

Wavelet features of two-dimensional images of the oculomotor system quadratic Volterra models transient characteristics for psychophysiological state assessment

Vitaliy Pavlenko^{1,*†}, Andrii Doroshuk^{1,†}

¹ Odessa Polytechnic National University, Shevchenko Ave. 1, 65044 Odesa, Ukraine

Abstract

This paper examines the application of a modified method of eye-tracking data processing for assessing an individual's psycho-physiological state. Based on data from "input-output" experiments with the horizontal movements of on-screen stimuli, a model of the human oculomotor system has been constructed in the form of a second-order Volterra polynomial. Wavelet coefficients with Daubechies 8 filter bank were calculated for images depicting the linear part of the model's transient function and a diagonal section of a quadratic function, a feature space was constructed using the moments of their distribution. Support Vector Machine classifiers were built for the experimental data in horizontal direction and for the augmented dataset containing original samples with additive 1, 3 and 5% level Gaussian noise. To reduce the dimensionality of the space, only pairs of features were considered, and a complete search was used to select the best pair. For the first time, a method has been developed for constructing diagnostic feature spaces based on two-dimensional images of the combined first- and second-order transient characteristics of oculomotor system models, which has made it possible to use synchronous spatio-temporal representation of transient characteristics for developing fundamentally new machine learning diagnostic models with accuracy 0.875 and F1-score 0.867 for original dataset, and accuracy 0.9375 with F1-Score 0.936881 on augmented dataset for recognition of psychophysiological states.

Keywords

eye tracking, Volterra polynomials, wavelet transform, machine learning, oculomotor system, modeling

1. Introduction

Today, eye-tracking technologies are widely used in neuroscience research. They make it possible to assess various properties of the human oculomotor system, which may be specific to an individual or depend on the individual's condition. Both the theoretical and practical applications of this technology require the development of new data analysis methods that are interpretable. One of the current research priorities is to study the characteristics of the oculomotor system's response to stimuli, which enable the development of a mathematical model of the system. This will simplify the diagnostics of this system and the study of its characteristics. It is particularly important when assessing a person's level of fatigue, which can be applied in various situations, such as determining eligibility for a particular job. This problem requires the construction of a feature space, which can be derived from "input-output" experimental data. Based on these features, it is possible to build classifiers that allow us to estimate the state.

A promising method for modeling the oculomotor system is to formulate its equation as a segment of a Volterra series. It allows the system to be described as a nonlinear dynamic model, taking into account both the linear and nonlinear components of the system's response. This method requires

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^{1*} Corresponding author.

[†] These authors contributed equally.



pavlenko_vitalij@ukr.net (V. Pavlenko); andrew.doroshuk@gmail.com (A. Doroshuk)



0000-0002-5655-4171 (V. Pavlenko); 0009-0008-2327-4975 (A. Doroshuk)



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only a few experiments using test signals of varying amplitudes. Unlike neural networks, this method preserves mathematical consistency.

To analyze the results, wavelet transforms were selected, as they allow the consideration of the frequency-time representation of transient processes in both the linear and nonlinear parts of the model, taking into account its characteristics at different levels of decomposition. The moments of the wavelet coefficient distributions are used, which makes it possible to account for all of them simultaneously at a single level of decomposition while maintaining a low-dimensional feature space for classification, thereby simplifying their practical application in the future.

The Support Vector Machine method was chosen to classify the results of these experiments involving eye motions.

Both the original and augmented datasets were used for horizontal movements dataset to investigate classifier metrics.

