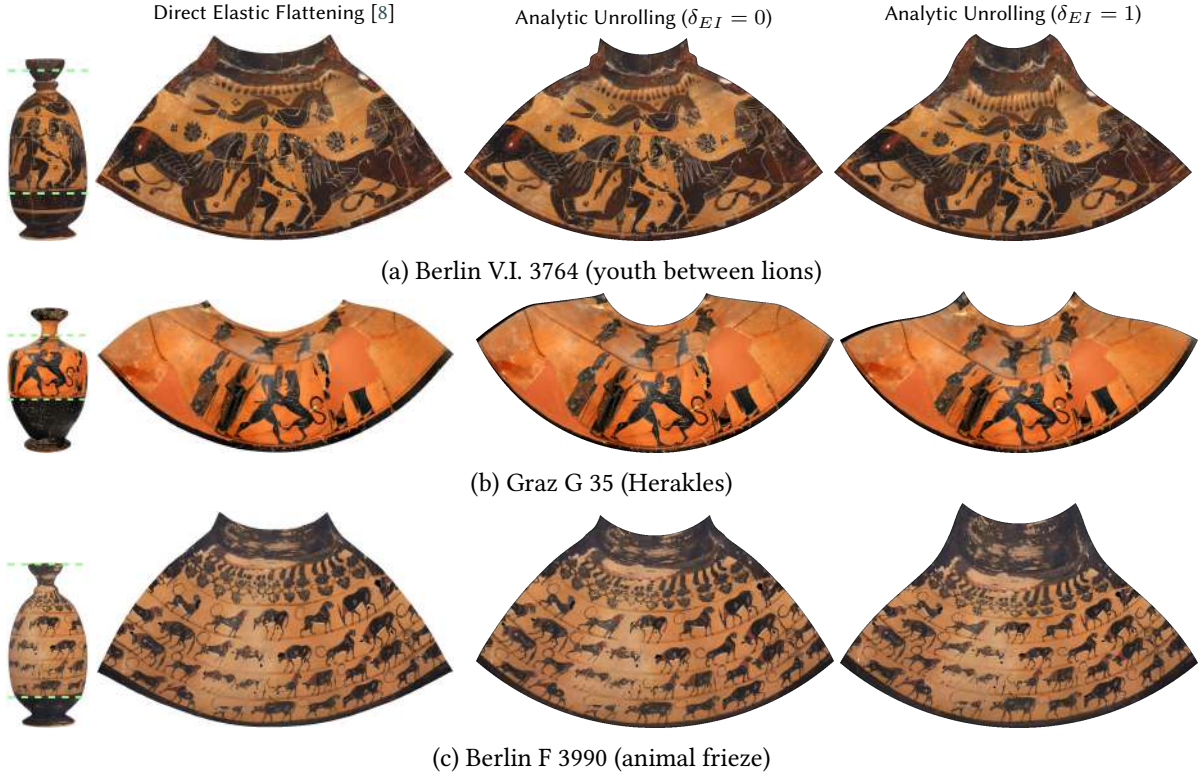


Figure 8: Mid/Right: Our method. Left: Direct Elastic Flattening [8] (© 2021 The Authors. Eurographics Proceedings © 2021 The Eurographics Association. Reproduced by kind permission of the Eurographics Association.)



Figure 9: Segmented results, radius factor optimized for minimal vertical distortion, $\alpha_{IF} = 0.25$, $\beta_U = 0$, $\delta_{EI} = 0$

5.5. Limitations

Because our model is low-dimensional, it is also much more rigid and unable to minimize local distortions in the way a physically-based approach can. This is often especially visible in the corner regions of the unrollings. More generally, our model can only approximate an ideal result, even if all parameters are perfectly optimized toward a certain measure of distortion.

The model also operates under the assumption of perfect symmetry around the upward axis in the target vessels. This often holds true, but is not a guarantee and can be strongly violated, e.g., for vessels reconstructed from several fractured pieces. Distortions introduced by such an asymmetry in the input object are not measurable and cannot be reflected in our distortion measures.

6. Conclusion and Future Work

In this work, we have presented a novel approach to creating 2D rollouts of rotational objects like pottery artifacts based on a low-dimensional mathematical model with a closed-form solution that is based on the Elastic Flattening approach. We demonstrated that this model can produce similar and even superior results. In addition, we introduced a variation of the base model aimed at globally

minimizing anisotropic distortions. Our model is significantly less computationally demanding, making it a more accessible and interactive alternative for archaeological research in everyday use.

Future research could extend this model in several ways. One interesting idea that does not involve increasing the number of parameters is to explore other functions to define the local interpolation factors. Further, using a radius factor parameterized by the height could additionally decrease distortions at the rollout corner areas. Another promising idea involves allowing the user to define specific local areas of interest with varying horizontal distortion behaviors to locally minimize anisotropic distortion.

Declaration on Generative AI

During the preparation of this work, the authors used ChatGPT, Grammarly in order to: Grammar and spelling check, Paraphrase and reword. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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A. Derivation of β_U Bounds

For a given horizontal contour $U(h_a)$ at vertical position h_a , the choice of the uniformity parameter β_U has to meet the constraint

$$U(x, h_a) = L(h_a) + A_{IF}(x) \cdot (O^C(h_a) - L(h_a)) > 0 \quad \forall x \in [-1, 1].$$

Substituting Equations 5, and 6, we get

$$L(h_a) + (3(\alpha_{IF} - \beta_U)x^2 + \beta_U) \cdot (O^C(h_a) - L(h_a)) > 0.$$

The extrema of the quadratic function $A_{IF}(x)$ within $x \in [-1, 1]$ are given at $A_{IF}(0)$ and $A_{IF}(\pm 1)$, and thus, due to symmetry, it suffices to consider two cases:

$x = 0$:

$$L(h_a) + \beta_U \cdot (O^C(h_a) - L(h_a)) > 0$$

leading to the first boundary

$$b_0(h_a) = \frac{-L(h_a)}{O^C(h_a) - L(h_a)},$$

and $x = 1$:

$$L(h_a) + (3(\alpha_{IF} - \beta_U) + \beta_U) \cdot (O^C(h_a) - L(h_a)) > 0$$

leading to the second boundary

$$b_1(h_a) = \frac{3\alpha_{IF}}{2} + \frac{L(h_a)}{2(O^C(h_a) - L(h_a))}.$$

Depending on the sign of $O^C(h_a) - L(h_a)$, $b_0(h_a)$ and $b_1(h_a)$ are either upper or lower bounds. Note that in the case $O^C(h_a) = L(h_a)$, the inequality above reduces to $L(h_a) > 0$, which always holds, as a zero or negative $L(h_a)$ implies a zero or negative vessel circumference. In this case, β_U may take any value.

Thus, the final bounds are defined as

$$\min(b_0(h_a), b_1(h_a)) < \beta_U < \max(b_0(h_a), b_1(h_a)).$$

In practice, globally conservative bounds $\forall h_a \in [0, h_a(H)]$ can be obtained by considering all h_a sampled during the mapping and determining the maximum over the lower bounds, and the minimum over the upper bounds.