A Formal Theory of Orders in Complex Organizations: A Defeasible Deontic Approach

Matteo Cristani[1,*]

[1]Department of Computer Science, University of Verona, Strada Le Grazie 15, 37134 Verona, Italy

Abstract

Orders are a fundamental mechanism through which duties are created, coordinated, and enforced in complex organizations. Despite their centrality in practical settings, they remain insufficiently analysed in formal frameworks for normative reasoning, particularly in Defeasible Deontic Logic (DDL). This paper addresses this gap by developing a formal theory of orders grounded in a predicate-based extension of DDL.

We adopt a hybrid perspective in which orders are understood as authoritative communicative acts occurring within an institutional context and characterised by their normative effects. In particular, orders are analysed as structured events linking an issuer, a set of receivers, and a set of normatively targeted agents, whose primary function is to generate or transmit obligations.

To support this analysis, we extend standard DDL with a quantifier-free predicate language capable of representing agents, roles, organizations, and their relations, while preserving computational tractability. A key contribution of the framework is the reconstruction of authority in terms of deontic meta-rules, which regulate the admissibility of norm-generating acts and allow a principled distinction between the issuance of an order and the validity of the resulting duty.

Within this setting, orders are formalised as rule-generating acts whose effects are defeasible and subject to priority relations. The framework is illustrated through a range of examples capturing direct orders, role-based assignments, broadcast communications, and delegation. The result is a unified account that integrates communicative structure, institutional competence, and normative reasoning, providing a foundation for the formal analysis of responsibility and accountability in multi-agent systems and legal domains.

Keywords

Defeasible Deontic Logic, Normative Multi-Agent Systems, Orders and Authority, Deontic Meta-Rules, Institutional Communication, Norm Generation, Legal Knowledge Representation, Agent Communication Languages, Normative Reasoning, Organizational Modeling

CEUR-WS.org/Vol-4307/paper19.pdf

1. Introduction

Orders play a central role in the functioning of complex organizations. They are the primary mechanism through which duties are assigned, coordinated, and enforced across structured hierarchies of agents, roles, and units. Despite their practical importance, the notion of an order remains surprisingly under-analysed from a formal perspective, especially in frameworks aimed at reasoning about norms, such as Defeasible Deontic Logic (DDL).

At an intuitive level, an order can be understood as a communicative act through which a duty is brought to bear on one or more agents. However, even this seemingly simple characterization conceals a number of conceptual distinctions that become crucial when attempting a formal treatment. In particular, an order is not merely a statement of obligation, nor merely a communication, but rather a structured institutional act connecting an issuer, a set of receivers, and a set of normatively targeted agents within an organizational context.

A natural starting point is to treat orders as primitive entities of the formal language. On this view, one introduces a predicate such as $\mathsf{Order}(i, a, \phi)$, representing that agent i orders agent a to bring about ϕ, and stipulates axioms linking orders to obligations, for instance by postulating that $\mathsf{Order}(i, a, \phi)$ entails $O_a\phi$. This approach has the advantage of simplicity and aligns with many computational treatments, where orders are taken as given inputs. However, it leaves unexplained what distinguishes

WOA 2026: 27th Workshop From Objects to Agents, June 15–17, 2026, Salerno, Italy

*Corresponding author.

matteo.cristani@univr.it (M. Cristani)

© 2026 Copyright for this paper by its authors. Use permitted under Creative Commons License Attribution 4.0 International (CC BY 4.0).

CEUR Workshop Proceedings ceur-ws.org ISSN 1613-0073

published 2026-10-30

an order from other communicative acts, such as requests, reminders, or reports. For example, the utterances "Evacuate the building immediately" and "You are required to evacuate the building" may be treated equivalently, even though only the former may constitute an order in a given organizational setting. As a result, the primitive approach risks conflating normatively distinct phenomena.

A more refined alternative consists in analysing orders as derived institutional facts. On this view, an order is not primitive, but arises when a communicative event satisfies certain conditions, such as being issued by an agent with appropriate authority, being directed to a relevant audience, and occurring within a recognized organizational structure. Formally, one may represent a communication event e together with predicates specifying its issuer, receivers, and content, and define $\mathsf{Order}(e)$ in terms of these components and the underlying authority relations. This approach allows one to distinguish, for instance, between a valid order and an unauthorized instruction, and to capture the role of organizational structure in conferring normative force. However, it also introduces a significant ontological overhead, requiring a rich representation of institutional relations that may obscure the core deontic mechanisms of interest.

A third possibility is to characterize orders primarily in terms of their normative effects, namely as acts that create or otherwise bring about duties. This perspective fits naturally within DDL, where the central concern is the derivation and interaction of obligations. For instance, one may identify an order with an event whose occurrence results in the obligation of an agent to achieve a certain state of affairs. Yet, such an approach tends to collapse important distinctions. The act of issuing a new directive, the activation of a standing duty in a specific context, and the mere transmission of an already established obligation all result in duties, but differ significantly in their organizational and normative roles. A purely effect-based characterization therefore lacks the granularity required for a comprehensive account of orders.

In this work, we adopt a hybrid approach that integrates the strengths of the latter two perspectives. We treat orders as communicative institutional acts occurring within an organizational setting, while taking their defining feature to be their connection to the creation, activation, specification, or authoritative communication of duties. In other words, an order is understood as a structured event in which an agent or body, acting with appropriate standing, communicates content that normatively targets one or more agents, thereby affecting their deontic position.

