

The error-based ranking method for evaluating the accuracy of solving problems in dynamic system modeling

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Abstract

The relevance of this research is determined by the growing need to improve the accuracy and reliability of computational methods used for modeling and control of dynamic systems described by differential and integral equations. The aim of this work is to develop an error-based ranking method for evaluating the accuracy of solving modeling and control problems and predicting the reliability of their computational implementation. The following tasks are addressed: development of the error-based ranking method, analytical formalization of the ranking procedure, construction of lower-bound estimates of the distribution functions of irreducible implementation errors, and numerical verification of the proposed approach. For the considered test problem, the computed parameters are $\|A\|_2 = 3.7$, $\|A^{-1}\|_2 = 16.956$, $\|b\|_2 = 8.934$ with $\mu=0.016$; $C=0.311$, while the assumed coefficient-setting accuracy is 0.3%. The resulting lower-bound distribution provides a guaranteed accuracy threshold of $C_r=0.625$ for the considered construction. These results demonstrate that the proposed ranking framework can transform repeated error-analysis calculations into a classification problem based on previously constructed reference classes. In practical computational environments, this approach can support the selection of computational tools and numerical algorithms according to required accuracy and can reduce the need for repeated high-cost error analyses. A broader statistical evaluation on multiple problem classes is required to quantify the computational savings and generalization capability of the method.

Keywords

dynamic systems; modeling; computational accuracy; irreducible error; accuracy assessment

1. Introduction

The development and refinement of methods for calculating and controlling the functioning processes and operating modes of technical systems – including the modeling and control mechanisms that constitute these systems – represent a significant scientific challenge with current practical relevance [1]. First and foremost, the relevance of this problem is linked to assessing the accuracy and reliability of the implementation of the technical (in particular, dynamic) systems under study during the stages of their analysis (modeling and identification) [2], as well as the synthesis of control laws for them [3].

Performing calculations to ensure the qualitative optimization of dynamic system parameters according to various criteria, as well as organizing effective production and operational control, is possible only by developing effective methods for analyzing the functioning processes and accuracy of the complex structures of dynamic systems that are being designed and developed [4]. The

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development of these methods gives rise to a complex set of scientific problems, the solution of which requires a series of scientific studies.

Scientific research is needed on the creation and implementation of methods and the computational algorithms derived from them for solving problems related to the analysis of operational processes, as well as for ensuring and controlling the accuracy of continuous objects and systems described by differential, integral, and algebraic equations. The objects of application for these methods and algorithms – whether existing or yet to be developed – include modeling and control systems, their elements and components, as well as specialized and general-purpose computing tools used to solve the corresponding problems (in particular, the modeling and control of dynamic systems).

2. Literature Review

An effective framework for describing dynamic processes in continuous systems when solving direct (modeling), reverse (control), and inverse (identification) problems are differential and integral equations [5]. This is due to the inherent nature of these types of mathematical expressions and the presence of an independent time parameter that characterizes the dynamic behavior of the systems under study. However, it should be noted that certain difficulties arise when solving differential and integral equations, which, in the course of modeling and controlling dynamic systems, are considered as mathematical models (MM) of these systems [6]. In particular, when solving differential equations, convergence problems may arise, and when solving integral equations, solutions may be ill-posed (for example, for Volterra and Fredholm equations of the first kind) [5]. One of the key indicators of the performance of computational tools used to solve differential and integral equations – for which there is currently no universal solution – is the assessment of the accuracy of solutions to modeling and control problems using the existing fleet of computers [7]. However, despite the widespread use of computing technology, the problem of assessing computational accuracy cannot be considered solved, and its relevance is growing due to the rapid development and proliferation of cybernetic tools for various purposes [8]. The severity of the problem lies in the complexity of analyzing computational errors, which leads to cumbersome analytical justifications and a large volume of computations required to obtain specific numerical data.

Both general-purpose and specialized computing technology (CT) tools are widely used to solve many technical and research problems. A characteristic distinction between the latter and general-purpose CT tools is the intentionally narrow class of algorithms implemented, which is focused on solving a limited range of applied problems [9]. Naturally, this is expected to yield a number of specific advantages (compared to general-purpose CT tools), which typically include one or a group of factors such as increased speed, a non-analytical approach to problem-solving (for analog specialized CT tools), reduced size and weight, and lower cost, among others [10]. It should be noted that the issue of accuracy is relevant for both general-purpose and specialized CT tools, albeit in slightly different forms depending on the type of tool and its operating principle; for example, the nature of initial errors [11].

