

Grayscale images segmentation peculiarities in multidimensional space of information features

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Abstract

An algorithm for grayscale image segmentation is presented. It implements a multi-stage process that includes stages of feature space expansion, complex orthogonalization, and type-1 and type-2 recursive clustering. It is shown that combining the potential of singular value decomposition methods for multi-stage signal purification and the advantages of complex representations for highlighting low-contrast boundaries, together with a fuzzy clustering apparatus, ensures effective segmentation. The influence of the type of type-2 fuzzy membership function on the sensitivity and reliability of the result is researched. Experimental results are provided for images of various physical nature.

Keywords

Image Segmentation, Type-2 Fuzzy Clustering, Multidimensional Space of Information Features, Complex Singular Subspaces

1. Introduction

Images generated by various information, tracking, and diagnostic systems are often of insufficient quality for reliable analysis. Automatic image segmentation is one of the fundamental and most complex tasks in computer vision. The process of dividing visual data into homogeneous content areas is critical for medical analysis, Earth remote sensing, non-destructive testing, and other applications. The effectiveness of subsequent recognition and high-level interpretation of scenes directly depends on the accuracy of object and detail boundary localization during the segmentation stage [1]. The main problem of classical image processing (filtering, segmentation, contrast enhancement) is related to spatial and brightness uncertainty at object boundaries, caused by noise, poor lighting, or blur. For its effective mathematical description, fuzzy sets are currently used, which can be interpreted as a specific type of encoding of the source data, depending on both the data type and the final processing goal [2-3]. Classical Type-1 Fuzzy Sets (FST1), proposed by L. Zadeh, allow modeling fuzziness-type uncertainty of object boundaries in segmentation problems. However, their effectiveness is insufficient when processing images with complex structures or in conditions of non-stationary noise, due to low noise immunity, sensitivity to spatial heterogeneity and illumination variations, high class interference, and the uncertainty of the membership function itself [4-6]. These drawbacks are partially removed by switching to Type-2 Fuzzy Sets (FST2) and using interval membership functions. However, this approach significantly increases the computational complexity of the models, and segmentation accuracy remains limited by the original image feature space. Using only a one-dimensional grayscale pixel brightness scale prevents clustering algorithms from differentiating objects with similar histograms but different spatial characteristics and texture structures [7]. To remove these drawbacks, modern algorithms employ the principle of expanding the information feature space (by extracting texture descriptors, wavelet coefficients, or gradient matrices). The resulting problem of the "curse of dimensionality" and data redundancy is traditionally

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decided by orthogonal projection methods, among which singular value decomposition (SVD) stands out as an effective tool for noise suppression and principal latent component extraction. Type-2 Fuzzy Clustering (T2FCM) is an advanced extension of the classical Fuzzy C-Means (FCM/T1FCM) algorithm. This approach is designed to model the second-order uncertainty that arises when the membership functions themselves contain elements of randomness or imprecision [8, 9]. Despite a significant number of publications on the use of SVD methods [10], complex spaces [11] or interval algorithms of the second type (IT2) [12] for processing visual data, there are practically no works devoted to their systematic interconnected integration. This paper examines the features of the algorithm for grayscale image segmentation, which implements a multi-stage process, including the following stages: descriptor extensions, complex orthogonalization, recursive clustering of type-1 and type-2, the purpose of which is to increase reliability and sensitivity.

2. Review of literature

The development of grayscale visual data processing methods over the past five years has been characterized by a shift from classical deterministic approaches to hybrid intelligent systems capable of operating under stochastic uncertainty. Three important areas of contemporary research aimed at increasing the reliability and sensitivity of image segmentation stand out: multidimensional feature space expansion, orthogonal texture profiling, and fuzzy modeling using type-2 fuzziness [13-15]. Methods for forming a redundant multidimensional ensemble of pixel features (including local descriptors of brightness, texture, and gradients) allow for the integration of the spatial component [16-19]. The transition to a complex representation, in which the modulus encodes local intensity, and the phase angle encodes the direction of structural changes and hidden boundaries, is effective in the task of detecting low-contrast objects [20-21]. A weighted combination of amplitude and phase components ensures mathematical invariance of detection to small-scale brightness fluctuations [22]. A consequence of generating multidimensional feature spaces based on source data is the problem of the "curse of dimensionality" and high correlation of new synthesized features. Orthogonal transformations, and in particular singular value decomposition (SVD), are used as a key tool to struggle data redundancy [23, 24]. In computer vision problems, SVD not only provides data compression but also acts as a controlled filter for high-frequency noise [25]. Analysis and selection of the most significant singular components corresponding to the maximum singular values allows for the effective removal of stochastic noise while preserving the fundamental structural and geometric properties of the image [26]. Recent studies prove that multiple iterative application of SVD at various stages of feature transformation (primary, intermediate, and final) consistently cleans the information signal, converting it into an optimal orthogonal basis for subsequent segmentation [27, 28]. The Fuzzy C-Means (FCM) fuzzy clustering algorithm is traditionally considered the most flexible tool for working with fuzzy class boundaries. However, the limitations of classical type-1 systems (T1FCM), which consist of rigid determinism of the membership functions themselves, make it unstable in the presence of significant noise and outliers, and incorrectly distribute data if clusters differ significantly in density or number of elements [29]. Type-2 fuzzy clustering (T2FCM) is a modern extension of classical T1FCM fuzzy methods, designed to model second-order uncertainty when the membership functions themselves contain elements of randomness or imprecision [30]. However, despite the high mathematical flexibility of T2FCM algorithms, their isolated application to image segmentation faces several fundamental limitations. First, their accuracy critically depends on the quality of the original feature space; one-dimensional brightness histograms do not allow differentiating objects with similar spectral but different textural characteristics. Second, the transition to second-order interval functions significantly increases computational complexity and sensitivity to high-intensity stochastic noise. Literature analysis shows that local image quality enhancement or isolated spatial feature extraction do not provide a comprehensive solution to the problem. Moreover, combining the potential of singular value decomposition (SVD) methods for multi-stage signal purification and the advantages of complex representations for highlighting low-contrast boundaries together with fuzzy clustering techniques poses a serious practical challenge:

such synthesis traditionally requires adapting a significant number of algorithm parameters to each specific application [31].

3. Problem statement

The purpose of this work is to describe the developed image segmentation algorithm that implements a multi-stage process (including the stages of descriptor expansion, complex orthogonalization, recursive clustering of type-1 and type-2), and to study the influence of the FMT2 used for increasing the reliability and sensitivity of clustering.