2. Literature review

Eye tracking has recently gained popularity as a tool for assessing various characteristics of an individual and their state based on experimental data. This technology has been used in research to assess the performance of individuals in various professions in the workplace. For example, paper [1] examined the attention of drillers under varying levels of mental workload. Eye tracking is also used to monitor astronauts [2] and pilots, particularly in the development of human-machine interfaces [3]. Other studies have described medical applications, such as for assessing Parkinson's disease [4], Alzheimer's disease [5, 6, 7], autism spectrum disorders [8, 9], dyslexia [10], for diagnosing disorders [11] and mild cognitive impairments [12]. Tahri Sqalli et al. [13] also mention other applications, such as assessing the visual expertise of cardiologists. There are also other applications [14], such as recognizing emotions [15] or mental states [16]. Eye-tracking data also makes it possible to assess fatigue [17], for example, in a traffic control center [18]. One of the distinguishing features of eye movements is their transient processes, including their nonlinear properties. The study [19] examines the use of models in the form of a Volterra series, which allows for the estimation of such properties.

3. Theoretical background

This study utilized data from an experiment in which eye movements were recorded in response to stimuli in the form of three-step signals of varying amplitude designed to represent a target point on the screen. The experiment is described in detail in previous studies [20, 21]. For the case, the signal amplitude a corresponds to the shift of the stimulus on the screen, where $a_1 = (1/3)X$, $a_2 = (2/3)X$, $a_3 = X$ (where X is the screen width in pixels). To assess transient processes in the system, datasets from Tobii Pro TX300 eye tracker were used, which recorded eye responses to stimuli from eight students in the morning and after classes with sampling rate 300 Hz [20].

3.1. Volterra model of human eye movement system

Volterra polynomials are a convenient tool for modeling nonlinear continuous-time dynamical systems, including oculomotor system [22, 23]. They describe the system's response as the sum of integrals of the following form:

$$\begin{aligned}
y_N(t) = \sum_{n=1}^N \hat{y}_N(t) = & \int_0^t w_1(\tau)x(t-\tau)d\tau + \\
& + \iint_{00}^{tt} w_2(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1d\tau_2 + \\
& + \iiint_{000}^{ttt} w_2(\tau_1, \tau_2, \tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1d\tau_2d\tau_3 + \dots,
\end{aligned} \tag{1}$$

where $y_N(t)$ is the output function of the model, N is the approximation order, $\hat{y}_N(t)$ is the n -th partial component of the model output function (an n -th order convolution integral), $w_n(\tau_1, \dots, \tau_n)$ is the n -th order Volterra kernel (weight function); t is the current time, τ_i is a point in the past that determines the memory of the system.

In this case, the nonlinear properties of the system are described by the corresponding multidimensional weight functions (Volterra kernels) under convolution integrals. To identify a nonlinear dynamic system with an unknown structure, it is necessary to find the Volterra kernels based on input-output experimental data. Algorithms for calculating Volterra kernels based on various test signals are described in detail in [24, 25]. In this paper the least-squares algorithm was used.

In this study, multi-step test signals were involved, which are suitable for identifying the human oculomotor system. One step signal can be represented as a Heaviside step function:

$$\theta(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{otherwise} \end{cases} \tag{2}$$

where t is time, when the stimulus is moved to the corresponding position on the screen.

These test signals make it possible to estimate the transient function of the linear part of the system, which is described by the following equation:

$$h_1(t) = \int_0^t w_1(t-\lambda_1)d\lambda_1 \tag{3}$$

where w_1 is first-order Volterra kernel.

The diagonal section of the quadratic transient function is described by the following equation:

$$h_2(t, t) = \int_0^t \int_0^t w_2(t-\lambda_1, t-\lambda_2)d\lambda_1d\lambda_2 \tag{4}$$

where w_2 is second-order Volterra kernel.

Figure 1 shows transient responses of the linear system part and diagonal sections of quadratic functions in the morning. For each individual experiment, its functions are represented by a solid line of a single color. A dotted line indicates the average value for the corresponding set of functions.