This perspective allows us to capture a variety of cases within a unified framework. A direct instruction such as "Submit the report by noon" can be seen as creating a new duty for a specific agent. A statement such as "All team leaders must attend the meeting" assigns a duty to agents satisfying a role condition. A broadcast communication like "Unit A advance, Unit B hold position" may be directed to a wide audience while normatively targeting only specific subgroups. Finally, a statement such as "Headquarters orders Unit C to withdraw" may function as the transmission of a duty already established elsewhere. These examples illustrate that orders are not reducible to a single pattern, but involve a combination of communicative structure and normative effect.

The framework developed in this paper builds on this hybrid understanding. We adopt a predicate-based representation that allows us to model agents, roles, communications, and duties in a structured way, while remaining within a quantifier-free setting in line with standard practice in defeasible reasoning. This design choice preserves the computational advantages of DDL, while significantly enhancing its capacity to represent the relational structure of normative systems in complex organizations.

The ultimate goal of this work is to provide a formal account of orders that supports the analysis of responsibility and accountability. By grounding orders in a clear ontology and linking them systematically to the emergence and transformation of duties, we lay the foundation for a derivation system in which such notions can be defined and studied rigorously. The subsequent sections will introduce the formal apparatus underlying this framework and explore its properties and applications.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on DDL, legal knowledge representation, agent communication, and proclamation. Section 3 motivates the predicate-based representation by analysing different varieties of orders and their representational requirements. Section 4 introduces the formal apparatus, including the quantifier-free predicate extension of DDL and the role of deontic meta-rules. Section 5 develops the proposed theory of orders

and illustrates it through representative examples. Finally, Section 6 concludes the paper and outlines future developments.

2. Background and Related Work

This section reviews the literature relevant to the representation of legal knowledge in order to discuss the current gaps in the literature with respect to the representation of orders.

The foundational DDL literature provides a well-developed framework for reasoning about obligations, permissions, violations, and normative conflict. Early work by Governatori and Rotolo develops modal extensions of defeasible logic for agency, intention, and obligation [1]. More recent expository work presents DDL as a computational formalism for normative reasoning and clarifies its role in modelling obligations, permissions, and temporal aspects of norm application [2]. This line is reinforced by studies on contrary-to-duty structures and violations [3], on strong and weak permissions [4], and on temporalised normative positions [5]. It is further extended by work on legal change, including abrogation and annulment [6], broader accounts of legal dynamics [7], and revision-oriented treatments of legal contraction and preference change [8, 9, 10].

A further development in this direction is provided by work on deontic meta-rules. Olivieri et al. [11] extend defeasible deontic logic with meta-rules capable of representing defeasible meta-theories with deontic modalities and different kinds of conflict among rules. This is especially relevant because it shows that the formalism can represent not only first-order obligations and permissions, but also higher-order normative constraints governing the use, priority, and interaction of norms. Taken together, these works establish DDL as a robust reasoning core for normative effects, conflict resolution, legal dynamics, and meta-level regulation.

At the same time, the literature remains focused primarily on normative statuses and their transformations rather than on the institutional and communicative acts by which such statuses are brought about. Even where legal dynamics and meta-rules are addressed, the object of analysis remains the norm, its force, or its interaction with other norms. Orders, by contrast, require attention to the act of issuance, the authority of the issuer, the structure of addressees, and the conditions under which a communication counts as normatively binding. Those issues are not yet central in the DDL literature.

Parallel to this, research on legal knowledge representation addresses the problem from a representational and interoperability perspective. Gordon et al. [12] formulate requirements for rule interchange languages in the legal domain, thereby distinguishing representational adequacy from proof-theoretic elegance. LegalRuleML subsequently provides an XML-based framework for representing legal rules, norms, and associated metadata [13]. Work by Lam et al. [14] then shows how norms represented in LegalRuleML can be translated into modal defeasible logic for automated reasoning. These contributions are important because they clarify how legal norms can be encoded, exchanged, and connected to executable reasoning systems. Yet they still focus on the representation of norms and related metadata rather than on orders as a distinct institutional category.

A broader jurisprudential background is supplied by work such as Rodríguez and van der Torre's study of deontic logic and legal systems [15], which situates defeasibility, conflict, conditional norms, and legal dynamics within a larger theoretical landscape. Methodological work on legal text encoding also contributes to this picture.

A complementary line of work emerges in agent communication languages and normative multi-agent systems. Early ACLs such as KQML and FIPA ACL provide performatives and message structures, but their semantics is often mentalistic. Singh's criticism of mentalistic ACL semantics and his move toward a social semantics based on commitments are therefore important [16, 17]. Commitment-based frameworks developed in subsequent work by Yolum et al. [18], and Alberti et al. [19] model communication in terms of publicly accessible social effects rather than private psychological states. This is a significant step toward legal and institutional analysis, since legal effects are not usually grounded in unverifiable mental attitudes. Research on normative multi-agent systems similarly incorporates norms into MAS architectures and offers a bridge between communication and deontic reasoning.

Nevertheless, the communication-oriented literature has a limitation complementary to that of DDL. It provides detailed treatments of performatives, protocols, and social commitments, but often lacks a sufficiently strong notion of institutional competence and legal power. Commitment updates do not, by themselves, explain why a communication is valid, when it is ultra vires, or how authority is tied to role and office. As a consequence, the communication side of the problem is better handled than in DDL, but the specifically legal-institutional dimension remains insufficiently articulated.

The closest bridge between these bodies of work is the literature on declarative power and proclamation. Gelati et al. [20] analyse declarative power as the capacity of an authorised agent to create normative positions by proclaiming them . This is the point in the literature where communication, institutional competence, and normative effect are most clearly brought together. Related work on institutional agency in defeasible logic continues this direction by modelling counts-as relations and institutionalised power within a defeasible framework [21]. These works are crucial because they show that certain communications do not merely convey information or update commitments; under the right institutional conditions, they change the normative situation itself.