4. Materials and Methods

Mathematically grayscale image of size $M \times N$ with L brightness levels (where $L = 256$) may be represented as a matrix:

$$G = \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1N} \\ g_{21} & g_{22} & \dots & g_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M1} & g_{M2} & \dots & g_{MN} \end{bmatrix}, \quad (1)$$

where g_{ij} is the brightness value of pixel (i, j) in the range $[0, L-1]$. As a result of using the T1FCM method, for each pixel we obtain membership functions $u_k(g_{ij}) \in [0, 1]$, $u_k(g_{ij})$ is the number of clusters (which is considered pre-set), and $k = 1, \dots, c$, for which the relation is valid:

$$\sum_{k=1}^c u_k(g_{ij}) = 1, (\forall i \in \{1, \dots, N; j \in \{1, \dots, M\}\}), \quad (2)$$

These functions map the input brightness value g_{ij} to the exact number, defining compliance with each of k clusters. The result is the synthesis of c new fuzzy parameters (type-1 fuzzy membership functions (MFT1)) obtained based on only one input parameter (brightness) for a grayscale image (Figure 1 a, b). Clusters are mapped to some property, such as object of interest, background, noise, boundary, and can be displayed as an image. Then the set of membership functions of the form:

$$UT1 = \{u_k(g_{ij})\}, i = 1, \dots, N, j = 1, \dots, M, k = 1, \dots, c \quad (3)$$

describes a new three-dimensional feature space in fuzzy space (an extension of the original feature space), where the first two dimensions are the spatial coordinates of the pixel (i, j) , and the third is the cluster k index with the value of the membership degree (Figure 1 c).

When transitioning to type-2 fuzziness, the uncertainty of the membership function itself (caused by noise, uneven illumination, blurred boundaries, or the algorithm) is taken into account. In this case, the exact numerical value $u_k(g_{ij})$ is replaced by a membership interval that is limited by functions describing its lower $(u_l)_k$ (Lower Membership Function (LMF)) and upper $(u_h)_k$ (Upper Membership Function (UMF)) boundaries. They define the minimum and maximum guaranteed degrees of pixel (i, j) matching to k -th cluster, respectively. The result of interval fuzzy clustering of T2FCM image G for each membership function type-1 has the form:

$$UT2 = \{(u_l)_k(u_k(g_{ij})), (u_h)_k(u_k(g_{ij}))\}, i = 1, \dots, N, j = 1, \dots, M, k = 1, \dots, c \quad (4)$$

Each pair $u_l^k, u_h^k, k = 1, \dots, c$ specifies an interval that mathematically describes the local Footprint of Uncertainty (FOU) for the brightness level of a particular pixel in the class k . This description forms an extended three-dimensional attributes space of size $M \times N \times c$, where instead of scalar quantities, each coordinate contains a second-order interval fuzziness vector (Figure 1 c). This allows for effective isolation of noise components of the image within the boundaries $[(u_l)_k, (u_h)_k]$, preventing them from influencing the final segmentation or contour extraction.

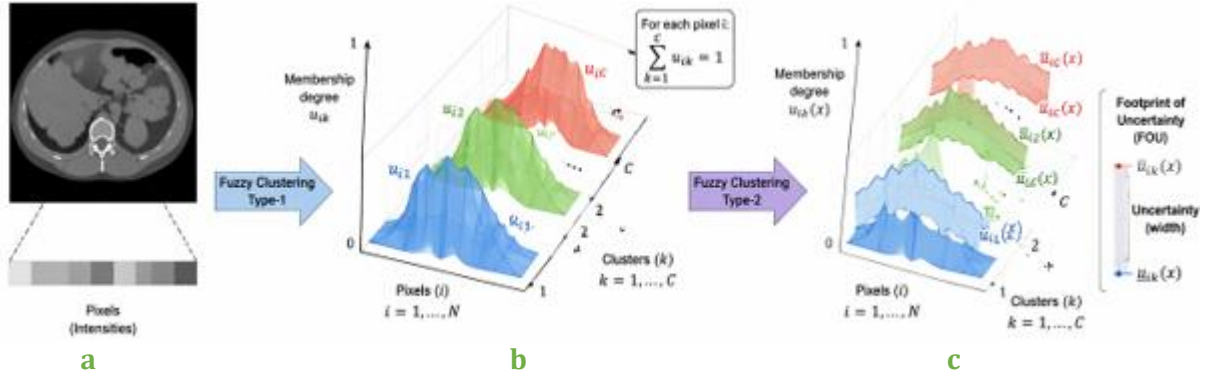


Figure 1: Expansion of the input feature space based on type-1 and type-2 clustering: (a) original image a brain CT scan; (b) 3D space membership functions of type 1; (c) 4D space membership functions of type 2 with FOU.

An effective method for determining of lower u_l and upper u_h membership functions is the next approach:

$$u_l(i, j) = u(g(i, j))^{\alpha(i, j)}, \quad (5)$$

$$u_h(i, j) = u(g(i, j))^{\frac{1}{\alpha(i, j)}}, \quad (6)$$

where $\alpha \in (1, \infty)$ is the transformation coefficient [32].

A pixel's membership value includes the pixel itself, its possible fuzziness range, and a weighting coefficient (amplitude) for each value within this range. In interval fuzzy sets of type-2, the additional coordinate (amplitude) is set to 1, significantly reducing the mathematical loading while preserving all the analytical advantages. While in FCM, a random extreme outlier distorts cluster centroids, in type-2 methods, this noise is effectively neutralized by the FOU zone and has practically no effect on the final position of the centers. In areas where clusters overlap significantly, type-2 methods more accurately separate objects through a flexible analysis of interval weights, minimizing false triggering. Before obtaining the final result, the algorithm requires an additional complex step: converting the type-2 fuzziness into classical type-1 fuzziness, and only then defuzzification (obtaining crisp numbers) occurs. To solve the problem of grayscale images sensitive segmentation, a complex algorithm combining methods of multidimensional expansion of the feature space, orthogonal transformations, and interval fuzzy clustering of type-1 and type-2 is proposed in [31]. The logical scheme of the algorithm is based on the sequential execution of four conceptual stages: preprocessing and formation of primary features, complex transformation, intelligent clustering, and synthesis of output images.

1. Primary feature generation and filtering stage. The initial stage of the algorithm is aimed at invariant data representation and the generation of a redundant space of information features.
 - Data normalization. The original grayscale image I is scaled to the range $[0, 1]$, which ensures the stability of subsequent computational procedures.
 - Primary feature space expansion. A 9-dimensional descriptor ensemble is formed from the scaled image using a windowing transform. This procedure allows for the local spatial-luminance characteristics of pixels to be taken into account.
 - Orthogonalization of the feature space. Singular Value Decomposition (SVD) is applied to the resulting 9-dimensional space. This allows for transition to a new basis, automatic selection of the most significant components with noise suppression, and the formation of the feature matrix I^{svd} , scaled to the range $[0, 1]$.
2. Complex expansion of the feature space. This stage of the algorithm aims to extract structural dependencies by transition to a complex plane.