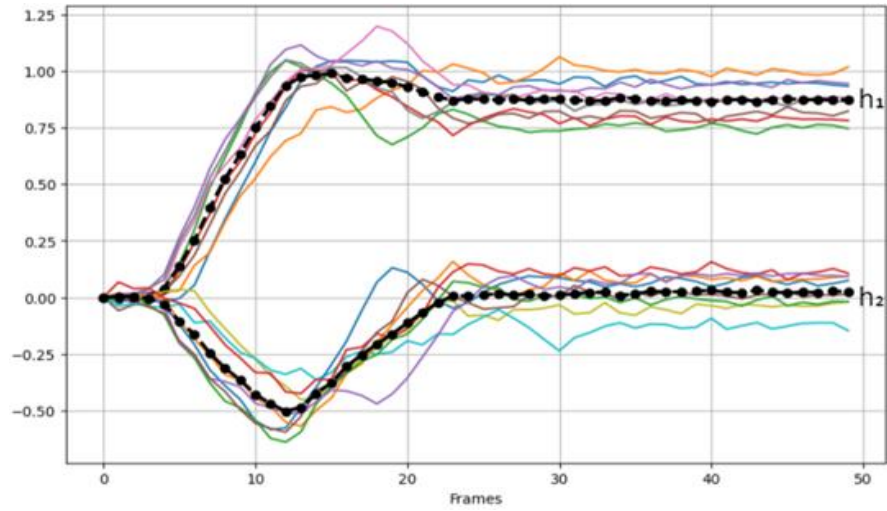


Figure 1: Transient function and diagonal section of quadratic transient characteristics for Volterra models of the morning experimental data (source: [26])

Figure 2 shows similar graphs for the evening experiments.

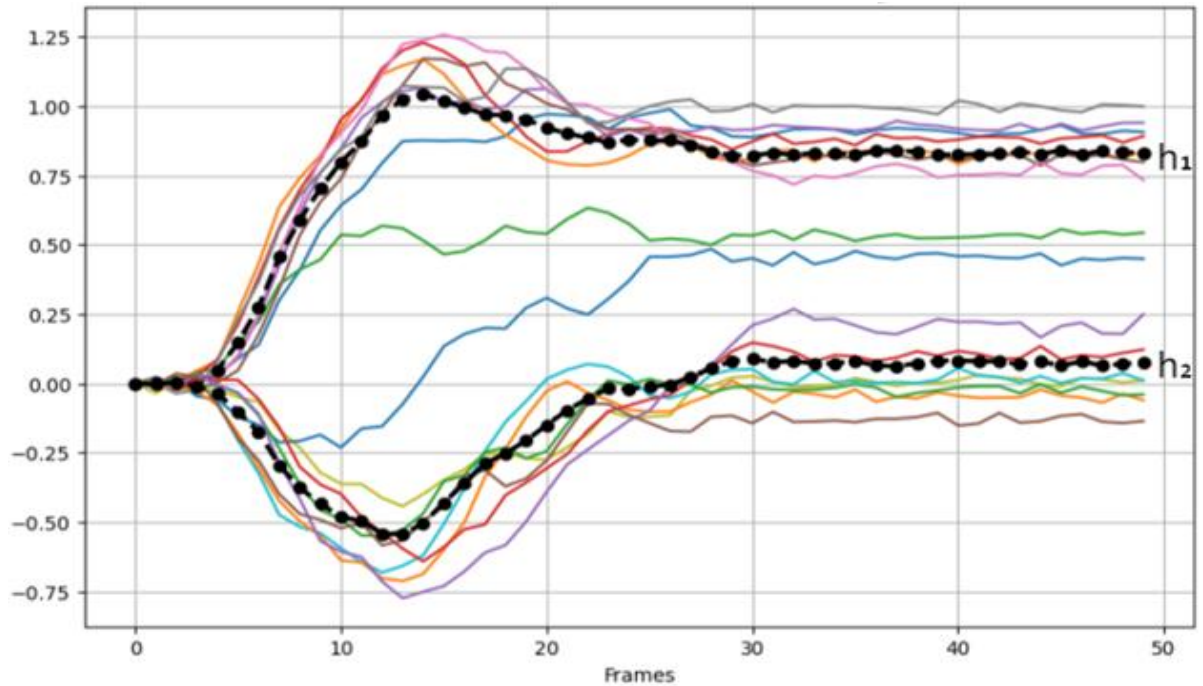


Figure 2: Transient function and diagonal section of quadratic transient characteristics for Volterra models of the evening experimental data (source: [26])

The figures show that the functions differ across the various experiments.