This proclamation line is the closest antecedent to a theory of orders, but it still stops short of one. It identifies the mechanism by which an institutional communication may create duties, permissions, or powers, yet it does not develop order as a fully articulated category in its own right. In particular, the literature does not sufficiently analyse the asymmetry between an audience that receives a communication and the subset of agents who become duty-bearers as a consequence of it. Nor does it fully derive the related chains of responsibility and accountability that follow from issuance, transmission, invalid exercise of power, or defective uptake.

The comparison with Public Announcement Logic is helpful here. The idea has originally been discusse dby Plaza [22] and further extended by investigation on Dynamic Epistemic Logic by Van Ditmarsch et al. [23]. PAL studies how public announcements update epistemic models by eliminating worlds incompatible with the announced proposition. Proclamation is structurally similar in being a public communicative event with system-level consequences, but the relevant update is normative rather than primarily epistemic. Its success depends on competence, validity conditions, and institutional recognition, rather than only on truthfulness and publicity. For this reason, PAL helps illuminate the dynamic aspect of proclamation, but it does not provide an adequate account of authority, legal validity, delegated power, or duty assignment.

Across these strands of literature, a stable pattern emerges. DDL offers a sophisticated account of normative reasoning, conflict resolution, legal dynamics, and even meta-level regulation. Legal knowledge representation research offers increasingly refined methods for encoding and exchanging legal norms. ACL and MAS research offer models of communication and social effect. The proclamation literature demonstrates that communication can itself be norm-creating when institutionally qualified. Yet these contributions remain only partially integrated. The literature still lacks a unified framework in which an order is treated as a distinct institutional communicative act characterised by an authorised issuer, a structured audience with designated addressees, conditions of validity and uptake, defeasible effects, and explicit consequences for responsibility and accountability. That, more precisely, is the remaining gap in the representation of orders.

3. The representation framework

A central design choice of our framework concerns the level of expressiveness required to adequately represent normative systems. As widely acknowledged in the literature, norms in legal and multi-agent settings are inherently relational: they refer to agents, roles, actions, objects, and the relationships among them. Early work on defeasible deontic reasoning already emphasized this point, proposing first-order representations to capture the structure of bureaucratic and legal regulations [24].

At the same time, much of the Defeasible Logic (DL) and Defeasible Deontic Logic (DDL) tradition has been developed at a propositional level, or has treated variables in rules as mere schemata for their ground instances [25, 26]. While this approach has clear computational advantages, it does not provide

a fully satisfactory account of the relational structure required in realistic normative domains.

In this work, we adopt an intermediate position. We extend DDL with a predicate-based language that allows for the explicit representation of relations among entities, while deliberately avoiding the introduction of quantifiers in the object language. Rules are thus interpreted as schemata over ground instances, in line with standard DL practice, but the underlying vocabulary is enriched to support predicates of arbitrary arity.

This choice is motivated by both conceptual and practical considerations. On the conceptual side, it enables the representation of structured norms such as obligations involving multiple agents and contextual conditions, without committing to a full first-order proof theory. On the practical side, it aligns with existing implementations of defeasible reasoning systems, which typically rely on grounding procedures and operate over function-free fragments [27, 2].

Moreover, recent applications of DDL to legal informatics and regulatory compliance further reinforce the need for predicate-level expressiveness. Approaches to legal coding and rule extraction from natural language routinely rely on structured representations involving entities and relations, even when the underlying reasoning engines ultimately operate on grounded rules [28].

In light of these considerations, our framework can be understood as a predicate-based, quantifier-free extension of DDL. It preserves the computational properties and operational intuitions of standard DL, while significantly enhancing its representational adequacy for normative reasoning in complex domains.

3.1. Varieties of Orders and Representational Requirements

In order to motivate the formal apparatus, we begin by surveying a range of examples of orders in natural language. The purpose of this analysis is to expose the structural diversity of orders and, correspondingly, the representational requirements that arise from each case. We proceed by distinguishing different types of orders, each illustrated by representative examples.

(1) Simple Direct Orders.

- "Submit the report by noon."
- "Close the door."
- "Turn off the machine immediately."

These are the simplest cases, involving a single action directed at an (often implicit) addressee, sometimes with temporal qualifications[1]. The fact that the addressee is implicit does not affect the nature of the order itself, as this is a matter of anaphoric reference in the sentence used to convey it. Accordingly, the following are equivalent instances:

- "John, submit the report by noon."
- "Team leader, coordinate the meeting."
- "Operator A, shut down reactor 3."

Here, the addressee is explicitly identified, and may correspond to an individual, a role-holder, or a member of an organizational unit.

Representational need. Orders cannot be represented as atomic propositions. The framework must support a structured representation involving at least an *agent*, an *action*, and possibly *temporal constraints*. It must also distinguish between *agents*, *roles*, and *organizational positions*, allowing duties to be assigned to differently characterized entities.

[1] As we further decline in Section 6, we did not deal with temporal and referential aspects of the orders in this paper, leaving it for further deepening.

(2) Orders with Multiple Addressees.

- "John and Mary, review the document."
- "All engineers must attend the briefing."
- "Unit A and Unit B, advance to checkpoint Delta."

These cases raise the question of whether duties are to be interpreted distributively or collectively, and whether the relevant group is explicitly enumerated or defined by a predicate.

