

Fractional-order modelling of texture changes and poroelastic state of irrigated soils with significant non-local interactions*

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Abstract

We propose a coupled 1D model for simulating long-term soil texture redistribution under irrigation that extends a previously developed space-fractional poroelastic model by incorporating particle transport for sand, silt, and clay particles. Each type of particles is governed by a kinetic attachment-straining-detachment system with advection velocity obtained from a fractional-order Richards equation. The simulated particle concentrations are used to update the hydro-physical parameters of the soil represented by the van Genuchten-Mualem model using the Rosetta software and the soil grain density via a known empirical texture-density relation, closing feedback loops between particle redistribution, water flow, and soil mechanics. The five coupled equations of the model are discretized using a finite-difference technique, share a single adaptive time step, and are solved jointly in a Picard iteration process with the application of the TFQMR algorithm to solve linear equation systems. The implementation uses OpenMP shared-memory parallelisation. Three-year simulations of a heavy clay loam soil column under a realistic irrigation schedule demonstrate that clay is the dominant mobile fraction with depletion rates exceeding those of the other types of particles more than by a factor of seven. The simulated texture change in the top 30 cm layer accelerates non-linearly in time. Decrease in fractional derivative orders amplifies predicted clay washout and soil displacement while leaving average moisture dynamics largely unchanged. The fractional-order operators provide a mechanism for fitting the predicted intensities of the modelled processes to the potentially observed anomalous behaviours.

Keywords

poroelasticity, particle transport, soil, irrigation, fractional-differential model

1. Introduction

Predicting long-term changes in soil properties under the influence of irrigation is an important problem in sustainable land management and precision agriculture. Repeated irrigation not only alters the moisture regime, but also, as a consequence, the mechanical composition of soil, the strain-stress state of soil skeleton, and soil density. Moreover, the movement of pore water can also mobilise and redistribute soil fine particles [1]. The resulting changes in soil texture and density progressively modify the soil's hydraulic properties creating a feedback between particle redistribution and water flow. Understanding and quantifying this feedback is essential for accurate prediction of the consequences of long-term irrigation.

Modelling water transport in unsaturated and partially saturated soils is classically based on the Richards equation, whose nonlinear character could be further complicated by the fractal micro-structure observed in natural soils. As argued in [2], the classical assumption of a linear change of a

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conserved quantity within a representative elementary volume is not fulfilled in fractal media. Thus, replacing the Taylor series by its fractional-order analogue yields conservation laws that are exact for power-law spatial distributions and therefore better suited to describe moisture dynamics in soils with non-local interactions [3]. Such non-local effects manifest themselves, in particular, in anomalous dispersion of moisture fronts [4], which cannot be captured by integer-order models.

Concurrent deformation of soil skeleton under changing pore-water pressures provides a further non-trivial coupling: compaction reduces porosity and modifies moisture capacity, while changes in moisture content alter the effective stress and hence the deformation field. The theory of poroelasticity provides the natural framework for this coupling. The significance of mechanical deformations for predicting ecological consequences of irrigation has been studied numerically for both integer-order and fractional-order models [5, 6], and the computational tools required to solve the resulting stiff, nonlinear systems have been discussed, particularly, in [7].

The transport of fine soil particles in porous media can be modelled at the pore and column scales using a kinetic attachment–straining–detachment framework first proposed in [8]. In this framework, the suspended particle concentration, the concentration of particles reversibly attached to grain surfaces, and the concentration of irreversibly strained particles are governed by a coupled advection–reaction system in which particle attachment and straining are driven by the local Darcy flux. The HYDRUS family of codes [9] implements this framework coupled to the integer-order Richards equation via a standard finite-element scheme and Picard iteration procedure. It has become the de facto reference for colloid transport in variably saturated media. However, HYDRUS treats a single representative colloid size, fixes the hydro-physical parameters throughout the simulation, and does not include mechanical deformations of soil skeleton.

The gap between these lines of research motivates the present contribution. We extend the space-fractional poroelastic model developed in [5] by incorporating the transport of three concurrent types of particles (sand, silt, and clay), whose redistribution continuously modifies the hydro-physical parameters of the soil, thus coupling the evolving texture of the soil back into the water flow and mechanical subproblems.

2. The fractional-order model with particle transport

2.1. Base poroelastic model

The base model, derived and studied in detail in [5], consists of two coupled fractional-order partial differential equations in one spatial dimension $z \in [0, L]$. Here, we consider them under the assumption of incompressible soil grains.