- Paired component analysis. From the matrix I^{svd} pairs of components are selected. Each pair forms the real and imaginary parts of complex numbers.
 - Generation of matrices A (modulus of a complex value) and Σ (average over the sum of angles for a complex value).
 - Synthesis of the matrix I^c . Based on the weighted sum of matrices A and Σ matrix I^c is formed, all components of which are scaled to the range $[0,1]$.
 - Secondary SVD transformation. To the matrix I^c orthogonal transformation (SVD) is applied for transition to a new feature space with subsequent automatic selection of the most significant components, which after merging with the matrix I^{svd} gives the matrix I^{svd-c} . This matrix is the input for fuzzy clustering.
3. Intelligent clustering based on type-2 fuzzy logic. The matrix I^{svd-c} serves as an input feature vector for the fuzzy segmentation procedure.
- The T2FCM algorithm. Clustering is implemented using the interval Type-2 Fuzzy C-Means (T2FCM) algorithm. A distinctive feature of this implementation is the usage of dynamic compression of the fuzzy membership function matrix.
 - Using type-2 fuzzy membership function (FMT2). FMT1 and FMT2 are used during model tuning. The final training iteration results in the optimized FMT2 matrix.
 - Recursive refinement of cluster structure. Interpretation of the matrix I^{svd-c} as FMT1, and the formation of the FMT2 matrix I^{tr} on its basis, which undergoes transformations similar to those performed for FMT1.
4. Orthogonal synthesis and restoration of output images. The final stage of the algorithm decomposes fuzzy features and visualizes the segmentation results. The output image is formed based on an orthogonal transformation (SVD), identifying the most significant components, synthesizing them, and using an adaptive histogram equalization algorithm.

In the work [31], the “upper” u_h and “lower” u_l membership functions were calculated by the next formulas:

$$u_{h_1}^{old}(i, k) = u(i, k)^{1-u(i, k)^{1+u(i, k)^{-1}_{+u_k-0.5}}}, \quad (7)$$

$$u_{l_1}^{old}(i, k) = u(i, k)^{1+u(i, k)^{1-u(i, k)^{-1}_{-u_k+0.5}}}, \quad (8)$$

$$u_{h_2}^{old}(i, k) = u(i, k)^{K+0.75+u_k^{-1}/2}, \quad (9)$$

$$u_{l_2}^{old}(i, k) = u(i, k)^{K+1.25+u_k^{-1}/2}, \quad (10)$$

$$\bar{u}_k^{-1} = (\bar{u}_k)^{1-\max(\bar{u}_k, 1-\bar{u}_k)}, \quad (11)$$

$$u_h(i, k) = (u_{h_1}^{old}(i, k) + u_{h_2}^{old}(i, k)) / 2, \quad (12)$$

$$u_l(i, k) = (u_{l_1}^{old}(i, k) + u_{l_2}^{old}(i, k)) / 2, \quad (13)$$

where $\bar{u}_k$ is the average membership for the k -th cluster, and $i \in [1, N]$, K is the coefficient (has a significantly impact the transformation results), values of which are recommended to be chosen within the range $[0, 0.2]$. Matrix a , representing FMT2, was obtained as follows:

$$a(i, k) = u_h(i, k) - u_l(i, k). \quad (14)$$

Thus, the calculation of u_h and u_l was reduced to averaging two different power functions $u_{h_1}^{old}$, $u_{h_2}^{old}$, $u_{l_1}^{old}$ and $u_{l_2}^{old}$. In this work, the calculation of four similar functions $u_{h_1}^{new}$, $u_{h_2}^{new}$, $u_{l_1}^{new}$ and $u_{l_2}^{new}$ is carried out in the next way:

$$u_{h_1}^{new}(i, k) = u(i, k)^{1-u(i, k)^{1-\text{sgn}(u^s(i, k)-0.5)}-K_n^1/(2.5-u^s(i, k))}, \quad (15)$$

$$u_{l_1}^{new}(i, k) = u(i, k)^{1+u(i, k)^{1+\text{sgn}(u^s(i, k)-0.5)}+K_n^1/(1.5-u^s(i, k))}, \quad (16)$$

$$u_{h_2}^{new}(i, k) = u(i, k)^{K_n^1 + (0.25 + 2 \cdot \bar{u}_k^{-1} + 0.5 - K_n^1)/(2 + K_n^1)}, \quad (17)$$

$$u_{l_2}^{new}(i, k) = u(i, k)^{K_n^1 + 0.5 + \bar{u}_k^{-1}(\bar{u}_k^{-1} + 0.5 + K_n^1)/(2 - K_n^1)}, \quad (18)$$

$$u^s(i, k) = (\bar{u}_k^{-1} + u(i, k))/2, \quad (19)$$

$$K_n^1 = (K_n)^{1+K_n/(1.4 + \bar{u}_k^{-1}(1 + 1/(3.75 + K_u)))}, \quad (20)$$

$$K_n = \frac{\bar{u}_k^{-1}}{K_u + (\bar{u}_k^{-1} + 0.5)/2}, \quad (21)$$

$$K_u = \frac{\bar{u}_k^{-3} \cdot \bar{u}_k^{-3} \cdot \bar{u}_k}{1 - \bar{u}_k}, \quad (22)$$

$$\bar{u}_k^{-3} = \left(\bar{u}_k^{-2}\right)^{1 - \bar{u}_k^{-2}/d_u}, \quad (23)$$

$$d_u = 10 + \text{sgn}(0.5 - \bar{u}_k) \cdot |0.5 - \bar{u}_k|^{1 - \bar{u}_k^{-2}}, \quad (24)$$

$$\bar{u}_k^{-2} = \frac{\max(\bar{u}_k, 1 - \bar{u}_k) + 0.5}{2}, \quad (25)$$

where $\bar{u}_k^{-1}$ is calculated by the formula (11). This method of calculation made it possible to eliminate the need of tuning the coefficient K , which is necessary for calculating functions $u_{h_1}^{old}$, $u_{h_2}^{old}$, $u_{l_1}^{old}$ and $u_{l_2}^{old}$ by the formulas (7) – (11) providing greater adaptability to functions $u_{h_1}^{new}$, $u_{h_2}^{new}$, $u_{l_1}^{new}$ and $u_{l_2}^{new}$. The calculation of u_h and u_l in this work is carried out similarly to the formulas (12) – (13):

$$u_h(i, k) = (u_{h_1}^{new}(i, k) + u_{h_2}^{new}(i, k))/2, \quad (26)$$

$$u_l(i, k) = (u_{l_1}^{new}(i, k) + u_{l_2}^{new}(i, k))/2, \quad (27)$$

and matrix a is calculated by the formula (14).