Thus, at present, when solving the vast majority of problems in the practice of modeling dynamic systems and synthesizing control for them, a contradiction is observed: while there is well-developed modeling and methodological support for MM of industrially important and environmentally significant dynamic systems [12], the issues of assessing the accuracy and reliability of solutions to the corresponding equations as models of dynamic systems, as well as detecting failures (errors) in the computational process, have not been sufficiently developed [13].

The proposed method therefore differs from conventional sensitivity analysis and direct error estimation by introducing an intermediate layer of reference accuracy classes and correction

coefficients. The objective is not to replace rigorous error bounds, but to use them to construct reusable accuracy classes that can support rapid assessment of new computational problems. The main research gap addressed in this work is therefore the absence of a unified procedure that combines probabilistic error characterization, guaranteed bounds, reference-class construction, and computational ranking for the subsequent rapid assessment of dynamic-system model implementations.

3. Purpose and Objectives of the Study

The aim of the study is to develop methods for predicting the accuracy and analyzing the reliability of the computational implementation of dynamic system models, represented by classes of differential and integral equations, when solving applied modeling and control problems based on them, by determining quality indicators for the computer, software, and algorithmic tools used. To achieve this research objective, the following problems were formulated and solved in this work:

- a method of error-based ranking was developed to evaluate the accuracy of solving problems involving the modeling and control of dynamic systems;
- “bottom-up” estimates of the distribution functions of the irreducible implementation error of mathematical models of dynamic objects, represented by differential and integral equations, in modeling and control problems were constructed (based on the error-based ranking method).

4. Materials and Methods

4.1. Analytical formalization of the ranking procedure based on the framework of differential and integral equations

Let us assume that the control theory tools are used to solve problems that can be represented in the following form:

$$A_k y = x, \quad (1)$$

where A_k is an operator that is an element of a certain set of possible operators for a specific problem (or type of operational theory tool); x, y are the sought-after solution and the known right-hand side (numbers, vectors, functions), which have their own limited domains of definition.

Initial errors in the initial data necessitate solving an approximate problem

$$\tilde{A}_k \tilde{y} = \tilde{x}, \quad (2)$$

where $\tilde{A}_k = A_k + \Delta A_k$, $\tilde{x} = x + \Delta x$.

The machine (approximate) solution can be expressed as:

$$\tilde{y} = R_m \tilde{x}, \quad (3)$$

where R_m is a computational algorithm, which represents a system of operations in general-purpose or specialized computing hardware, and therefore incorporates the entire set of primary instrumental errors, as well as the error of the implemented solution method.

The solution error is, obviously, defined as:

$$\Delta y = \tilde{y} - y, \quad (4)$$

can only be found if an exact solution is known, which limits the practical application of (4) as a computational formula to cases involving the solution of test (control) problems.

Rearranging (4), we obtain

$$\Delta y = R_m \tilde{x} - A_k^{-1} x, \quad (5)$$

which can be taken as the initial error equation.

Expressing the solution error as the result of transforming the primary errors q_s ($s=1, 2, \dots, n$)

$$\Delta y = F[q_s], \quad (6)$$

it can be stated that the transformation F is determined by two main factors: the properties of the problem being solved and the accuracy characteristics of the computational tool, which is a well-known issue that causes numerous complications in the analysis of computational accuracy [14].

Given the large number of initial errors, the significant computational effort involved in carrying out the above procedures, and the complexity of the initial relationships (5), the following algorithm can be followed:

1. Select a solution accuracy metric $T[\Delta y]$ that provides sufficiently comprehensive information about the error and can be effectively determined through numerical experiments.
2. Perform a series of computer calculations (experiments) to establish the relationships between $T[\Delta y]$ and the initial errors q_s ($s = 1, 2, \dots, n$) across a set of possible problems. It is entirely reasonable in this case to use a grid of reference problems that “discretely” represent and cover the entire set of problems being solved, which ultimately allows for the use of an interpolation approach when investigating the accuracy of any of the possible problems that do not coincide with any of the reference problems, but for which the closest (in terms of accuracy) reference problem can be found.
3. Identify, in the course of these calculations, groups of closely related dependencies that combine individual problems into accuracy classes. In this context, it is possible to automate the classification process using recognition methods, where the computer itself groups problems (dependencies) and identifies similarity criteria.
4. Identify the accuracy metrics of the problem

$$Q_\xi = Q_\xi(A_k, x); (\xi = 1, 2, \dots, \xi \leq s), \quad (7)$$

that allow, using a simple rule C , the K -th problem to be assigned to the L -th class based on accuracy parameters

In this case, it can be stated that the method considered below [6] is based on the principle of ranking by task errors and CT tools, which consists in evaluating solution errors using associative dependencies between the accuracy metrics of interest, and the primary errors obtained as a result of the preliminary process of solving the modeling problem, as well as by classifying the problems being solved according to their accuracy using a system of reference examples.