The fractional Richards equation for the water head $h(z, t)[m]$ is as follows:

$$(C(h) + \theta(h)C_w\rho_w g) \frac{\partial h}{\partial t} + D^\alpha(\theta(h) \frac{\partial u_z}{\partial t}) = D^\alpha(k(h)D^\alpha(h - z)) + S \quad (1)$$

where $C(h) = \partial\theta/\partial h[1/m]$ is the specific moisture capacity, $k(h)[m/s]$ is the hydraulic conductivity, $u_z(z, t)[m]$ is the vertical displacement of the solid skeleton, $\rho_w[kg/m^3]$ is the density of water, $g[m/s^2]$ is the acceleration of gravity, $C_w[m^2/N]$ is the compressibility of water, $S[1/s]$ is the source and sink term that particularly models plant transpiration, and the space-fractional operator is

$$D^\alpha f(z) = \frac{\Gamma(2 - \alpha)}{z^{1 - \alpha}} \quad {}^c D_z^\alpha f(z), 0 < \alpha \leq 1 \quad (2)$$

with ${}^c D_z^\alpha$ denoting the Caputo derivative with respect to z .

The fractional consolidation equation for the displacement u_z and the displacement velocity $v_z = \partial u_z / \partial t$ [m/s] is as follows:

$$\rho_{mix} \frac{\partial v_z}{\partial t} = (\lambda + 2\mu) D^\alpha (D^\alpha u_z) - B \rho_w g D^\alpha \left(\frac{\theta(h)}{n} h \right) + \rho_{mix} g \quad (3)$$

where λ, μ [Pa] are the Lamé parameters of the soil skeleton, B [–] is the Biot coefficient, n [–] is the porosity, and $\rho_{mix} = (1 - n)\rho_s + n\rho_w$ [kg/m³] is the bulk density of the saturated mixture with solid grain density ρ_s .

The functions $\theta(h)$ and $k(h)$ are parametrised by the van Genuchten–Mualem model [10] with parameters $(\theta_r, \theta_s, \alpha_{vG}, n_{vG}, K_s, \eta)$. In the base model [5] these parameters are fixed in time. Their dynamic update due to particle redistribution constitutes the key new contribution described in Section 2.3.

2.2. Particle transport

We consider three types of soil particles indexed by $p \in \{1, 2, 3\}$ corresponding to sand ($p=1$), silt ($p=2$), and clay ($p=3$). For each type, three concentrations are introduced: $c^{(p)}(z, t)$ [–] is the suspended concentration of particles per unit volume of pore water; $s_a^{(p)}(z, t)$ [–] is the concentration of reversibly attached particles per unit bulk volume; $s_s^{(p)}(z, t)$ [–] is the concentration of irreversibly strained particles per unit bulk volume.

The governing equations follow the kinetic attachment–straining–detachment framework given in [11]:

$$\frac{\partial(\theta c^{(p)})}{\partial t} = - \frac{\partial(q c^{(p)})}{\partial z} - \frac{\partial s_a^{(p)}}{\partial t} - \frac{\partial s_s^{(p)}}{\partial t}, \quad (4)$$

$$\frac{\partial s_a^{(p)}}{\partial t} = \lambda_a^{(p)} c^{(p)} q - k_{det}^{(p)} s_a^{(p)}, \quad (5)$$

$$\frac{\partial s_s^{(p)}}{\partial t} = \lambda_s^{(p)} c^{(p)} q \quad (6)$$

where $q(z, t) = -k(h) D^\alpha (h - z)$ [m/s] is the Darcy flux obtained from the solution of the Richards equation (1), $\lambda_a^{(p)}$ [1/m] is the attachment coefficient, $\lambda_s^{(p)}$ [1/m] is the straining coefficient, and $k_{det}^{(p)}$ [1/s] is the detachment coefficient. These three coefficients are different for each particle types, reflecting the distinct sizes and surface chemistries of sand, silt, and clay particles.

The non-negativity constraints $c^{(p)} \geq 0$, $s_a^{(p)} \geq 0$, $s_s^{(p)} \geq 0$ are enforced: the source terms in (5)–(6) are set to zero whenever the corresponding concentration would become negative.

The first equation (4) is a nonlinear hyperbolic conservation law driven by the advective flux $q c^{(p)}$. The second and third equations (5)–(6) are stiff ordinary differential equations parametrically coupled through $q c^{(p)}$. The system (4)–(6) is of the same class as the one studied in [12] for a single type of particles under saturated or variably saturated flow. Our study extends it to three concurrent size classes whose combined spatial redistribution modifies the soil hydro-physical properties dynamically.