Representational need. The framework must support *group-level representation*, including the distinction between *individual, collective, and role-based assignment* of duties, and allow duties to be associated with sets of agents.

(3) Broadcast Orders with Subset Targeting.

- "To all personnel: security team secure the perimeter, medical team prepare triage."
- "Unit A advance, Unit B hold, Unit C retreat."

In these cases, a single communicative act is directed to a broad audience, while different parts of its content normatively target distinct subgroups. This structure has been previously captured, at least in part, by the **Procl** operator introduced in [5], which models the public communication of normative content.

Representational need. The framework must distinguish between *receivers of the communication* and *normative addressees*, and allow a *single order to generate multiple, independently targeted duty assignments*.

(4) Temporally Qualified Orders.

- "Complete the task within two hours."
- "Do not enter the area until clearance is given."
- "Remain at your post until relieved."

These orders involve deadlines, durations, or termination conditions.

Representational need. The representation must incorporate *temporal structure* within the content of duties, including deadlines and persistence conditions.

(5) Orders Activating or Communicating Pre-existing Duties.

- "Evacuate the building now!" (in an emergency)
- "Apply protocol Delta."

In these cases, the order does not create a new duty, but activates or specifies a previously established one.

- "Headquarters orders Unit C to withdraw."
- "You are required by regulation to file this report."

In these cases, the speaker communicates a duty whose source may lie elsewhere.

Representational need. The framework must distinguish between *duty creation*, *duty activation or specification*, and *duty communication*, and must allow the representation of a separation between the *issuer of the communication* and the *source of normative authority*.

(6) Delegation Orders.

- "Captain, instruct your unit to retreat."
- "Manager, assign someone to handle this case."

These are higher-order orders, whose content concerns the generation of further duties or orders.

Representational need. The framework must support *nested normative structures*, where the content of an order may itself involve the creation, transmission, or assignment of further obligations.

3.2. Discussion of Representational Needs

We are now in a position to consolidate the previous analysis into a more precise account of orders, and to clarify the scope of the formal framework developed in this paper.

We begin with an informal characterization that captures the general intuition guiding our approach:

An order is an authoritative communicative act, performed within an organizational context, whereby an agent or institutional body conveys content that normatively targets one or more agents, resulting in the creation, activation, specification, or authoritative communication of a duty assigned to them.

This definition reflects the hybrid perspective discussed above, combining the communicative nature of orders with their normative effects. In particular, it explicitly distinguishes four possible ways in which an order may affect the normative position of its addressees: (i) by creating a new duty, (ii) by activating an existing norm, (iii) by specifying a generic duty into a concrete one, and (iv) by communicating a duty whose source lies elsewhere.

While this general formulation is adequate at the conceptual level, a closer inspection reveals that these four cases differ significantly in their representational and logical requirements.

The case of *creation* is the most direct: an order gives rise to a new obligation for its addressees. This aligns naturally with the core mechanisms of Defeasible Deontic Logic, where rules are used to derive obligations from given conditions.

The other three cases, however, introduce additional layers of complexity.

Activation presupposes the existence of a background normative structure, where duties are defined in a conditional or latent form and become applicable only when certain triggering conditions are satisfied. Representing this requires a clear distinction between the existence of a norm and its applicability in a given context.

Specification involves the transition from a general or abstract duty to a more concrete one, often through contextual refinement. This requires mechanisms for linking different levels of abstraction within the normative system, and raises issues concerning inheritance, exception handling, and contextual interpretation.

Communication, finally, concerns the transmission of duties whose normative source lies elsewhere. In this case, the agent issuing the order does not generate the duty, but conveys it with authoritative force. This introduces a separation between the *source of the norm* and the *issuer of the communicative act*, and requires the representation of chains of transmission and institutional channels.

Given these differences, a fully general treatment of orders would require a significantly richer formal apparatus, combining deontic reasoning with mechanisms for context management, abstraction, and institutional communication. While such an extension is of clear interest, it would also considerably increase the complexity of the framework.

For this reason, in the present work we focus primarily on the case of *duty creation*, treating it as the core mechanism underlying orders, and leaving the other cases as directions for future development. This choice is justified not only by considerations of tractability, but also by the observation that the creation of duties provides the foundational layer upon which the other mechanisms can be built.

On this basis, we adopt the following more precise characterization, which will guide the formalization:

An order is an authoritative communicative act by which an agent, within a given scope of addressees, either generates a duty for one or more agents or transmits a duty whose normative source lies elsewhere. Authority consists, on the generative side, in the permission to impose such duties, and, on the transmissive side, in the power to communicate them with normative effect. The validity of the resulting duty is not built into the bare notion of order, but is determined by deontic meta-rules governing the admissibility and priority of norm-generating acts.

This formulation makes explicit several key aspects. First, authority is understood as a structured notion: it consists in a permission to generate duties, combined with a power to transmit them, both constrained by a scope of addressees. Second, it separates the act of issuing an order from the validity of the resulting duty, which is instead governed by higher-level meta-rules.

The decision to focus on duty creation in the core formalism should be read in light of the representational requirements identified in the previous section. As shown there, even the simplest cases of orders already require a structured representation of agents, actions, roles, and temporal constraints. More complex cases, such as broadcast orders with subset targeting, delegation, and the distinction between issuer and addressee, already push the expressive limits of a tractable formal system.

In this setting, introducing at once the full range of phenomena associated with activation, specification, and communication would risk obscuring the central contribution of the framework. By isolating the generative aspect of orders, we obtain a clean and well-founded core upon which the additional mechanisms can later be layered in a principled way.