3.2. Image Formation and Wavelet Decomposition

To simultaneously take into account both the linear transient response and the diagonal section of the quadratic transient response, images were generated that contain their respective curves, where the horizontal axis represents time and the vertical axis represents amplitude. An orthogonal bank of filters with a finite impulse response is used to implement the wavelet [28]. Figure 3 shows the impulse responses of the filters.

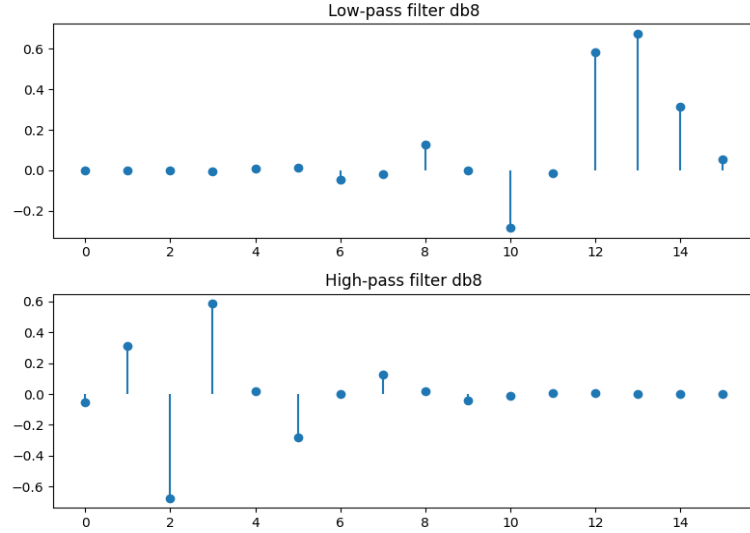


Figure 3: Impulse responses of filters for wavelet decomposition

As a result of successively applying a low-pass filter (L) and a high-pass filter (H) in different combinations to the rows and columns of the image, we obtain four images with names LL, LH, HL, and HH. Quadratic splines [27] were used to interpolate the discrete values obtained from the eye tracker. Since the image is divided into four coefficients at each level of decomposition, square images with a side length of 1024 pixels were generated, which ensures sufficient accuracy. A db8 implementation of Daubechies 8 transform from the PyWavelets library in Python was used.

4. Feature space overview

To reduce the dimensionality of the features to two scalar values – thereby reducing the number of computations – five moments of their distributions were used, as shown in Table 1.

Table 1
Features for classification

Name	Formal definition
Mean	$\mu = E[X]$
Variance	$Var(X) = E[(X - \mu)^2]$
Skewness	$skew(X) = \frac{E[(X - \mu)^3]}{E[(X - \mu)^2]^{3/2}}$
Kurtosis	$Kurt(X) = \frac{E[(X - \mu)^4]}{E[(X - \mu)^2]^2}$
5 central moment	$Mom5(X) = E[(X - E(X))^5]$

For classification based on wavelet coefficients, the moments of the distributions of each of the resulting images were used for the second level of decomposition. This makes it possible to assess at which frequencies the differences between classes reach the greatest values, as well as which image details (vertical, horizontal, or diagonal) are most useful for constructing the feature space. The scipy library was used to calculate the moments.

5. Classification

In all cases, an SVM implementation with a radial basis function kernel from the scikit-learn library was used.

For dataset consisting of 16 experiments (eight in the morning and eight in the evening), which correspond to different levels of fatigue (fatigue is higher in the evening), as well as for augmented feature set, where 1%, 3%, and 5% white noise was added to each feature, thereby expanding the dataset to 32 instances of each class. For each of them, the Stratified k-fold procedure with 4 folds was used, and the Accuracy and F1-Score average metrics between folds were evaluated. Parameter C was set to 0.01 for regularization and to prevent overfitting, and a gamma scale was used.