At the soil surface ($z=0$) we set zero suspended concentration when the flux is directed into the domain (downward flow – precipitation or irrigation):

$$c^{(p)}|_{z=0}=0, q(0, t)>0. \quad (7)$$

When the flux is directed upward ($q(0, t) \leq 0$), a zero-flux condition is applied, i.e. no particles are extracted through the surface.

At the lower boundary ($z=L$), a zero-gradient condition is imposed:

$$\frac{\partial c^{(p)}}{\partial z}|_{z=L}=0. \quad (8)$$

The initial concentrations of suspended and strained particles are set to zero:

$$c^{(p)}(z, 0)=0, s_s^{(p)}(z, 0)=0, z \in [0, L]. \quad (9)$$

The initial attached concentration $s_a^{(p)}(z, 0)=s_{a,0}^{(p)}$ is set uniformly and represents the initial texture of the soil (in fractional units summing to unity over all three types of particles).

2.3. Dynamic parametrisation

The key extension with respect to both the base model [5] and the other mentioned models is that we consider hydro-physical properties of the soil not fixed but evolving in time as the simulated particle concentrations change.

In each node i we define the total volumetric fraction of each type of particles as the sum of its three concentration components weighted by moisture content:

$$\Phi_i^{(p)}=\theta_i c_i^{(p)}+s_{a,i}^{(p)}+s_{s,i}^{(p)}, p=1,2,3. \quad (10)$$

The local normalized mass fractions of sand, silt, and clay are then equal to

$$f_i^{(p)}=\frac{100\Phi_i^{(p)}}{\Phi_i^{(1)}+\Phi_i^{(2)}+\Phi_i^{(3)}}, p=1,2,3. \quad (11)$$

The five parameters ($\theta_{r,i}, \theta_{s,i}, \alpha_{vG,i}, n_{vG,i}, K_{s,i}$) of the van Genuchten-Mualem model at node i are then computed from $(f_i^{(1)}, f_i^{(2)}, f_i^{(3)})$ using version 3 of the Rosetta pedotransfer function [13] implemented as a Python module. To avoid redundant computations, the update is performed only when the rounded integer values of $(f_i^{(1)}, f_i^{(2)}, f_i^{(3)})$ change from one time step to the next.

This feedback constitutes a coupling from the particle transport submodel to the hydraulic submodel: the evolving texture of the soil continuously modifies the water retention and hydraulic conductivity functions, which in turn alter the Darcy flux that drives further particle transport. An analogous mechanism was proposed analytically for a single particle type problem in [14] but without dynamic recalculation and without coupling to soil mechanics.

To form the feedback from particle redistribution to the mechanical submodel, the soil grain density ρ_s entering the consolidation equation (3) is also updated as a function of the local silt content. Following the empirical relationship given in [15], the grain density in the node i is computed as

$$\rho_{s,i}=1000(1.567 - 8 \times 10^{-3} f_i^{(2)})[kg/m^3]. \quad (12)$$

This relation is updated at every time step, so that sediment-enriched layers acquire a lower grain density while sand-dominated layers acquire a higher one, with a consequent modification of the bulk momentum balance.

3. Numerical scheme for the particle transport submodel

For the particle transport submodel we use the same non-uniform time grid, uniform space grid, and a Picard iteration approach as in the base model [5]. The grid has the form

$$T=\{(z_i, t_j): z_i=\Delta_z i, i=0, \dots, N, \Delta_z=L/N, t_j=t_{j-1}+\tau_j, t_0=0\}. \quad (13)$$

The notation $f_i^{j,k}$ denotes the approximation of a function f at node i , time level j , and Picard iteration k .

Further we roughly follow the scheme given in [16].

The attachment and straining equations (5)–(6) are discretised by the implicit Euler method at the k -th Picard iteration as

$$\frac{s_{a,i}^{(p),j,k} - s_{a,i}^{(p),j-1}}{\tau_j} = \lambda_a^{(p)} c_i^{(p),j,k-1} q_i^{j,k-1} - k_{det}^{(p)} s_{a,i}^{(p),j,k}, \quad (14)$$

$$\frac{s_{s,i}^{(p),j,k} - s_{s,i}^{(p),j-1}}{\tau_j} = \lambda_s^{(p)} c_i^{(p),j,k-1} q_i^{j,k-1}. \quad (15)$$

The non-negativity constraints are enforced by setting the right-hand sides of (14) and (15) to zero whenever $s_{a,i}^{(p),j,k-1} + \tau_j (\lambda_a^{(p)} c_i^{(p),j,k-1} q_i^{j,k-1} - k_{det}^{(p)} s_{a,i}^{(p),j,k-1}) < 0$.