The algorithm described in the work [31] has also undergone a number of changes:

1. Scaling the original grayscale image I to the range $[0, 1]$.
2. Interpretation of the resulting scaled image I^1 as FMT1, and calculation FMT2 (only functions $u_{h_2}^{new}$ and $u_{l_2}^{new}$ are used) based on it. FMT2 is interpreted as an image and scaled to the

range $[0,1]$ resulting in the image I^2 . In this case, the value $\overline{I^1}$ (the average on image I^1) is used as $\overline{u}_k$.

3. If the condition is met:

$$\overline{I_1^2} > 0.5, \quad (28)$$

where $\overline{I_1^2}$ is calculated by the formula:

$$\overline{I_1^2} = \left(\overline{I^2} \right)^{1 + \left(\overline{I^1} \right)^{1 + \overline{I^2} / (3.7 + \overline{I^2})}}, \quad (29)$$

where $\overline{I^2}$ is the average on the image I^2 , then $\overline{I_1^2}$ is updated in the next way:

$$\overline{I_1^2} = \frac{\overline{I_1^2}}{4.5 + \overline{I_1^2}}, \quad (30)$$

and image I^1 is used to expand the space of information features. Otherwise, the image I^2 is used to expand the space of information features. Adding this step allows us to increase the detail before generating a set of additional information features for dark images.

4. Performing window transformation (window size 3x3, increasing the size of the processing window leads to a smoothing effect of the signal and a decrease in the resolution of the algorithm; in addition, it slows down the execution of SVD transformation and increases memory costs) brightness values of image I^2 or I^1 (depending on the fulfillment of condition (28)) and forming in that way a 9-dimensional ensemble interpreted as image I^3 including the brightness of neighbouring pixels to take into account for spatial features. It is the first stage of expanding the input data space.

5. Making SVD transformation of the matrix I^3 and following automatic selection of the most significant components [14] based on the coefficient vector C calculating as follows:

$$C_i = \frac{\left| \sum_{j=1}^{1+c} V_s(i, j) \right| + \left| \sum_{j=1}^{1+c} V_s(j, i) \right|}{2}, i \in [1, 1+c], \quad (31)$$

where V_s contains the right singular vectors for the SVD. Vector C is sorted in descending order. A difference vector dC (contains the differences for each neighbouring pair in the sorted vector C) is also created. The threshold value dC_a (is used to determine the number of the most significant elements of the left singular vector matrix) is calculated by the next formula:

$$dC_a = \frac{\sum_{j=1}^c \frac{dC_j}{c} + \frac{dC_{\min} + dC_{\max}}{2}}{2}, \quad (32)$$

where $dC_{\min}$ and $dC_{\max}$ are the minimum and maximum elements of vector dC , correspondingly. The selected indices correspond to the original indices in vector C prior to sorting I^{svd} .

6. As a result of step 5, the multidimensional image I^{svd} is formed, all components of which, except the first, are scaled to the range $[0,1]$. The first component is replaced with image I^4 , which is calculated as a weighted sum of images I^1 and I^2 by the formula:

$$I^4(x, y) = I^1(x, y) \cdot \left(1 - \overline{I_1^2} \right) + I^2(x, y) \cdot \overline{I_1^2}. \quad (33)$$

This replacement is performed to reduce the effect of over-detailing in subsequent steps of the algorithm.

7. The second stage of data space expansion involves generating matrices A , B^1 and B^2 for each pair of the matrix I^{svd} components with indices k , l by the next formulas:

$$A = \left| I_k^{svd} + j I_l^{svd} \right|, \quad (34)$$

$$B^1 = \text{angle}(I_k^{svd} + jI_l^{svd}), \quad (35)$$

$$B^2 = \text{angle}(I_l^{svd} + jI_k^{svd}). \quad (36)$$

Compared to the algorithm in [31], the angle matrices are used individually rather than as a sum. These matrices are then scaled to the range $[0,1]$, and a new component of the matrix I^c (the total number of components equals the number of all possible pairs of I^{svd} components multiply 2) is computed in the next way:

$$I_j^c = A \cdot (1 - K_2) + B^1 \cdot K_2, \quad (37)$$

$$I_{j+1}^c = A \cdot K_2 + B^2 \cdot (1 - K_2), \quad (38)$$

$$K_2 = K_1 + \bar{B}^{1-\bar{A}} - \bar{A}^{1-\bar{B}}, \quad (39)$$

$$K_1 = K_0 + \frac{0.5 - \bar{I}^1}{1 + \frac{\bar{I}^1 + K_0}{2}}, \quad (40)$$

where $\bar{A}$ is the average values of matrix A and K_0 is a constant parameter (values in the range $[0.4, 0.6]$ were used in experiments, this coefficient significantly affects segmentation sensitivity). As the value $\bar{B}$ in the formula (39) the average on the matrix B^1 is used, when the value K_2 is applied in the formula (37), and the average on the matrix B^2 , when K_2 is used in the formula (38). Using K_1 instead of K_0 in equation (39) allows for the consideration of specific characteristics of the processed image.

8. Based on three-dimensional matrix I^c , which is interpreted as FMT1, FMT2 is calculated (only functions $u_{h_2}^{new}$ and $u_{l_2}^{new}$ are used), which is interpreted as image and scaled to the range $[0,1]$ resulting in the multidimensional image I_1^c . In this case, averages on corresponding components of the image I^c are used as the value $\bar{u}_k$. The transition to FMT2 allows for increased segmentation sensitivity.

9. Making SVD transformation for matrix I_1^c components with subsequent the most significant components selection according to the algorithm described in section 5 and scaling selected components to the range $[0,1]$. The matrix I_2^c is a result of this action.

10. Merging matrices I^{svd} and the I_2^c most significant components into the matrix I^{svd-c} .

11. Interpretation of all components of the matrix I^{svd-c} , except first, as FMT1, on the basis of which FMT2 is calculated (only functions $u_{h_1}^{new}$ and $u_{l_1}^{new}$ are used). In this case, averages on corresponding components of the matrix I^{svd-c} are used as the value $\bar{u}_k$. As a result, the matrix I_1^{svd-c} is formed, which is the input data for fuzzy clustering. This allows, on the one hand, to increase the sensitivity of segmentation, and on the other hand, to avoid unnecessary over-detailing, which could result from applying this transformation to the matrix I^{svd} before matrix I^c forming.