4.2. The error-based ranking method in the implementation of mathematical models of dynamic systems in modeling and control problems

The literature describes methods for analyzing accuracy [15] in the numerical solution of systems of linear algebraic equations

$$\mathbf{Ax} = \mathbf{b}, \quad (8)$$

where $\mathbf{A}$ is a nonsingular matrix of order n ; $\mathbf{b}$ and $\mathbf{x}$ are n -dimensional vectors representing the right-hand side and the sought solution, respectively; among these, the most widely used are: methods of classical accuracy theory (sensitivity theory [12]), performing intentionally more accurate computations when solving the problem, and applying analytical estimates of the form of inequalities [16].

The ranking principle, the analytical formalization of whose procedures is discussed in Section 2.1, offers certain possibilities for overcoming the complications mentioned above. These opportunities lie in the fact that the bulk of the computations involved in accuracy analysis are performed in advance, resulting in tables (or graphs) that reflect how accuracy metrics depend on initial errors for different classes of problems. Data prepared in this way make it possible to obtain numerical characteristics of errors fairly quickly, with minimal computation, provided that the specific problem being solved can be “successfully” classified into one of the “pre-ranked” classes.

To develop a ranking method for the implementation of dynamic system MM in modeling and control problems [5], we will analyze the irreducible error of the solution to system (8), which arises when considering the system in an excited state:

$$(\mathbf{A} + \Delta\mathbf{A})(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{b} + \Delta\mathbf{b}, \quad (9)$$

where $\Delta\mathbf{A} = \{\Delta a_{ij}\}, i, j = \overline{1, n}$ is the matrix of initial errors in the coefficients of matrix $\mathbf{A}$; $\Delta\mathbf{b} = \{\Delta b_i\}, i = \overline{1, n}$ is the vector of errors (excitation terms) in the right-hand sides; $\Delta\mathbf{x}$ is the vector of solution errors.

As an initial estimate of the residual error, we choose the norm of the error vector [6]

$$\|\Delta\mathbf{x}\| \leq \|\mathbf{A}^{-1}\| \cdot \|\boldsymbol{\varepsilon}\|, \quad (10)$$

where $\boldsymbol{\varepsilon} = \mathbf{A}(\mathbf{x} + \Delta\mathbf{x}) - \mathbf{b}$ is the vector of inconsistencies.

Due to the use of the representation of the error vector $\boldsymbol{\varepsilon}$ in the form

$$\boldsymbol{\varepsilon} = \mathbf{D}\boldsymbol{\varepsilon}_0 + \mathbf{m}_{\boldsymbol{\varepsilon}}, \quad (11)$$

where $\boldsymbol{\varepsilon}_0$ is a random normal vector with mean zero and a correlation matrix of unity, the following estimate can be obtained for this vector; $\mathbf{D}$ is a diagonal matrix whose elements are defined as the variances of the corresponding random variables; $\mathbf{m}$ is a vector whose elements are defined as the mathematical expectations of the corresponding random variables

$$\|\Delta\mathbf{x}\| \leq \|\mathbf{A}^{-1}\|(\|\mathbf{D}\| \cdot \|\boldsymbol{\varepsilon}_0\| + \|\mathbf{m}_{\boldsymbol{\varepsilon}}\|), \quad (12)$$

which, like the corresponding estimate for the relative error, can already be used for matching systems of linear algebraic equations by accuracy, where the class criterion is the order of the system n , the accuracy parameter of the class is the distribution function $\|\boldsymbol{\varepsilon}_0\|$, and the accuracy parameters of the problem are the values $\|\mathbf{A}^{-1}\|, \|\mathbf{D}\|$ and $\|\mathbf{m}_{\boldsymbol{\varepsilon}}\|$.