For the suspended concentration equation (4), the time derivative is discretised implicitly while the advective flux $qc^{(p)}$ is discretised by a first-order upwind scheme. Introducing the notation $\phi_i^{(p)} = q_i c_i^{(p)}$ for the advective flux at node i , the upwind discretisation at the interior node i ($1 \leq i \leq N - 2$) is as follows:

$$\left. \frac{\partial(qc^{(p)})}{\partial z} \right|_{z_i} \approx \begin{cases} \frac{\phi_i^{(p)} - \phi_{i-1}^{(p)}}{\Delta_z}, & q_i > 0, \\ \frac{\phi_{i+1}^{(p)} - \phi_i^{(p)}}{\Delta_z}, & q_i \leq 0. \end{cases} \quad (16)$$

At sonic points where the sign of q changes between adjacent nodes, an entropy-satisfying flux is used. Specifically, when $q_i > 0$ and $q_{i-1} \leq 0$, the left flux is replaced by the arithmetic average $\phi_{i-1/2}^{(p)} = \frac{1}{2} (q_i c_i^{(p)} + q_{i-1} c_{i-1}^{(p)})$, and analogously for the right flux when $q_i \leq 0$ and $q_{i+1} > 0$.

Combining (14)–(16), the full discretisation of the suspended concentration equation at the interior node i , time level j , and Picard iteration k is

$$\frac{\theta_i^{j,k} c_i^{(p),j,k} - \theta_i^{j-1} c_i^{(p),j-1}}{\tau_j} = - \frac{\partial(qc^{(p)})}{\partial z}_{z_i}^{j,k-1} - \frac{s_{a,i}^{(p),j,k} - s_{a,i}^{(p),j-1}}{\tau_j} - \frac{s_{s,i}^{(p),j,k} - s_{s,i}^{(p),j-1}}{\tau_j} \quad (17)$$

where the advective divergence on the right-hand side uses the solution at Picard iteration $k - 1$ for the explicit linearisation.

The boundary condition (7) is applied by setting $c_0^{(p),j,k}=0$ when $q_0^{j,k-1}>0$; the zero-gradient condition (8) is applied by setting $c_N^{(p),j,k}=c_{N-1}^{(p),j,k}$.

4. Overall numerical procedure

All five solvers (Richards, consolidation, and three particle transport systems) share a single time step τ_j and a joint Picard iteration. At each iteration, the following sequence is executed:

1. The linearised Richards equation (1) is discretized and the resulting linear equation systems are solved by the TFQMR algorithm with accuracy threshold ε_H using the Darcy flux from the previous Picard iteration (the numerical scheme is detailed in [5]).
2. The linearised consolidation system (3) is discretized and the resulting linear equation systems are solved by the TFQMR algorithm with accuracy threshold ε_Z (the numerical scheme is detailed in [5]).
3. For each type $p=1,2,3$ of particles the transport system (14), (15), (17) is solved using the TFQMR algorithm with accuracy threshold ε_P .
4. The parameters of the van Genuchten-Mualem model and grain density are updated as described in Section 2.3.
5. The mean-square change between the solution on successive Picard iterations is evaluated over all five variable fields; if this change is below the threshold ε_{Pic} the iteration process is considered converged.

All five solvers are synchronised to the minimum time step required by any of them and a relaxed Mann iteration with relaxation parameter $\omega \in (0,1]$ is used to stabilise convergence of the Picard iteration process.

The adaptive time-stepping procedure follows [17]. The time step τ_j is multiplied by a factor $\mu_\tau=1.25$ if the number of TFQMR iterations at the first Picard iteration is less than 66% of the maximum allowable number for all five solvers, and divided by μ_τ if any solver fails to converge within that maximum. After a step reduction, the Picard iteration is restarted from the values at time level $j - 1$. An irrigation-period constraint further limits $\tau_j \leq \tau_{irr}$ (default 60s) whenever the surface irrigation flux is active in order to capture the rapid wetting-front dynamics and associated particle mobilisation. Hard bounds $\tau_{min} \leq \tau_j \leq \tau_{max}$, $\tau_{min}=10^{-3}s$, $\tau_{max}=86400s$ are enforced throughout.