In the following, we proceed to define the formal language and inference mechanisms corresponding to this core notion of order.

4. Formal Framework

4.1. Basics of Defeasible Deontic Logic

Defeasible Deontic Logic (DDL) is a rule-based non-monotonic formalism designed to represent and reason about normative systems, particularly in contexts where obligations may conflict and exceptions are pervasive. It extends Defeasible Logic (DL) with modal operators capturing deontic notions such as obligation and prohibition, while preserving the computational efficiency characteristic of DL.

At its core, DDL is based on a propositional language built from a set of atomic propositions and standard logical connectives. In addition, it introduces a deontic operator O to represent obligations. A literal is either an atomic proposition or its negation, and deontic literals are expressions of the form $O\phi$, where ϕ is a (classical) literal.

Note that the derivation mechanism of DDL is based on *proof tags* and *proof conditions* and is fundamentally grounded into the computation of extensions. Therefore, the introduction of a specific semantics of orders shall require a disclosure of the composition and re-composition of constraints derived by the specific usage of the communicative acts, temporalized assertions and reference to existing regulations, which are all left to further development.

A DDL theory is typically defined as a tuple:

$$\mathcal{D} = (F, R, >),$$

where:

- F is a set of facts,
- R is a set of rules,
- $>$ is a superiority relation over rules, used to resolve conflicts.

Rules are the central component of the system and are divided into different types, reflecting varying strengths of inference. In particular, one distinguishes:

- *Strict rules*, of the form
$$A \to \phi,$$
representing classical, non-defeasible implications.
- *Defeasible rules*, of the form
$$A \Rightarrow \phi,$$
representing conclusions that hold in the absence of stronger, conflicting information.
- *Defeaters*, of the form
$$A \rightsquigarrow \phi,$$
which do not support conclusions directly, but may prevent opposing conclusions from being drawn.

In the deontic extension, rules may also derive obligations. For example, a defeasible rule

$$A \Rightarrow O\phi$$

expresses that, under conditions A, ϕ is normally obligatory. Conflicts between obligations, such as $O\phi$ and $O\neg\phi$, are resolved through the superiority relation over the rules that support them.

The semantics of DDL is proof-theoretic and is based on the notion of *defeasible provability*. Conclusions are derived through a constructive process that takes into account the available rules, the facts, and the resolution of conflicts via the superiority relation. The system distinguishes between different proof statuses, such as provability and refutability, allowing for a fine-grained treatment of normative reasoning under uncertainty and exception.

A key feature of DDL is its ability to handle contrary-to-duty (CTD) obligations, where secondary obligations arise in response to violations of primary ones. This is typically achieved through chains of defeasible rules, where the violation of a norm triggers additional normative consequences.

Overall, DDL provides a flexible and computationally tractable framework for modeling normative systems with exceptions, priorities, and dynamic interactions among obligations. In the following subsection, we extend this propositional setting to a predicate-based language capable of representing agents, organizations, and temporal aspects, as required by the analysis of orders developed in this paper.

4.2. Predicate-Based Extension to Defeasible Deontic Logic

The analysis developed in the previous sections highlights the need for a richer representational language than the propositional one underlying standard Defeasible Deontic Logic (DDL). In particular, the representation of structured entities and their relations requires the introduction of predicates, while preserving the overall architecture of DDL.

To this end, we extend the propositional language of DDL by introducing a quantifier-free predicate language. Let $\mathcal{C}$ be a finite set of constants, and $\mathcal{P}$ a finite set of predicate symbols, each with an associated arity. The basic expressions of the language are *ground atoms* of the form:

$$P(c_1, \ldots, c_n),$$

where $P \in \mathcal{P}$ and $c_1, \ldots, c_n \in \mathcal{C}$.

A *literal* is either a ground atom A or its negation $\neg A$, and the set of literals is denoted by $\mathcal{L}$. As in standard DDL, the language is extended with the deontic operator O, yielding *deontic literals* of the form $O\phi$, where $\phi \in \mathcal{L}$. No variables or quantifiers are admitted; all expressions are fully grounded.

Based on this language, a theory retains the same structure as in the propositional case, that is, a tuple $\mathcal{D} = (F, R, >)$, where $F \subseteq \mathcal{L}$ is a set of facts, R is a finite set of rules, and $>$ is a superiority relation over R. Rules are defined as in standard DDL, with strict rules of the form $A_1, \ldots, A_n \to \phi$, defeasible rules of the form $A_1, \ldots, A_n \Rightarrow \phi$, and defeaters of the form $A_1, \ldots, A_n \rightsquigarrow \phi$, where each $A_i \in \mathcal{L}$ and $\phi \in \mathcal{L} \cup \{O\psi \mid \psi \in \mathcal{L}\}$.