Issues regarding the computation of the distribution functions of certain norms of the vector $\boldsymbol{\varepsilon}_0$ were considered in [5] from the perspective of using available tabulated functions (the normal distribution function, the distribution χ^2). For the purpose of ranking, however, it seems appropriate to tabulate the distribution functions of various norms of the vector $\boldsymbol{\varepsilon}_0$ for different systems of linear algebraic equations. In this case, the values of the distribution functions $\|\boldsymbol{\varepsilon}_0\|_1$ can be calculated using the formula

$$F_{\|\epsilon_0\|_{1,n}}(\tau) = \left(\frac{2}{\sqrt{2\pi}} \int_0^\tau e^{-(t/2)^2} dt \right)^n \quad (13)$$

the value of the distribution function $\|\epsilon_0\|_2$ is given by the formula

$$F_{\|\epsilon_0\|_{2,n}}(\tau) = \frac{2}{\sqrt{2\pi}} \int_0^\tau [F_{\|\epsilon_0\|_{2,n-1}}(t) e^{-(\tau-t/2)^2}] dt \quad (14)$$

The existence of these distribution functions makes it easy to construct a lower bound for the distribution functions $\|\Delta x\|$ ($\|\Delta x\|/\|x\|$) (or a confidence limit for the error) by scaling the independent variable of the corresponding distribution function of the quantity $\|\epsilon_0\|$ by $\|A^{-1}\| \cdot \|\mathbf{D}\| [(\|A^{-1}\| \cdot \|\mathbf{D}\|)/\|x\|]$ and shifting the origin to the left by $\|A^{-1}\| \cdot \|\mathbf{m}_\epsilon\| [(\|A^{-1}\| \cdot \|\mathbf{m}_\epsilon\|)/\|x\|]$.

It can be shown that for the distribution of the vector under consideration (the distribution), the random variables $T_i(A)$ and $\|\epsilon_0\|_i$ are independent when $i=n$ and dependent when $i = \overline{1, (n-1)}$. However, the dependence in the latter case is weak, and therefore it can be neglected in practical calculations. Thus, knowing the distribution functions of the variables $T_i(A)$ and $\|\epsilon_0\|_i$, one can determine the distribution function of the variable $\|\Delta x\|_i$.

The above describes a method for using correction coefficients to rank by error (in essence, by accuracy) when implementing mathematical models of dynamic systems by constructing sets (classes) of matrices Q_j , $j = \overline{1, m}$, and determining $F_{T_{ij}}(\tau)$, $j = \overline{1, m}$. In doing so, the following conditions (class conditions) must be satisfied:

- class quality condition—the distribution function of the correction coefficients of the matrices from the fixed set Q_j , $j = \overline{1, m}$ must be, in a certain sense, close to one another, and the degree of closeness (or measure of dispersion) is a characteristic of the quality of this class Q_j ;
- class discrimination condition—the existence of a criterion for determining whether a specific problem belongs to a certain class, which can actually be computed (class criterion).

In this context, the practical algorithm for solving the problem of constructing the set of matrices Ξ and subsequently partitioning it into classes Q_j , $j = \overline{1, m}$, can be presented as follows:

1. The class criterion is defined a priori. In other words, at the first stage of the solution, it is necessary to a priori (conditionally) partition the initial set of matrices Ξ into m classes Q_j , $j = \overline{1, m}$. However, m should not be chosen too small, since it is clear that the more classes are a priori defined by the selected criterion (i.e., the smaller the degree of variation among matrices within a single class), the higher the likelihood of obtaining classes of “good” quality. On the other hand, the number m should not be too large, since a significant number of a priori defined classes leads to a substantial computational load. Solving practical problems has shown that it is advisable to set $m \approx 10$.
2. $F_{T_{ij}}(\tau)$, and the quality measure C_0 for each class Q_j are defined as $j = \overline{1, m}$ for the selected norms i ($i = \overline{1, n}$). Regarding the computation of $F_{T_{ij}}(\tau)$, and C_{ij} , the following should be noted. The analytical calculation of these characteristics is quite complex; therefore, it is advisable to use numerical methods of analysis in the calculations, which necessitates replacing the set of matrices Q_j , $j = \overline{1, m}$, with a finite subset Q_{jN} , consisting of N representative matrices of the set Q_j . It also makes sense to define, instead of $F_{T_{ij}}(\tau)$, the

following distribution function $F_{\bar{T}_{ij}}^*(\tau)$ (i.e., the estimate $F_{\bar{T}_{ij}}(\tau)$), such that $p\{F_{\bar{T}_{ij}}^*(\tau) \leq F_{\bar{T}_{i(A)}}(\tau), A \in \mathbf{Q}_j\}$.