The matrix assembly and TFQMR iterations for all five solvers are parallelised for the execution in a multi-threaded environment using OpenMP, following the approach described in [5]. Each linear system solution runs inside a single parallel region where matrix assembly, matrix–vector products, and coefficient updates are distributed across threads with explicit synchronisation with several non-

thread-safe steps restricted to a single thread - cache initialisation and calls into the embedded Python interpreter. The van Genuchten–Mualem parameter update embeds a Python interpreter directly in the process and is batched and cached: a combined query is submitted only for grid nodes whose rounded particle concentration has actually changed, cutting the number of pedotransfer function evaluations substantially over a multi-year run.

The algorithms were implemented in C++. The source code is available at https://github.com/sevaboh/fr_sss.

5. Computational experiments

5.1. Simulated scenarios and solver settings

To evaluate the long-term redistribution of soil texture fractions under repeated surface irrigation, we solve the boundary-value problem for the proposed model under various settings for a soil column of depth $L=1\text{m}$ over a total simulation time of $T_m \approx 2.85$ years.

Three scenarios were considered: classical integer-order case $\alpha=\beta=1$ without poroelastic simulations (1) and with them (2); full model in the fractional-order case $\alpha=\beta=0.9$ (3).

The initial hydro-physical and mechanical parameters are summarised in Table 1 representing a possible case of heavy clay loam soil.

Although the parameters of the van Genuchten–Mualem model and the Lamé parameters for soils are widely determined experimentally and reported in the literature, the amount of data on soil particle transport parameters is limited. Therefore, particle transport parameters for the sand were taken within the ranges reported in [8]. Detachment coefficient for the silt was first taken equal to the maximal value reported in [8] for the sand. The values of the attachment and straining coefficients for the clay were taken within the ranges reported in [18]. The values of other parameters were interpolated.

The detachment coefficients reported in the literature that were measured in laboratory conditions for the colloid transport in soils have much higher values than the expected ones regarding the detachment of soil particles. Hence, we fit these values to roughly represent the changes in soil clay content within the three-year irrigation experiment reported in [19] – decrease by 0.38% in top 30 cm layer. To achieve such an order of decrease, the values reported for colloids should have been divided by 10^5 resulting in the values given in Table 1.

Evapotranspiration $E(t)$ and precipitation $P(t)$ were linearly interpolated between the values given for the first instant of each calendar month as follows: $P=(1.7 \times 10^{-8}, 1.3 \times 10^{-8}, 1.6 \times 10^{-8}, 2.1 \times 10^{-8}, 2.2 \times 10^{-8}, 1.1 \times 10^{-8}, 8.9 \times 10^{-9}, 1.4 \times 10^{-8}, 2.7 \times 10^{-8}, 2.9 \times 10^{-8}, 2.0 \times 10^{-8}, 1.7 \times 10^{-8})$, $E=(1.1 \times 10^{-8}, 1.4 \times 10^{-8}, 2.1 \times 10^{-8}, 3.0 \times 10^{-8}, 3.9 \times 10^{-8}, 4.9 \times 10^{-8}, 5.1 \times 10^{-8}, 4.8 \times 10^{-8}, 3.8 \times 10^{-8}, 2.7 \times 10^{-8}, 1.6 \times 10^{-8}, 1.1 \times 10^{-8})$. It was assumed that evapotranspiration flux passes only through the soil surface, leading to $S=0$ in (1). The annual cycle was applied periodically. The simulated scenario corresponds to the climatic water deficit of $\approx 376 \text{ mm yr}^{-1}$ that must be compensated by irrigation.

The irrigation event was initiated when the average water head in the top 30 cm layer decreases below -2m . The irrigation event was terminated when this value starts to exceed -0.5m . The irrigation flux applied at the surface was equal to 3.6 mm hour^{-1} . The groundwater depth was set at a fixed level of 1.5m . The initial time step was set to $\tau_0=0.01\text{s}$, the maximum allowable time step to $\tau_{max}=3600\text{s}$ (1 hour), and the minimum allowable time step to $\tau_{min}=10^{-8}\text{s}$.

Table 1
Soil hydro-physical and mechanical parameters.