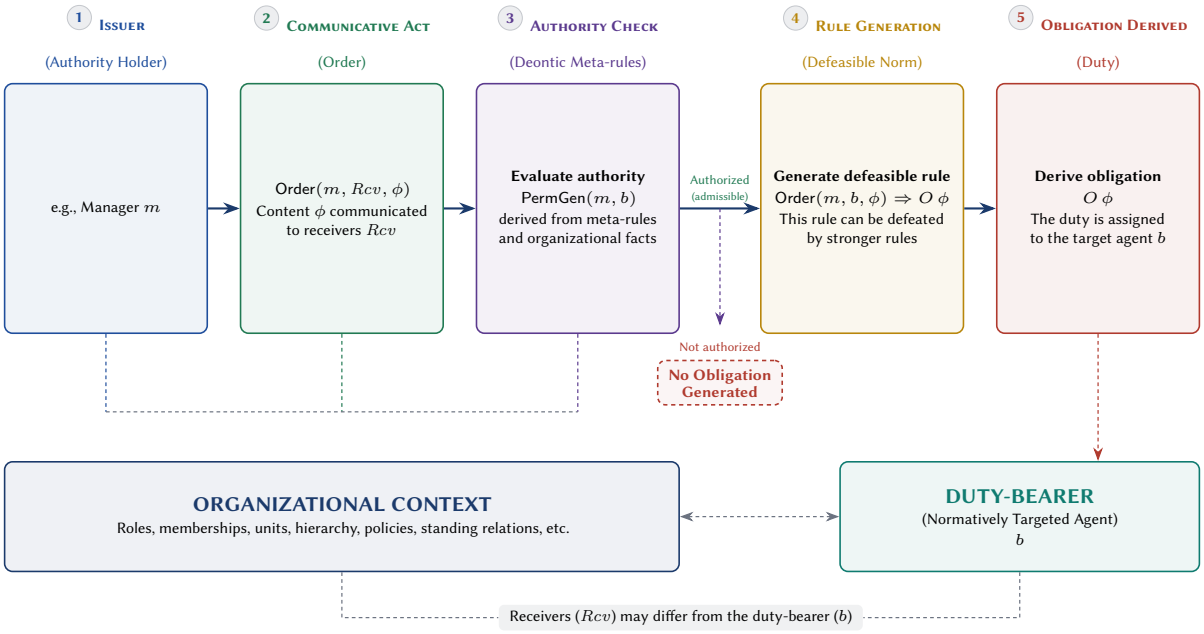

Figure 1: The mechanism of an order as an authoritative communicative act within a predicate-based Defeasible Deontic Logic framework. **Issuer**: agent or body with standing authority. **Order**: communicative act carrying content ϕ to its receivers. **Authority check**: meta-rules decide whether the issuer may generate norms for the target. **Rule generation**: if authorized, a defeasible norm-generating rule is introduced. **Obligation**: the rule, if undefeated, yields an obligation for the duty-bearer.

The extension is therefore purely syntactic: propositional atoms are replaced by structured, ground literals, while the definition of rules and the organization of theories remain unchanged. This allows one to represent relational information within the same inferential framework, without introducing variables, quantification, or additional mechanisms of instantiation.

In this sense, the extension is conservative with respect to the original framework, which is recovered as the special case in which all predicates are nullary.

A crucial component of the ontology concerns the notion of *generation power*, that is, the conditions under which an agent is permitted to generate duties for others. In the present framework, this is not captured directly at the object level, but rather through a layer of *deontic meta-rules*, following the approach developed in [11]. These meta-rules do not prescribe actions in the domain, but regulate the admissibility of norm-generating acts themselves, by specifying which agents are permitted to introduce rules of a given form and with given targets. In this sense, authority is reconstructed as a higher-order normative notion: an agent has the power to generate duties for a given set of addressees if and only if there are applicable meta-rules licensing the corresponding rule introduction. This allows one to separate the act of issuing an order from its normative validity, which is determined by the meta-level constraints governing the production of norms. As a consequence, the generation of duties is not taken as a primitive effect of orders, but as the outcome of a regulated process, where only those orders that are supported by appropriate meta-rules give rise to valid obligations.

The corresponding proof conditions for the extended system are defined in direct analogy with those of standard Defeasible Deontic Logic, and are omitted here for the sake of space.

Figure 1 illustrates the core mechanism underlying orders in our framework. An order originates as a communicative act performed by an issuer within an organizational context, and is directed to a set of receivers. Its normative effect, however, depends on an authority check governed by deontic meta-rules, which determine whether the issuer is permitted to generate duties for the targeted agents.

If the authority conditions are satisfied, the order gives rise to a defeasible norm-generating rule, whose application results in an obligation assigned to the duty-bearer. Otherwise, no normative effect is produced. The figure also highlights the structural distinction between receivers of the communication

and the agents who are normatively targeted, as well as the role of the organizational context in grounding authority relations.

5. A Theory of Orders in DDL

In this section, we introduce the formal apparatus underlying our theory of orders. Building on the predicate-based extension of Defeasible Deontic Logic (DDL), we define the ontology, the corresponding signature, and the mechanisms by which orders give rise to duties. Particular attention is devoted to the notion of authority, which is reconstructed in terms of deontic meta-rules regulating the generation of norms.

5.1. Ontology and Signature

The representation of orders in complex organizations requires a structured ontology capturing agents, institutional entities, and the relations among them. To this end, we introduce the following categories of predicates.

Agents and Institutional Structure. We assume a finite set of constants denoting agents and institutional entities. The following predicates are used:

- $\mathsf{Agent}(a)$: a is an agent,
- $\mathsf{Org}(o)$: o is an organization,
- $\mathsf{Role}(r)$: r is a role,
- $\mathsf{Member}(a, o)$: agent a belongs to organization o,
- $\mathsf{InRole}(a, r)$: agent a occupies role r.

These predicates provide the basic structure required to represent organizational settings and role-based interactions. In this paper we also avoid to discuss the, actually very relevant, issue of *temporal representations* that we have introcuced above. This is left to further developments.

Orders and Normative Content. Orders are represented as structured predicates linking an issuer, an addressee, and a content:

$$\mathsf{Order}(a, b, \phi)$$

where a is the issuer, b the addressee, and ϕ a ground literal representing the content of the order. Duties are represented using the deontic operator O, as in standard DDL, so that $O\phi$ denotes that ϕ is obligatory.