3. Define the classes $\mathbf{Q}_j, j = \overline{1, m}$ for which $C_{ij} < C_0$, where C_0 is the minimum class quality estimate. We will assume that there are m_1 such classes. Next, by appropriately modifying the class criterion, we partition the set $\mathbf{Y}_s \mathbf{Q}_s$ into m_{s_1} subsets $\mathbf{Q}_{s_1}, s_1 = \overline{1, m_{s_1}}, s_1 = \overline{1, m_{s_1}}$, such that $m_{s_1} > m_1$ holds. After that, calculations are performed for stage 2, then stage 3, and so on, until the quality estimates of the classes being formed are no worse than the minimum. This iterative process of constructing classes with a given quality may converge in the sense of $m_1 \rightarrow 0$ for large l (where l is the number of iterations back to Stage 2).
4. Using cluster analysis methods, such as the “k-means” method [10], m classes are grouped based on a specified distance function between classes and a class merging criterion (distance threshold). The classes resulting from this grouping form subclasses. The minimum quality score among the classes belonging to a subclass can serve as a measure of the subclass’s quality. Solving test problems shows that performing such a classification of the matrices $\mathbf{Q}_j, j = \overline{1, m}$ allows for a more efficient (compared to [14]) estimates of the distribution function $\|\Delta \mathbf{x}\|_i$ by determining the class $\mathbf{Q}_j$ to which a specific matrix $\mathbf{A}$ belongs and constructing the distribution function of the values $\bar{T}_{ij} \|\mathbf{A}^{-1}\|_{i\sigma} \|\mathbf{e}_0\|_i$, for which it is necessary to have the distribution function $F_{\bar{T}_{ij}}(\tau)$ of the value $\bar{T}_{ij} \|\mathbf{e}_0\|_i$. Assuming that the values $\bar{T}_{ij}$ and $\|\mathbf{e}_0\|_i$ are independent, the distribution function of their product can be estimated relatively easily using the volume formula:

$$F_{T_{ij}\epsilon}(\tau) = p\{\bar{T}_{ij} \|\mathbf{e}_0\| \leq \tau\} > \frac{1}{R} \sum_{r=1}^R F_{\|\epsilon_0\|} \left(\frac{\tau}{P_r} \right) = F_{\bar{T}_{ij}}^*(\tau), \quad (15)$$

where R is the number of intervals into which the interval $[0,1]$ is divided; P_r is the r/R is the quartile.

4.3. Constructing “lower” estimates of the distribution laws of the non-removable error based on the use of guaranteed estimates

Hereinafter, we will assume that the excitation elements $\Delta \mathbf{A}$ and $\Delta \mathbf{b}$ are random variables with known probability characteristics. The lower-bound distribution law for the quantities $\|\Delta \mathbf{x}\|$ and $\|\Delta \mathbf{x}\|/\|\mathbf{x}\|$ (i.e., those that determine the accuracy of the solution to equation (8)) can be obtained by determining the distribution law of the right-hand side of the corresponding guaranteed error. Without loss of generality, we will consider the following estimates:

- for absolute errors

$$\|\Delta \mathbf{x}\| \leq \frac{\|\mathbf{A}^{-1}\| (\|\Delta \mathbf{A}\| \cdot \|\mathbf{x}\| + \|\Delta \mathbf{b}\|)}{1 - \|\mathbf{A}^{-1}\| \cdot \|\Delta \mathbf{A}\|}, \quad (16)$$

- for relative errors

$$\frac{\|\Delta \mathbf{x}\|}{\|\mathbf{x}\|} \leq \frac{\|\mathbf{A}^{-1}\| \cdot \|\mathbf{A}\| \left(\frac{\|\Delta \mathbf{A}\|}{\|\mathbf{A}\|} + \frac{\|\Delta \mathbf{b}\|}{\|\mathbf{b}\|} \right)}{1 - \|\mathbf{A}^{-1}\| \cdot \|\Delta \mathbf{A}\|}, \quad (17)$$

In the first case, the random variable of interest is a function of two random variables $\|\Delta \mathbf{A}\|, \|\Delta \mathbf{b}\|$ and non-random parameters $\|\mathbf{A}^{-1}\|, \|\mathbf{x}\|$. In the second case, it is a function of the same

random variables and non-random parameters $\|\mathbf{A}^{-1}\|, \|\mathbf{A}\|, \|\mathbf{b}\|$. It should be noted that the values $\|\mathbf{A}^{-1}\|, \|\mathbf{x}\|$ are, as a rule, unknown.