Parameter	Notation	Value
Residual moisture content	θ_r	0.118684
Saturated moisture content	θ_s	0.444
van Genuchten model shape parameter	n_{vG}	1.327
Inverse air-entry pressure	a_{vG}	0.00762 m ⁻¹
Saturated hydraulic conductivity	K_s	1.108 10 ⁻⁶ m s ⁻¹
Exponent in Mualem equation	η	0.678
Specific storage	$C_w \rho_w g$	10 ⁻³ m ⁻¹
Initial porosity	n_0	0.44
Lamé parameters	λ, μ	(19.8, 13.2) MPa
Initial texture (sand/silt/clay)	$S_{a,0}^{(p)}$	(0.25, 0.35, 0.4)
Attachment coefficients (sand/silt/clay)	$\lambda_a^{(p)}$	(0.0005, 0.00075, 0.001) m ⁻¹
Detachment coefficients (sand/silt/clay)	$k_{det}^{(p)}$	(1.66 10 ⁻¹² , 1.16 10 ⁻¹¹ , 8.16 10 ⁻¹¹) s ⁻¹
Straining coefficients (sand/silt/clay)	$\lambda_s^{(p)}$	(0.015, 0.0125, 0.01) m ⁻¹

The TFQMR linear solver was run with a maximum of 4000 iterations and accuracy thresholds $\varepsilon_H=10^{-12}$ for the Richards equation, $\varepsilon_Z=10^{-14}$ for the consolidation system, and $\varepsilon_P=10^{-20}$ for the particle transport system. The Picard iteration with a relaxation factor of $\omega=0.5$ was terminated when the root-mean-square deviation between solutions at successive iterations decreased below $\varepsilon_{Pic}=10^{-10}$. For all other solver settings, we used the default values described in Section 3.1.

5.2. Convergence and execution time

The problem in a fractional-order case with the poroelastic coupling was solved for the period of 347 days with varying grid size $N=50,100,150,200$ with and without usage of multi-threading parallelism (maximum of 4 threads on AMD Ryzen 3 5300U CPU). To assess the difference between the solutions obtained on different space and time grids (due to the adapting time-stepping procedure) the following metric was used. We considered only the period of initial drying up to $K_2=125$ days where no irrigation events occur and the functions are changing smoothly. The values of the sought-for functions were averaged over the top 30 cm layer and interpolated (nearest-neighbour technique) in time points $T_j, j=0,1,\dots,K_2$ at the beginning of each day; for the water head, the displacements, and the mass fractions of particles we calculated the metric

$$d_p(N_1, N_2) = \sqrt{\frac{\frac{1}{K_2} \sum_{j=0}^{K_2} (f_n^{(p)}(T_j, N_2) - f_n^{(p)}(T_j, N_1))^2}{\max_{j=0}^{K_2} f_n^{(p)}(T_j, N_1) - \min_{j=0}^{K_2} f_n^{(p)}(T_j, N_1)}} \quad \text{where } f_n^{(p)}(T_j, N), p=1,2,3,4,5 \text{ are the corresponding}$$

function values obtained on the grid of the size N , $N_1 < N_2$ are the consequent grid sizes; $\max_p d_p(N_1, N_2)$ is considered as the overall difference metric that represents a maximal per cent of the range where the function changes covered by average absolute local difference between the solutions obtained on different grids.

The obtained execution times and the values of the mentioned difference metric are given in Table 2. Starting from $N=100$, the execution time exhibited linear increase with respect to N that is a consequence of the adapting time-stepping procedure based on the convergence of the TFQMR algorithm. This effectively restricts the number of performed TFQMR iterations leading to the linear

dependency of execution time on N . For $N=50$, auxiliary operations, particularly external calls for Rosetta Python module, take up notable amount of time making execution time per one grid node higher. Speed-up due to multi-threaded execution on 4 threads increased with N reaching 2.17 for $N=200$. As the changes in the solutions became less than 1% in the considered metric for $N=150$ and higher, for further computational experiments, the soil column was discretised using $N=150$ uniformly spaced nodes, giving a spatial step $\Delta z \approx 6.66$ mm.

Table 2

Execution times and difference metric between the solutions obtained with the consequent grid sizes

Grid size	Execution time,s		Difference metric
	no multi-threading	with multi-threading	
50	348	336	
100	1321	864	1.88%
150	4746	2457	1.00%
200	8364	3850	0.71%

5.3. Simulation results

The simulated dynamics of clay content at the depths $z=0,0.3,0.6m$ are presented in Figure 1 for the case of Scenario 2 (integer-order model with poroelasticity taken into account). The results show that the influence of irrigation diminishes with depth and no visible changes in clay content at the depth of $0.6m$ was simulated. The simulation also revealed notable changes localized in time in the top layer of soil due to strong water potential gradients during irrigation in summer and dry periods in autumn – beginning of winter. However, general trend is a linear washout of clay particles.