12. Performing fuzzy clustering of the scaled matrix I_1^{svd-c} by T2FCM method [19] with fuzzy membership function dynamic compression [3] (involves initially setting a larger number of fuzzy clusters, which is dynamically decreased during training by merging close clusters). For closeness determination weighted Euclidean distance is used. It's calculated (for the distance between the centers of the k -th and l -th clusters) in the next way:

$$d^0(k, l) = \sqrt{\sum_{j=1}^q (S_{u_k} \cdot v^t(k, j) - S_{u_l} \cdot v^t(l, j))^2}, \quad (41)$$

where values S_{u_k} and S_{u_l} are calculated as:

$$S_{u_j} = \frac{\overline{u_j}}{u_{\min}}, (\forall j \in \{1, \dots, c\}), \quad (42)$$

where $\overline{u_j}$ is the average membership function value for the j -th fuzzy cluster, and $u_{\min}$ is the minimum value of the vector $\overline{u}$ of average membership values for each cluster.

During each training iteration t , the MFT2 matrix a^t is calculated based on formulas (14) – (27). The final membership function matrix a^t is interpreted as a multidimensional image.

13. Obtaining matrix I^u as the columns union of matrices I_1^{svd-c} and a^t .

14. Interpretation I^u as FMT1, and FMT2 calculation based on it by formulas (14) – (27). As the result the matrix I^{T2} is formed.

15. Applying SVD transformation to the matrix I^{T2} . After this, the most significant components are selected from the left singular vectors, as it was described in step 5. The formation of grayscale image I^s uses a weighted sum of selected significant components based on the values of the vector C , corresponding to the selected significant components of the left singular vectors, scaled so that their sum equals 1.

16. Generating two output images: I^{out} and I^{out-c} . The first output is obtained by applying the adaptive histogram equalization method (using a uniform transformation function) to image I^s . The second output image, I^{out-c} , is formed based on a pair of matrices: I^s and the first component of matrix I_1^{svd-c} using formulas (37) – (40), but the coefficient K_2 is computed in the next way:

$$K_2 = K_1 + \overline{B}^{1-\overline{B}} - \overline{A}^{1-\overline{A}} \quad (43)$$

for both weighted sums of matrices A , B^1 and A , B^2 (let's define them as S^1 and S^2 , correspondingly). After that, the image I^{out-c} is formed based on the weighted sum of S^1 and S^2 usage in the next way:

$$I^{out-c}(x, y) = S^1(x, y) \cdot (0.5 + \Delta_s) + S^2(x, y) \cdot (0.5 - \Delta_s), \quad (44)$$

$$\Delta_s = \overline{S^1} - \overline{S^2}, \quad (45)$$

where $\overline{S^1}$ and $\overline{S^2}$ are the averages for matrices S^1 and S^2 correspondingly, with subsequent scaling of the resulting image to the range $[0, 1]$ and applying adaptive histogram equalization (using a uniform transformation function).

5. Results

The described algorithm was applied for the segmentation of grayscale low-contrast images of various physical origins, including computed tomography (CT) scans of the human brain (Figure 2 a), which was used as a test image due to the known location of the objects of interest, X-ray tomography of the eye (Figure 3 a), MRI of the cervical spine (Figure 4 a), CT scan of the chest (Figure 5 a), and X-ray of the cervical spine (Figure 6 a).

These medical images were obtained using standard research methods; however, their quality is not enough sometimes for objective analysis. The source images have different level of contrast and contain significant dark, light, blurred, or noisy areas, where gray level changes are below the visual perception threshold. Due to the psychophysical limitations of human vision, target objects are indistinguishable under such conditions. The experimental sample verifies the algorithm's performance on images with various characteristics.

The experimental results of applying the described algorithm were obtained with the following control parameters: $m = 2$, $\varepsilon \leq 10^{-5}$, initial number of fuzzy clusters $c = 9$ (the experiments were conducted on medical images, so, this amount is more than enough), $K = 0.075$ (for the algorithm

presented in the work [31]), $K_0 = 0.4803$. When using the algorithm described in [31], MFT2 was calculated using formulas (7) – (14), and when using the algorithm described in this work, using formulas (14) – (27).

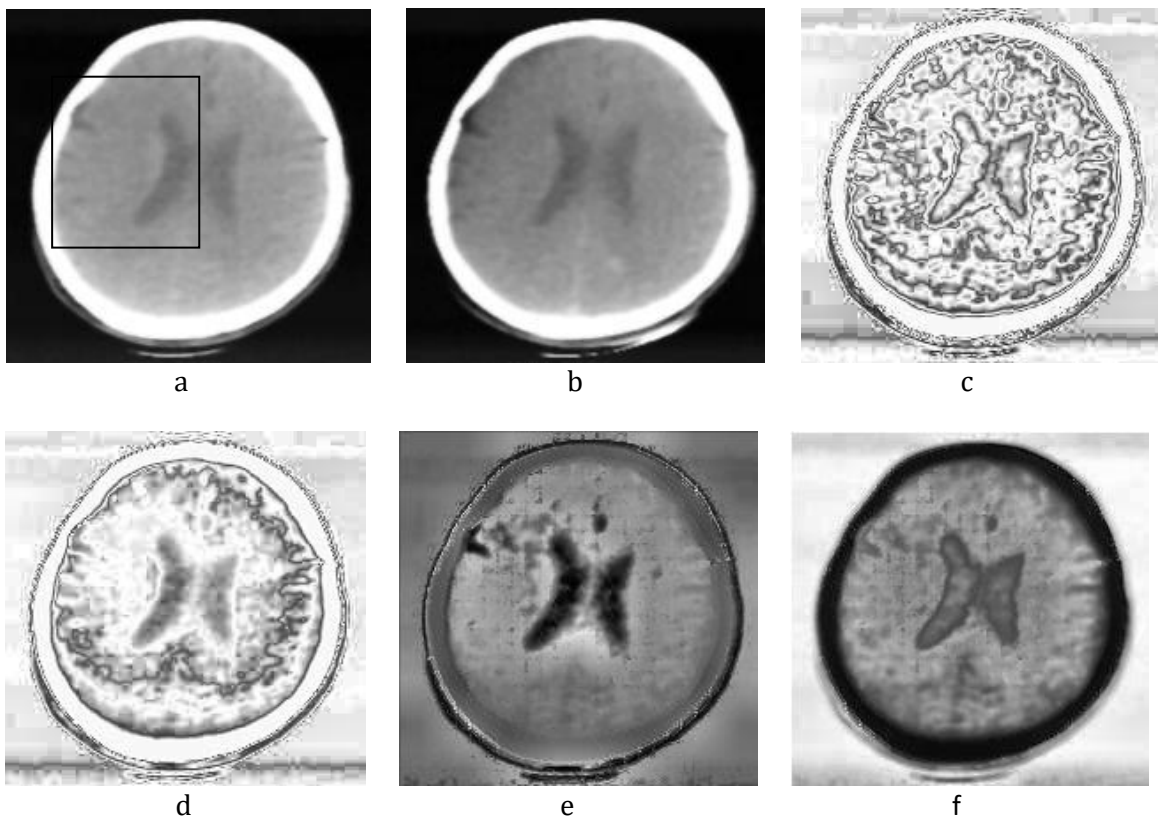
It was found that for some images, the best results could be obtained without applying step 14 (step 10 for algorithm, presented in [31]) using matrix I^u in step 15 instead of I^{T^2} .