To solve the given problem, it is necessary to know the distribution functions of the random variables $\|\Delta\mathbf{A}\|, \|\Delta\mathbf{b}\|$. To find them, we propose the following procedures.

1. Let there be an n -dimensional vector $\Delta\mathbf{b}$, whose components Δb_i and $i = \overline{1, n}$ are random variables with distribution functions $F_i(\tau)$. We need to:

– Find $F_{\|\Delta\mathbf{b}\|_1}(\tau)$:

$$F_{\|\Delta\mathbf{b}\|_1}(\tau) = \begin{cases} \prod_{i=1}^n [F_i(\tau) - F_i(-\tau)], & \tau > 0; \\ 0, & \tau \leq 0. \end{cases} \quad (18)$$

– Determine $F_{\|\Delta\mathbf{b}\|_2}(\tau)$:

$$F_{\|\Delta\mathbf{b}\|_2}(\tau) = P\left\{\sum_{i=1}^n |\Delta b_i| \leq \tau\right\} \quad (19)$$

2. Let there be an $(n \times n)$ matrix $\Delta\mathbf{A}$, whose components Δa_{ij} and $i, j = \overline{1, n}$ are independent continuous random variables with distribution functions $F_{ij}(\tau)$. We need to:

– determine $F_{\|\Delta\mathbf{A}\|_1}(\tau)$:

$$F_{\|\Delta\mathbf{A}\|_1}(\tau) = \begin{cases} \prod_{i=1}^n F_i^*(\tau), & \tau > 0; \\ 0, & \tau \leq 0. \end{cases} \quad (20)$$

where $F_i^*(\tau)$ is the distribution function of the random variables ξ_i .

– Determine $F_{\|\Delta\mathbf{A}\|_2}(\tau)$:

$$F_{\|\Delta\mathbf{A}\|_2}(\tau) = \begin{cases} \prod_{j=1}^n F_j^{**}(\tau), & \tau > 0; \\ 0, & \tau \leq 0. \end{cases} \quad (21)$$

where $F_j^*(\tau)$, $j = \overline{1, n}$ is the probability mass function of the random variables.

A coarser but easily computable lower bound can be constructed by determining the probability of landing within a rectangle with sides parallel to the coordinate axes and vertices at the points $(\tau_{1min}, \tau_{2min}), (c, \tau_{2min}), (\tau_{1min}, \alpha), (c, \alpha)$. It is advisable to take the point of intersection of the lines $\frac{\tau_1 + \tau_2}{\mu - \tau_1} = const$ and $\tau_1 = \gamma \tau_2$ as the point (c, α) , where $\gamma = \tau_{1min}$ (Fig. 1, a, b).

5. Experimental Setup and Results

Let the transfer function of a dynamic system in matrix form (in accordance with expression (8)) be given by:

$$\begin{bmatrix} 1 & 1.116 & 1 & 0.833 \\ 0.7 & 1 & 0.8 & 0.7 \\ 0.6 & 0.8 & 1 & 0.9 \\ 0.5 & 0.7 & 0.9 & 1 \end{bmatrix} \cdot \mathbf{x} = \begin{bmatrix} 2.584 \\ 1.850 \\ 2.250 \\ 2.250 \end{bmatrix} \quad (23)$$

It is also known that the errors in setting the coefficients, regardless of their values, are distributed according to a normal distribution with parameters $m=0$ and $D=10^3$ in the interval $(\pm 10^{-3})$, which corresponds to a coefficient setting accuracy of $\Delta=0.3\%$. For MM (23), taking into account the given parameters m , D , and accuracy Δ , the corresponding norms were calculated: $\|\mathbf{A}\|_2 = 3.7$; $\|\mathbf{A}^{-1}\|_2 = 16.956$; $\|\mathbf{b}\|_2 = 8.934$ and the values: $\mu=0.016$; $C=0.311$.

It is necessary to find a lower bound for the distribution function $\|\Delta \mathbf{x}\|_2 / \|\mathbf{x}\|_2$.