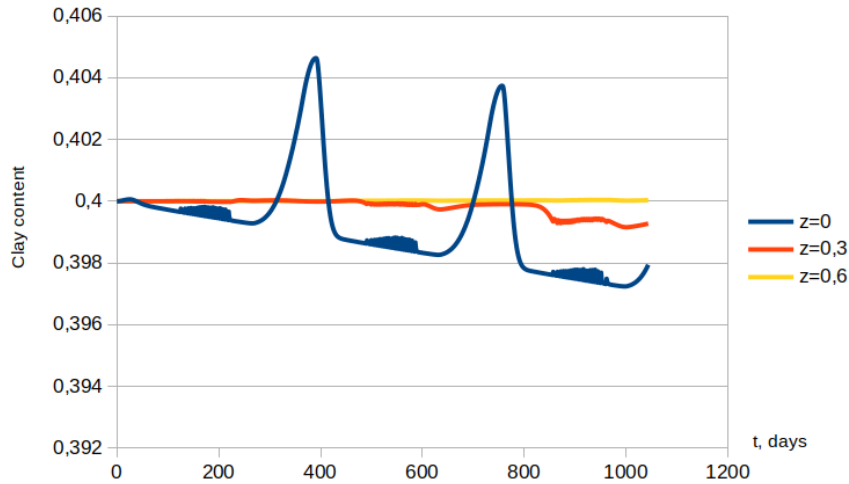


Figure 1: The dynamics of clay content for Scenario 2

The dynamics of the clay content at the depth $z=0.3m$ is given in Figure 2 for all three considered modelling scenarios. It can be seen that the consideration of the influence of soil strain-stress state on water movement leads to slower particle washout while the consideration of non-local interactions leads to much stronger increase of the washout.

The dynamics of the average water head values in the 30 cm layer shows similar tendencies as reported in [5]: the consideration of soil strain-stress state yields a slightly faster predicted drying of the soil with an increase in the amount of influence over time. A much stronger and uniform

influence on drying speed can be controlled by the orders of fractional derivatives if the observed water dynamics does not follow classical behaviour.

As a metric for comparing the solutions between the simulated scenarios we used the total relative difference in the form $\varepsilon_1 = \sqrt{\frac{\sum_{j=0}^K (f_{1,j} - f_{2,j})^2}{\sum_{j=0}^K f_{1,j}^2}}$ where K is the number of points, in which the solutions $f_{1,j}$ and $f_{2,j}$ are calculated according to the two different scenarios. In the considered settings, for the dynamics of average water head values the total relative differences between the three scenarios were in general minor and did not exceed 0.52%.

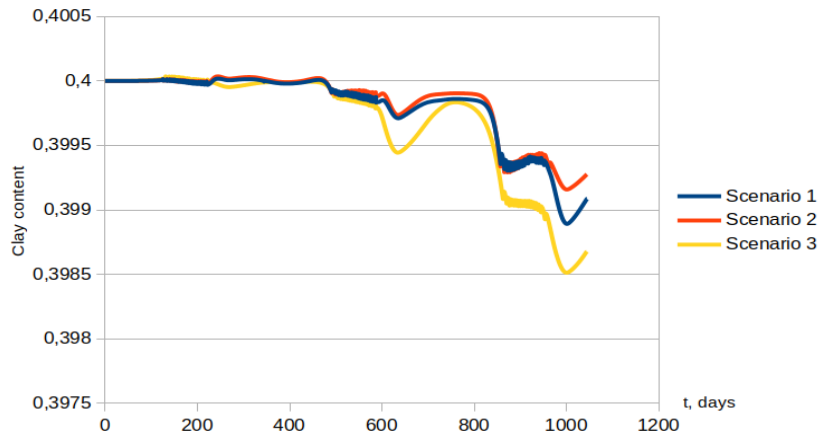


Figure 2: The dynamics of clay content in $z=0.3m$

Soil displacements in the moments $T=700,1000$ days do not exceed $0.06mm$ but exhibit a large relative influence (15.2% with a notable increase) of the consideration of non-local interactions. Changes with time in particle content in 30 cm layer are given in Table 3 for Scenarios 1 and 3. The results show that clay is by far the most mobile fraction. Across all three scenarios and all time points, clay shows depletion rates >7 times larger than for other types of particles. This size-selectivity is physically consistent with the smaller hydrodynamic diameter and lower settling velocity of clay particles, which render them more susceptible to downward advective transport before reattachment or straining can immobilise them.