Due to the limitations of automated metrics, which cannot adequately describe the insufficient or excessive detail of objects of interest in various image regions, this paper uses visual assessment of segmentation results. Visual assessment by experts (radiologists, surgeons) is currently the primary validation criterion for medical AI-systems prior to their clinical implementation.

The algorithm proposed in this article was implemented in the MATLAB R2016a environment. The developed software consists of a fuzzy image processing toolbox, including the T2FCM algorithm, and a set of scripts that implement the method proposed in this work.

Analysis of the results demonstrates that the described algorithm enhances segmentation detail and image contrast. For example, Figure 2 a presents a brain CT scan used for diagnosing the presence of a hematoma and its affected region (outlined by a rectangle). Figure 2 b shows a brain CT scan obtained with a radiopaque contrast agent usage. In spite of enhancing the visibility of the target region by increasing image contrast, their application poses potential health risks. Therefore, it is preferable to improve image quality through advanced image processing techniques rather than relying on invasive substances. The results of applying the standard FCM algorithm with $c = 9$ and $c = 5$ (Figure 2 c and Figure 2 d, respectively) to the original image (Figure 2 a) are visualized based on adaptive histogram equalization of the maximum membership function. In this case, only the hematoma was identified, but not the area of its influence, which was caused by the over-detailing of the obtained images.

Application of the algorithm presented in [31] (Figures 2e and 2f) significantly improved the selection of objects of interest and eliminated unnecessary detail. When visualized in the complex space of orthogonal membership functions, the method exhibits greater sensitivity to low-contrast regions of the image compared to Figure 3e, despite a lower contrast level. Applying the algorithm presented in this paper, similar results in identifying objects of interest (Figures 2 g, 2 h) were obtained. However, in the image I^{out-c} (Figure 2 h) the hematoma influence area is better highlighted than in Figure 2 f. Formation I^{out-c} without performing step 14 (Figure 2 i) allows us to select both objects of interest and the structure of the whole image.



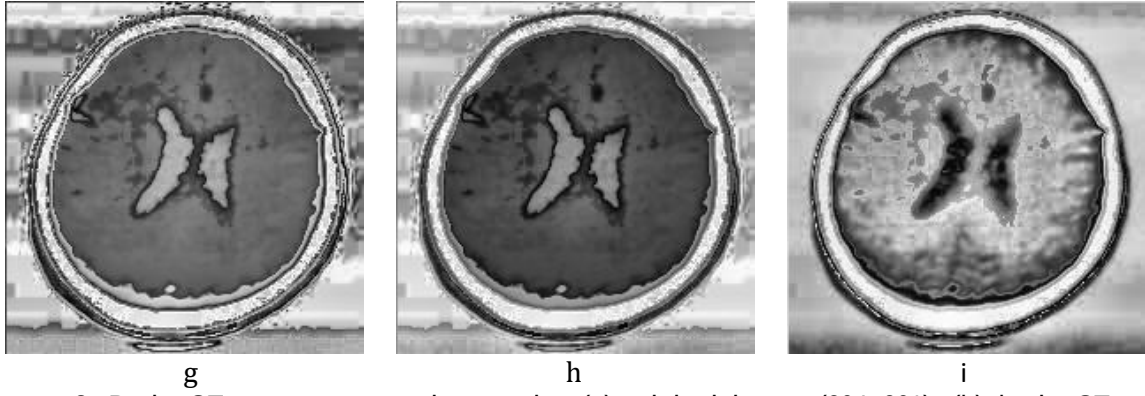


Figure 2: Brain CT scan segmentation results: (a) original image (204×201); (b) brain CT scan obtained using a radiopaque contrast agent; using the standard FCM algorithm with different numbers of fuzzy clusters: (c) $c=9$; (d) $c=5$; proposed in [31] algorithm: (e) orthogonal membership function space (I^{out}); (f) complex orthogonal membership function space (I^{out-c}); proposed in this paper algorithm: (g) I^{out} ; (h) I^{out-c} ; (i) I^{out-c} without step 14.

When segmenting the image in Figure 3 a, the usage of the proposed method (Figure 3 c) made it possible to clearly identify the structure of the image, as well as remove the background noise that is observed when using the algorithm described in [31] (Figure 3 b).

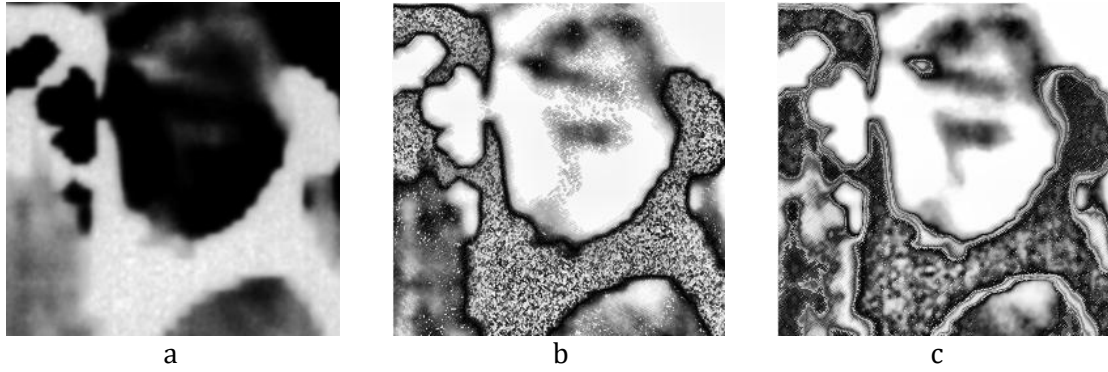


Figure 3: Segmentation results of X-ray tomography of the eye: (a) original image (255×255); (b) proposed in [31] algorithm, I^{out-c} ; (c) proposed in this paper algorithm, I^{out-c} .