Norm Generation and Transmission. In line with the discussion above, we distinguish between the generation and the transmission of duties:

- $\mathsf{Gen}(a, b, \phi)$: agent a generates a duty for b with content ϕ,
- $\mathsf{Tr}(a, b, \phi)$: agent a transmits a duty to b.

In the present work, we focus on the generative case, leaving the systematic treatment of transmission for future developments.

5.2. Authority and Meta-Rules

A crucial component of the framework concerns the notion of authority, understood as the permission to generate duties within a given scope. Rather than representing authority directly at the object level, we follow the approach based on deontic meta-rules developed in [29].

Meta-rules regulate the admissibility of norm-generating acts by specifying which agents are permitted to introduce rules of a given form. In this sense, authority is reconstructed as a higher-order normative notion: an agent is authorized to generate duties for a given set of addressees if and only if there exist applicable meta-rules licensing the corresponding rule introduction.

Thus, the generation of duties is not taken as a primitive effect of orders, but as the outcome of a regulated process:

$$\text{Meta-rules} \;\Rightarrow\; \text{Admissible rules} \;\Rightarrow\; \text{Duties}.$$

This separation allows one to distinguish between the issuance of an order and the validity of the resulting duty, which depends on the meta-level constraints governing norm production.

5.3. Orders as Rule-Generating Acts

Within this framework, orders are interpreted as acts that introduce defeasible rules. In particular, an order of the form $\mathsf{Order}(a, b, \phi)$ corresponds to the introduction of a rule:

$$\mathsf{Order}(a, b, \phi) \Rightarrow O\phi$$

or, more explicitly, to a defeasible rule whose antecedent captures the relevant conditions and whose consequent introduces the obligation.

The applicability of such rules is subject to the meta-rules governing authority. Only those orders that are supported by appropriate meta-rules give rise to valid obligations.

5.4. Examples

We illustrate the framework with fully specified examples. Each case explicitly defines the ontology, the meta-rules governing authority, the order, and the resulting obligation. Examples reported here also show that the orders which we can construct at this level of details in the language structure are by no means enough to develop a complete agentive model so far, for we need at least reference and temporalization.

Example 1: Direct Order within an Organization.

Ontology

$$\mathsf{Agent}(m),\ \mathsf{Agent}(j),\ \mathsf{Member}(m, \mathsf{Org1}),\ \mathsf{Member}(j, \mathsf{Org1}),\ \mathsf{InRole}(m, \mathsf{Manager})$$

Meta-rule (Authority)

$$\mathsf{InRole}(a, \mathsf{Manager}) \wedge \mathsf{Member}(b, \mathsf{Org1}) \Rightarrow \mathsf{PermGen}(a, b)$$

Order ⇒ Obligation

$$\mathsf{Order}(m, j, \mathsf{SubmitReport}(j)) \Rightarrow O\,\mathsf{SubmitReport}(j)$$

Conclusion

$$\boxed{O\,\mathsf{SubmitReport}(j)}$$

Example 2: Role-Based Order.

Ontology

$$\mathsf{InRole}(t, \mathsf{TeamLeader}),\ \mathsf{Member}(t, \mathsf{Org1})$$

Meta-rule (Authority)

$$\mathsf{InRole}(a, \mathsf{Manager}) \land \mathsf{InRole}(b, \mathsf{TeamLeader}) \Rightarrow \mathsf{PermGen}(a, b)$$

Order ⇒ Obligation

$$\mathsf{Order}(m, t, \mathsf{CoordinateMeeting}(t)) \Rightarrow O\,\mathsf{CoordinateMeeting}(t)$$

Example 3: Multiple Addressees.

Ontology

$$\mathsf{Member}(j, \mathsf{Org1}),\ \mathsf{Member}(k, \mathsf{Org1})$$

Orders ⇒ Obligations

$$\mathsf{Order}(m, j, \mathsf{Review}(j, doc)) \Rightarrow O\,\mathsf{Review}(j, doc), \quad \mathsf{Order}(m, k, \mathsf{Review}(k, doc)) \Rightarrow O\,\mathsf{Review}(k, doc)$$

Example 4: Broadcast Order with Subset Targeting.

Ontology

$$\mathsf{Member}(a, \mathsf{UnitA}),\ \mathsf{Member}(b, \mathsf{UnitB}),\ \mathsf{InRole}(c, \mathsf{Commander})$$

Meta-rule (Authority)

$$\mathsf{InRole}(c, \mathsf{Commander}) \land \mathsf{Member}(x, u) \Rightarrow \mathsf{PermGen}(c, x)$$

Orders ⇒ Obligations

$$\mathsf{Order}(c, a, \mathsf{Advance}(a)) \Rightarrow O\,\mathsf{Advance}(a), \quad \mathsf{Order}(c, b, \mathsf{Hold}(b)) \Rightarrow O\,\mathsf{Hold}(b)$$

Example 5: Delegation Order.

Ontology

$$\mathsf{InRole}(g, \mathsf{General}),\ \mathsf{InRole}(c, \mathsf{Captain})$$

Meta-rule (Authority)

$$\mathsf{InRole}(g, \mathsf{General}) \land \mathsf{InRole}(c, \mathsf{Captain}) \Rightarrow \mathsf{PermGen}(g, c)$$

Order

$$\mathsf{Order}(g, c, \mathsf{Gen}(c, x, \mathsf{Retreat}(x)))$$

Interpretation

This order imposes on c the obligation to generate duties for members of its unit.