For x_1 :

$$F_{x_1}(\tau_1) = \frac{A_1 \cdot \|\mathbf{A}\|_2}{\sqrt{2\pi} \cdot 1.2D} \int_0^{\tau_1} \exp \left[-\frac{\left(\tau - \frac{3.2D}{\|\mathbf{A}\|_2} \right)^2}{21.44D^2 \frac{1}{\|\mathbf{A}\|_2^2}} \right] d\tau; 0 \leq \tau_1 \leq 3.24 \cdot 10^{-3} \quad (24)$$

Similarly, we obtain for x_2 :

$$F_{x_2}(\tau_2) = \frac{A_1}{\sqrt{2\pi} \cdot 1.2D} \int_0^{\tau_1} \exp \left[-\frac{(\tau - 3.2D \cdot C)^2}{21.44D^2 C^2} \right] d\tau; 1.0 \cdot 10^{-3} \leq \tau_2 \leq 4.73 \cdot 10^{-3} \quad (25)$$

All numerical experiments were implemented in Python 3.13.15 using NumPy 2.3.x and SciPy 1.16.1. The experiments were executed on an Intel Core i7-13700K CPU with 16 GB of RAM running Ubuntu 24.04 LTS (64-bit). A fixed random seed of 42 was used for reproducibility. For each experimental scenario, 50 independent stochastic replications were performed. The reported estimates are based on the resulting empirical distributions, with 95% confidence intervals calculated for the principal performance indicators.

Figure 2 shows the distribution function (for the normal distribution) $F_{x_H}(\tau)$ in accordance with the expression

$$F_{x_H}(\tau) = F_{x_1}(C)[F_{x_2}(\alpha) - F_{x_2}(\alpha')] + F_{x_1}(C')[F_{x_2}(\alpha'') - F_{x_2}(\alpha)] + F_{x_1}(C'')F_{x_2}(\alpha''), \quad (26)$$

which takes into account the probability of landing within two additional rectangles with vertices (C', α'') , $(0, \alpha'')$, $(0, \alpha)$, (C', α) , (C', α') , (C, α'') , $(C', 0)$, where

$$C' = \frac{1}{3}C; \alpha' = \frac{2}{3}\alpha; C'' = \alpha \left[\gamma + \frac{1}{3(1+\tau)} \right]; \alpha'' = C \left[\frac{\gamma(1+\tau)}{3\gamma} \right]. \quad (26)$$

The point $C_r=0.625$ in Figure 2 corresponds to the guaranteed estimate.

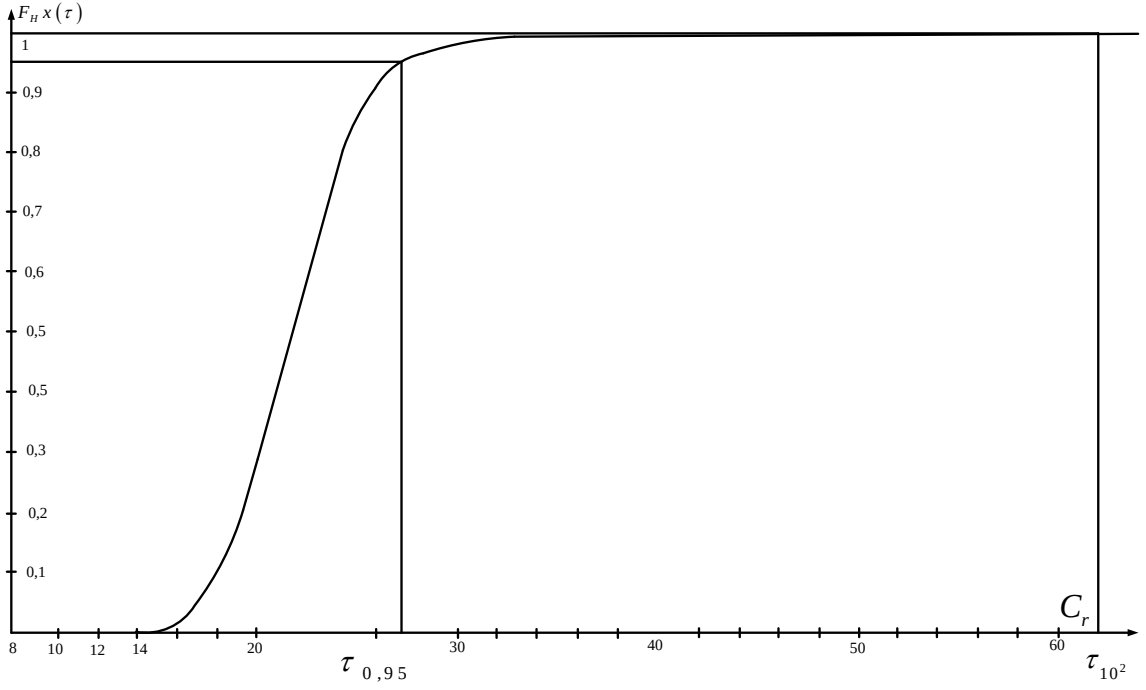


Figure 2: Distribution function (for the normal distribution).