Applying the proposed algorithm to the image in Figure 4 a (Figure 4 c) allowed us to clearly distinguish the structure of various objects in the image and eliminate the excessive detail that occurs when using the method presented in [31] (Figure 4 b). It is also worth noting better selection in Figure 4 c of both the structure of individual vertebrae and the adjacent tissues. Similarly, segmentation of the image in Figure 5 a using the proposed method (Figure 5 c) resulted in a higher-contrast image (with clearer highlighting of the boundaries of various objects) compared to using the method described in [31] (Figure 5 b), as well as significantly better delineation of the structure of objects in the lower central part of the frame.

When segmenting the image in Figure 6 a (despite the low quality of the original image), both methods (Figure 6 b, 6 c) demonstrated the detection of both the overall frame structure and initially indistinguishable objects (e.g., the eyes in the upper left part of the image), including the object of interest (the staple at the bottom of the spine). However, the proposed method (Figure 6 c) produced a brighter image with more uniform background, as well as better representation of the contours of the vertebrae and the lower back of the skull. To assess the background uniformity in the resulting images (Figures 6 b, 6 c), the area marked with a black square was scanned with a sliding window measuring 32 x 32 pixels. The scan was performed from left to right, top to bottom, along a zigzag trajectory. Graphs of the differences in average intensity values for adjacent windows ($\Delta \bar{w}$) are shown in Figure 6 d (the upper curve corresponds to Figure 6 b, the lower curve corresponds to Figure 6 c). Analysis of the nature of the curves and the average absolute values of these differences (0.07725

is for the original method and 0.07178 is for the proposed one) confirms a more uniform and even background formation in Figure 6 c.

The performance of the developed algorithm was compared to the method described in [31]. Theoretical complexity estimates are of little use, as the actual running time is critically dependent on the input data. The main contributors to computational costs are the fuzzy clustering (T2FCM) and singular value decomposition (SVD) stages, the duration of which is determined by the input image size and the number of generated informative features.

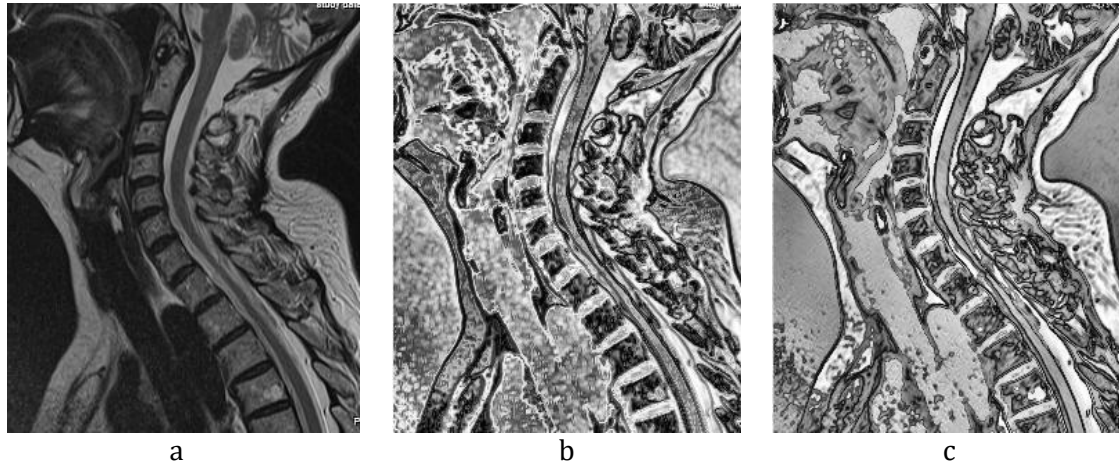


Figure 4: Cervical spine MRI segmentation results: (a) original image (253×310); (b) proposed in [31] algorithm, I^{out} w/o step 10; (c) proposed in this paper algorithm, I^{out} w/o step 14.

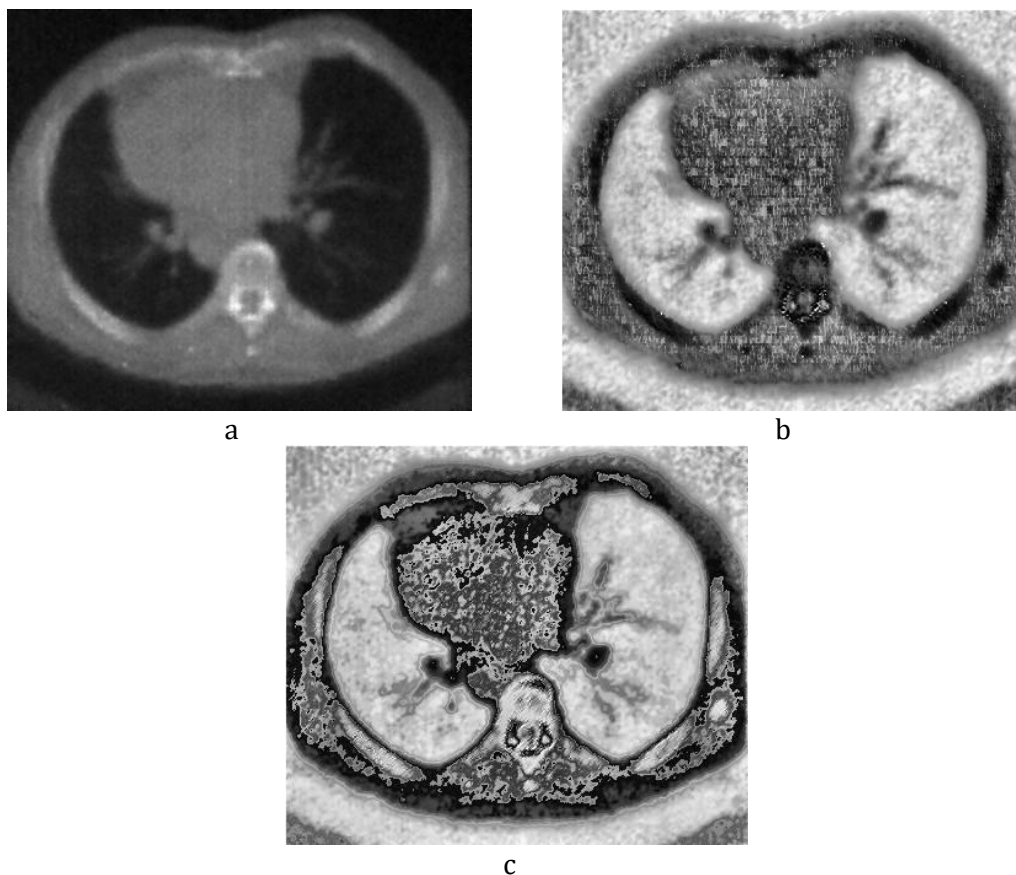


Figure 5: Segmentation results of CT scan of the chest: (a) original image (378×327); (b) proposed in [31] algorithm, I^{out-c} ; (c) proposed in this paper algorithm, I^{out-c} .