Table 3
Changes with time in particle content in 30 cm layer

	Scenario 1			Scenario 3		
	sand	silt	clay	sand	silt	clay
Average content in 30 cm:						
T=350 days	0.24999	0.34998	0.39984	0.24999	0.34997	0.39980
T=700 days	0.24998	0.34988	0.39908	0.24998	0.34986	0.39893
T=1000 days	0.24997	0.34972	0.39780	0.24997	0.34971	0.39767
Change compared to the initial state:						
T=350 days	0.0008%	0.0055%	0.0385%	0.0010%	0.0070%	0.0489%
T=700 days	0.0047%	0.0327%	0.2299%	0.0054%	0.0378%	0.2654%
T=1000 days	0.0112%	0.0781%	0.5477%	0.0118%	0.0827%	0.5804%
Average change	0.0055%	0.0387%	0.2720%	0.0061%	0.0425%	0.2982%

Texture change accelerates non-linearly with time when 30 cm layer is considered aggregately. This suggests that progressive clay depletion from the upper layer reduces the capacity for re-adsorption of suspended particles, accelerating downward flux over time. This process can result from a positive feedback through the dynamic update of the functions representing hydro-physical properties of the soil. Also, the consideration of non-local interactions through fractional-order operators consistently produces the largest fluxes that are mathematically controllable and, thus, can be used for fitting the model in the case of large deviations from the classical behaviour.

The absolute magnitude of the texture change over three years is small but physically meaningful. The model therefore predicts that texture redistribution would become hydraulically significant on timescales of decades of continuous irrigation, consistent with field observations such as those in [1] where accumulation of clay particles below 40 cm layer is reported for rice cultivation in paddy fields along with significant differences in clay particles distributions between paddy and non-paddy fields cultivated during 300 years, but not less.

6. Conclusions

We present the model that is an extension of the fractional-order poroelastic model given in [5] that incorporates simultaneous transport of three types of soil particles – sand, silt, and clay – with their redistribution continuously modifying the functional representation of hydro-physical properties and the density of the soil. The resulting simulation framework was implemented as a multi-threaded code using OpenMP with all subsystems sharing a single adaptive time step and a joint Picard iteration.

Three-year simulations of a heavy clay loam soil column under a realistic irrigation schedule with a climatic water deficit of $\approx 376 \text{ mm yr}^{-1}$ produced the following findings.

First, clay is the dominant mobile fraction: its depletion rate in the top 30 cm layer significantly exceeds that of other particle types in agreement with the size-selective transport physics of the used attachment–straining–detachment particle transport model.

Second, texture change in the top 30 cm layer accelerates non-linearly with time, suggesting a positive feedback through the dynamic update of the hydro-physical parameter functions: as clay is progressively depleted from the upper layer, the re-adsorption capacity decreases, amplifying the downward flux in subsequent periods.

Third, the three modelling scenarios revealed that the inclusion of poroelastic coupling slightly reduces the predicted particle washout, while the use of fractional-order spatial operators with $\alpha=\beta=0.9$ consistently amplifies it. The fractional-order case also produces a 15.2% larger soil displacement compared to the integer-order poroelastic case, with the gap growing in time, whereas the effect on average water head remains below 0.53% across scenarios. This confirms that fractional-order operators provide a mathematically controlled mechanism for adjusting the predicted intensity of both mechanical and transport responses to non-classical anomalous behaviours observed in soil, the structure of which can be characterized as fractal.

The absolute simulated texture changes over three years are modest (up to 0.58% of initial clay content in the top 30 cm layer), but the non-linear trend indicates possible significant redistribution on the decadal timescales typical for long-term irrigation.

Future work could be performed to address validation against column-scale experimental data and the extension of the model to two spatial dimensions to capture lateral heterogeneity formed where drip irrigation techniques are used. Sensitivity analysis, either mathematically rigorous or

numerical, e.g. by Sobol indices technique, is another direction of a separate research regarding the proposed model that could increase its practical significance.

Declaration on Generative AI

During the preparation of this work, the authors used Claude for: Drafting content. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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