5.1. Results for movements in horizontal direction

The classification results for the best feature pairs in the original dataset are shown in Table 2, and those for the augmented dataset are shown in Table 3. The classification boundary plots for the respective datasets are shown in Figure 4 and Figure 5 , respectively.

Table 2

Results for original horizontal dataset classification

Feature pair	Accuracy	F1-Score
L2_LH_mean & L1_LH_skew	0.8750	0.866667
L2_LH_skew & L1_LH_skew	0.8750	0.866667
L1_LH_skew & L1_LH_kurtosis	0.8125	0.808333
L1_LH_skew & L1_HH_mean	0.8125	0.808333
L2_LL_skew & L1_LH_skew	0.8125	0.8

It is visible that horizontal components (LH coefficients) yield good results. Each pair among the top contains one of the first-level LH coefficient components, indicating that at these frequencies for this level of decomposition the difference between the classes is most noticeable.

Table 3

Results for augmented dataset classification

Feature pair	Accuracy	F1-Score
L2_HH_mom5 & L1_LH_skew	0.9375	0.936881
L1_LH_skew & L1_LH_kurtosis	0.921875	0.921195
L2_HH_skew & L1_LH_skew	0.921875	0.920044
L2_LL_skew & L1_LH_skew	0.90625	0.905509
L1_LH_mean & L1_LH_mom5	0.90625	0.905258

For both the augmented and original datasets, the best pair of features contains skew of first-level LH coefficients.

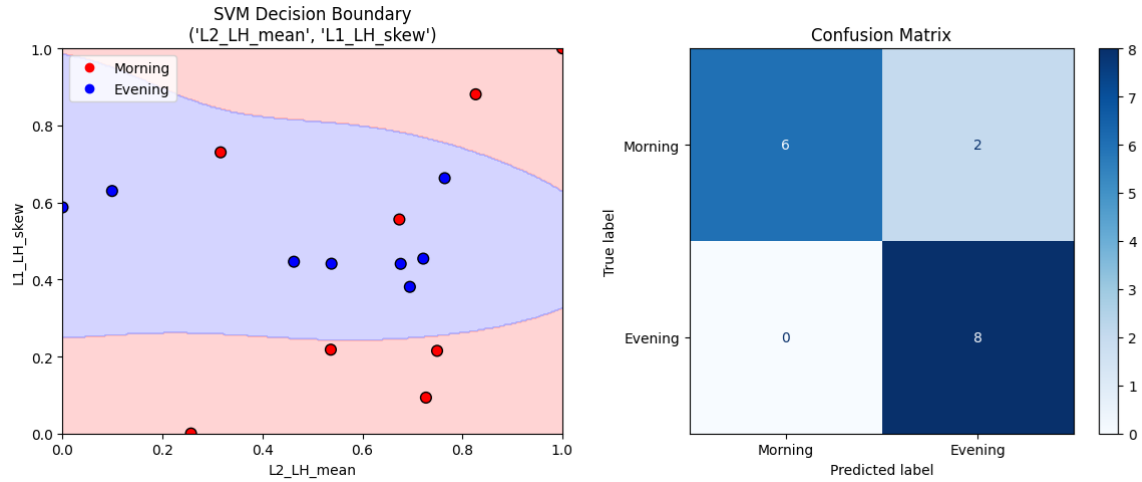


Figure 4: Feature space, decision boundary and confusion matrix for original horizontal movement dataset

Both figures show that the boundary between the classes is elliptical in shape. This indicates that the classes cannot be separated linearly. The use of an SVM with a Gaussian kernel allows to separate the classes successfully.