Let's compare the running time for the image with the largest dimension (Figure 6 a) on a Dell Inspiron N5110 laptop (CPU: Intel Core I3-2330m (2 cores / 4 threads, 2.2 GHz), RAM: 2x4 GB DDR3 PC10600, dual channel, timings: 9-9-9-24-1T) under Windows 8.1 x64 control.

The formation of 4 final images (I^{out} , I^{out-c} with and w/o step 14 (step 10 for the basic method)) took 463 sec and 731 sec for the proposed and the basic methods [31], correspondingly (despite the fact that step 7 of the proposed algorithm forms twice as many additional information features). According to the HWInfo64 v.8.50 program, when running the proposed method, the CPU load was 26-55% with a maximum of about 90%, and the RAM load increased by 1.66 GB, when running the basic method [31], the CPU load was 26-52% with a maximum of 95%, and the RAM load increased by 1.4 GB. The obtained data shows that the proposed algorithm completes the task 1.58 times faster than the basic method with comparable computing power and memory consumption.

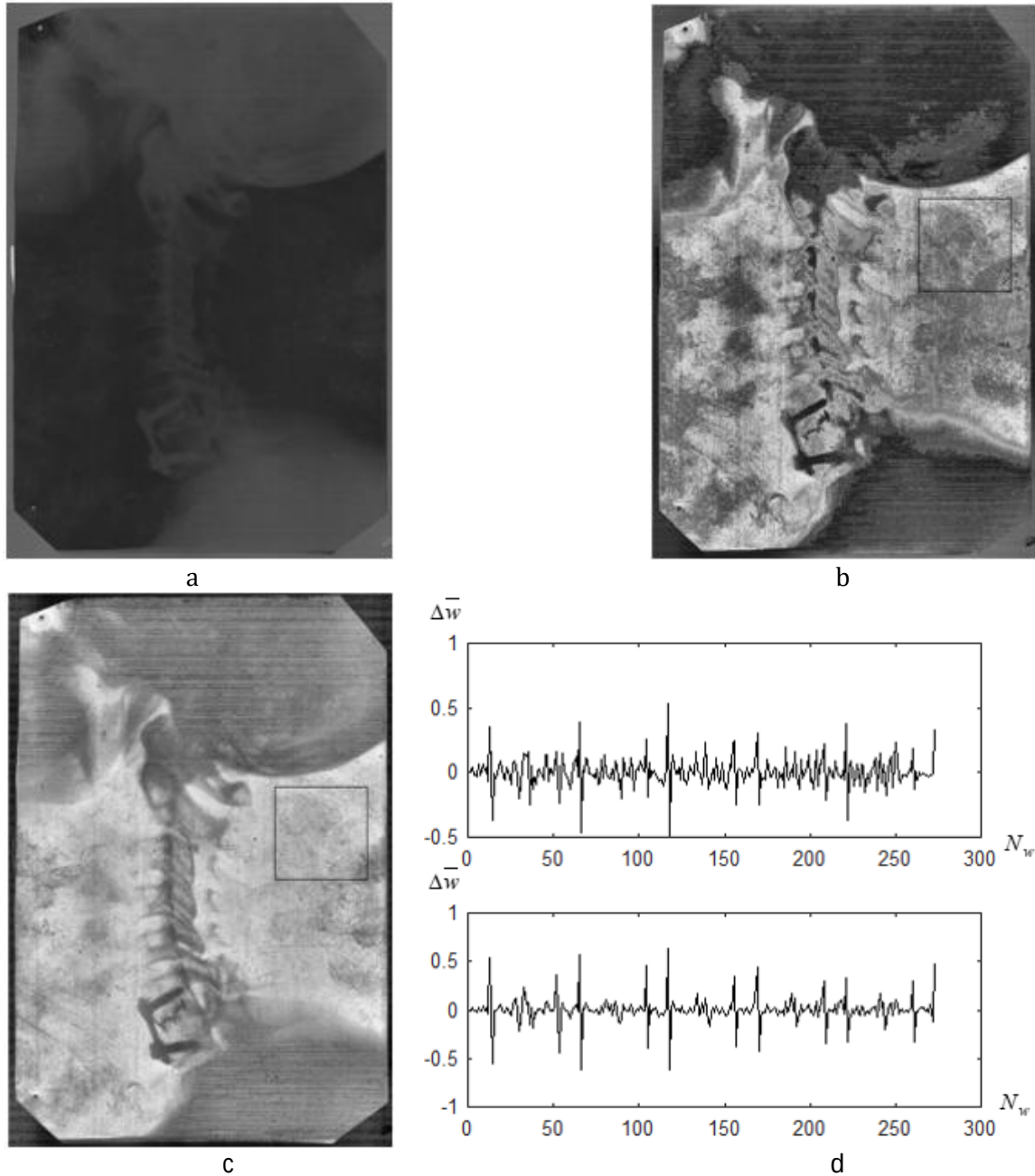


Figure 6: Cervical spine X-ray segmentation results: (a) original image (1344×1936); (b) proposed in [31] algorithm, I^{out-c} w/o step 10; (c) proposed in this paper algorithm, I^{out-c} w/o step 14; (d) graph of the difference in local average values of adjacent windows in the selected background fragment Figure 6 b (top) and Figure 6 c (bottom)

Conclusions

The proposed algorithm increases the sensitivity and resolution of grayscale medical images of various physical types (MRI, CT, and digital radiographs) segmentation while reducing noise. A new

multidimensional feature space is formed recursively based on FMT1 and FMT2. Orthogonalization allows the feature ensemble to be analyzed as a single entity. The usage of a complex space implements an anisotropic filtering effect by simultaneously considering the amplitude (intensity) and phase (gradient/boundary direction) of the signal. This reduces noise in internal homogeneous regions while simultaneously increasing the contrast of adjacent segments and the clarity of their boundaries. Adaptive calculation of control parameters ensures automatic algorithm tuning to the characteristics of the input data. Synthesizing the resulting image using orthogonalization minimizes the computational complexity of the fuzziness type reduction. Despite the algorithmic complexity of the proposed method and its dependence on input data, its performance is comparable to basic approaches with a higher reliability of results.

Promising directions for future research are the exploration of different orthogonalization methods for membership function ensembles.

Declaration on Generative AI

During the preparation of this work, the authors used Green AI tool in order to: generate images on Figure 1. The authors also used Gemini 3.5 Flash to improve text translation, paraphrasing, and reformulation. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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