6. Discussion

The numerical study demonstrates that the proposed error-based ranking framework can be used to construct lower-bound distributions of the irreducible implementation error from probabilistic models of input perturbations.

Errors are ranked in the form of p quartiles (i.e., equally sized groups of values, ordered from minimum to maximum, and obtained with probability p) of the corresponding distribution functions $\|\epsilon_0\|_i$ (i – the quartile number), which makes it possible to construct confidence interval estimates for the errors. When ranking errors, the introduced correction coefficients are taken into account: first-order (which yield upper bounds and allow for the construction of upper estimates of the distribution function $\|\Delta \mathbf{x}\|_i \geq \|\epsilon_0\|_i / \|\mathbf{A}\|_i$, where $\|\mathbf{A}\|_i$ is the matrix of primary errors in the MM problem formulation of a dynamic system) and second-order (which determines the possibility of an error falling within the region Ω of possible values of the elements ϵ – the vector of inconsistencies caused by the numerical solution of the equations forming the MM of the dynamic system).

Compared with classical sensitivity analysis and direct high-precision recomputation, the proposed approach changes the computational workflow rather than replacing the underlying error theory. Its principal advantage is the reuse of previously computed information. At the same time, this advantage introduces an important limitation: the reliability of the prediction depends on the representativeness and quality of the reference classes. If a new problem is substantially different from the reference set, the resulting estimate may not be sufficiently accurate.

A second limitation concerns the probabilistic assumptions used in the construction of the error distributions. The current formulation assumes specified statistical characteristics of the input errors and, in several steps, relies on independence or weak-dependence assumptions. These assumptions should be verified or stress-tested when the method is applied to real computational systems.

The current numerical study is also limited by the number of test cases. Therefore, the present results demonstrate feasibility rather than providing a statistically complete validation of the ranking

method. A broader evaluation should include multiple system dimensions, conditioning levels, error distributions, reference-class configurations, and repeated stochastic experiments. Such experiments would allow the computational savings, coverage probability, confidence intervals, and generalization capability of the method to be quantified.

Overall, the proposed approach is best interpreted as an accuracy-assessment layer that complements existing numerical error analysis. Its potential value lies in converting repeated error estimation into a reusable classification procedure, while retaining the analytical structure of guaranteed error bounds.

7. Conclusion

Accuracy has been defined as one of the key quality indicators of computational resources (both software-algorithmic and hardware) when they are used to solve applied problems. The possibility of analytically formalizing the procedure for ranking (i.e., classifying by accuracy) errors arising during the numerical implementation of dynamic systems modeled using differential and integral equations in the simulation and control of these systems has been demonstrated. An algorithm for ranking primary errors by accuracy is proposed, which makes it possible to automate this process.

A procedure is proposed for a method of ranking errors in the implementation of dynamic systems in modeling and control problems. Unlike the application of the well-known sensitivity theory or the performance of more precise calculations when solving the problem (which requires more cumbersome calculations comparable in computational complexity to the solution of the problem at hand, or even exceeding it), the use of the proposed analytical estimates in the form of inequalities should be considered economical (in terms of computational effort) and sufficient (in terms of absolute values) for further analysis and decision-making regarding the accuracy of the obtained solution.

“Lower-bound” estimates are proposed for the distribution functions of the irreducible implementation error of digital systems of dynamic objects, represented by differential and integral equations, in modeling and control problems using the error-ranking method. The construction of these estimates is based on the application of guaranteed estimates for the absolute and relative errors arising in the numerical implementation of dynamic systems; in particular, we propose the construction of a priori and a posteriori estimates of the distribution functions of the unavoidable error. The correctness of the proposed accuracy-based ranking method is confirmed by a numerical study on constructing “lower-bound” estimates of the distribution functions of the residual error when solving problems of modeling and controlling dynamic systems.

Declaration on Generative AI

While preparing this paper, the authors used Gemini 3.1 Pro to check grammar and spelling and to format the reference list according to the journal’s requirements. After using these services, the authors reviewed and edited the content as necessary and assume full responsibility for the content of the publication.

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