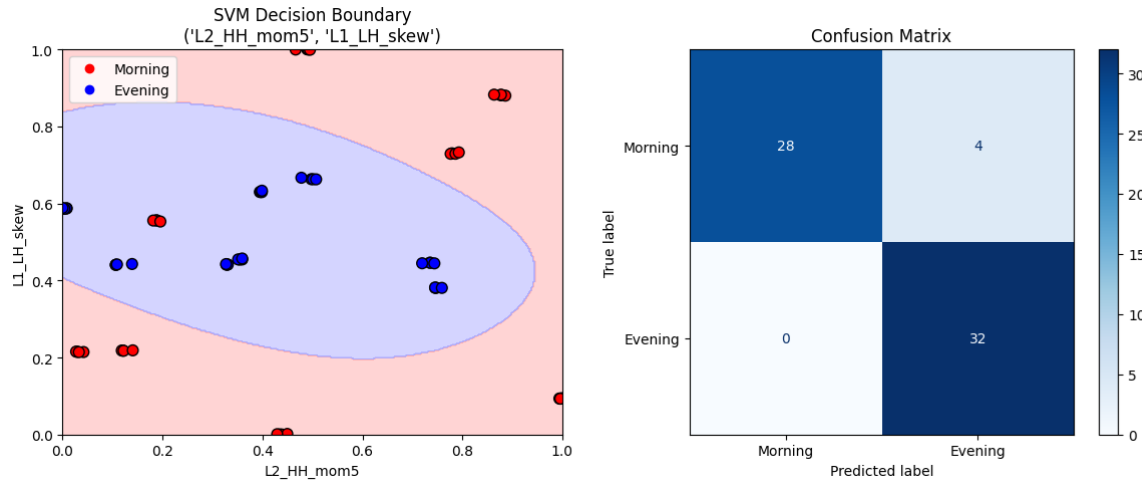


Figure 5: Feature space, decision boundary and confusion matrix for augmented horizontal movement dataset

It is clear that the accuracy differs between the original and augmented datasets. This suggests that a small number of experiments is insufficient to successfully generalize the properties of the data to the entire class.

6. Discussion

As a result of this work, the separability of classes in spaces based on the moments of wavelet coefficients was evaluated. The results show that, for decomposition level two, the first-level coefficients have a significant influence. The classes are separable in each of the spaces, but not linearly. A classification accuracy of at least 90% was achieved for augmented experiments dataset, which may indicate their suitability for fatigue assessment tasks based on multiple eye-tracking experiments. To prevent overfitting, stratified k-fold cross-validation and decision boundary visualization were used.

The results show that augmentation improves classification accuracy to 0.9375 compared to the 0.875 for the original dataset.

Using the described algorithm makes it possible to simultaneously account for both the linear and nonlinear responses of the system, while using only two scalar features, which simplifies its practical application.

Among the data points, the best pair for each of the two datasets contains skew of coefficients, that indicates the characteristics of the data distribution. Further experiments are needed to verify these results.

7. Conclusions

The paper proposed a modified method for individual's psycho-physiological state assessment based on eye-tracking data. Second-order Volterra model was constructed using step test signals in the form of stimuli on the white screen and least-squares algorithm. Novelty of the work lies in the application of wavelet transforms for images depicting transient characteristics of Volterra quadratic models were calculated using Daubechies 8 orthogonal filter bank. Eye movements in horizontal direction were examined during experiments using multistep test signals. As a result of this work, the classification spaces of eye-tracker data were investigated to assess fatigue levels based on wavelet coefficient moments. It was found that among the features used in this study to determine the fatigue level, the first-level wavelet decomposition coefficients provided the best quality, reaching accuracy 0.9375 for augmented dataset and 0.8750 for original one. Among the moments, skew was the most significant. The use of spline interpolation of transient characteristics makes it possible to apply this algorithm to eye trackers with different sampling rates.

Further research may include applying the results of this study to diagnostics in various settings, investigation of movements in different directions, statistical significance testing, for example on the findings on different categories of participants, the use of non-binary classifiers to assess different levels of fatigue, and using new types of stimuli as test signals — for example, three-dimensional stimuli [6]. Studying the effect of head movements during eye tracking could be considered another direction of research, since these movements alter the nature of the noise in the acquired signal and may affect the results [29].

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Declaration on Generative AI

During the preparation of this work, the authors used ChatGPT-5 for formatting assistance. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

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