5.5. Discussion

The formalization captures orders as rule-generating acts within a predicate-based DDL framework. By grounding authority in deontic meta-rules, it separates the issuance of an order from the validity of the resulting duty, providing a principled basis for reasoning about norm generation in organizational settings.

More complex cases—such as activation, specification, and transmission of duties—can be incorporated by extending the framework with additional constructs, and are left for future work.

6. Conclusions and Future Work

This paper has proposed a formal theory of orders within a predicate-based extension of Defeasible Deontic Logic. By analysing orders as authoritative communicative acts with normative effects, we have bridged a gap between three traditionally separate strands of research: deontic reasoning, legal knowledge representation, and agent communication.

The central contribution of the work lies in the integration of communicative structure and normative generation. Orders have been characterised as structured events linking issuers, addressees, and content, whose effects are captured through the introduction of defeasible rules. A key aspect of the framework is the reconstruction of authority via deontic meta-rules, which regulate the admissibility of norm-generating acts and provide a principled distinction between the issuance of an order and the validity of the resulting obligation.

The proposed predicate-based, quantifier-free extension of DDL enables the representation of relational structures typical of complex organizations, while maintaining the computational properties of defeasible reasoning. Through formal examples, we have shown how the framework can capture a variety of organizational patterns, including role-based authority, multiple addressees, broadcast orders with subset targeting, and delegation.

At the same time, the present work deliberately focuses on the core case of duty generation. Several important dimensions of orders remain to be explored in future developments. First, the systematic treatment of duty activation, specification, and transmission requires the introduction of mechanisms for context management, abstraction, and institutional channels. Second, temporal aspects of orders, including deadlines, persistence conditions, and dynamic evolution of obligations, call for an explicit integration with temporal extensions of DDL.

A further line of research concerns the formal analysis of invalid or defective orders, including cases of ultra vires acts, conflicting authorities, and failures of uptake. This is closely related to the study of responsibility and accountability, which can be developed on top of the present framework by tracing the chains of norm generation and transmission.

Finally, the framework opens the way to practical applications in legal informatics, compliance systems, and normative multi-agent systems, particularly in safety-critical and organizationally complex domains. Integrating the proposed model with executable reasoning systems and rule interchange formats represents an important step toward bridging formal theory and real-world deployment.

Overall, the theory developed in this paper provides a foundational layer for the formal treatment of orders, upon which richer models of institutional interaction and normative dynamics can be systematically constructed.

Declaration on Generative AI

During the preparation of this work, the authors used ChartGPT 5.3 to paraphrase and reword, perform grammar and spelling checks. After using these tools/services, the authors reviewed and edited the

content as needed and take full responsibility for the publication's content.

References

[1] G. Governatori, A. Rotolo, Defeasible logic: Agency, intention and obligation, in: Deontic Logic in Computer Science, volume 3065 of *Lecture Notes in Computer Science*, Springer, 2004, pp. 114–128. doi:10.1007/978-3-540-25927-5_8.

[2] G. Governatori, Practical normative reasoning with defeasible deontic logic, in: Reasoning Web. Learning, Uncertainty, Streaming, and Scalability, volume 11078 of *Lecture Notes in Computer Science*, Springer, 2018, pp. 1–25. doi:10.1007/978-3-030-00338-8_1.

[3] G. Governatori, A. Rotolo, Logic of violations: A gentzen system for reasoning with contrary-to-duty obligations, Australasian Journal of Logic 4 (2006) 193–215.

[4] G. Governatori, F. Olivieri, A. Rotolo, S. Scannapieco, Computing strong and weak permissions in defeasible logic, Journal of Philosophical Logic 42 (2013) 799–829. doi:10.1007/s10992-013-9295-1.

[5] G. Governatori, A. Rotolo, G. Sartor, Temporalised normative positions in defeasible logic, in: Proceedings of the 10th International Conference on Artificial Intelligence and Law (ICAIL), ACM, 2005, pp. 25–34. doi:10.1145/1160717.1160722.

[6] G. Governatori, A. Rotolo, Changing legal systems: Legal abrogations and annulments in defeasible logic, Logic Journal of the IGPL 18 (2010) 157–194. doi:10.1093/jigpal/jzp075.

[7] G. Governatori, A. Rotolo, Logics for legal dynamics, in: Logic in the Theory and Practice of Lawmaking, Springer, 2015, pp. 323–356. doi:10.1007/978-3-319-19575-9_12.

[8] G. Governatori, A. Rotolo, F. Olivieri, S. Scannapieco, Legal contractions: A logical analysis, in: Proceedings of the 14th International Conference on Artificial Intelligence and Law (ICAIL), ACM, 2013, pp. 110–119. doi:10.1145/2514601.2514609.

[9] G. Governatori, F. Olivieri, S. Scannapieco, M. Cristani, Superiority based revision of defeasible theories, in: Rule Interchange and Applications, volume 6403 of *Lecture Notes in Computer Science*, Springer, 2010, pp. 104–118. doi:10.1007/978-3-642-16289-3_10.

[10] G. Governatori, F. Olivieri, M. Cristani, S. Scannapieco, Revision of defeasible preferences, International Journal of Approximate Reasoning 104 (2019) 205 – 230. doi:10.1016/j.ijar.2018.10.020.

[11] F. Olivieri, G. Governatori, M. Cristani, A. Rotolo, A. Sattar, Deontic meta-rules, Journal of Logic and Computation 34 (2024) 261–314. doi:10.1093/logcom/exad047.

[12] T. F. Gordon, G. Governatori, A. Rotolo, Rules and norms: Requirements for rule interchange languages in the legal domain, in: Rule Interchange and Applications, volume 5858 of *Lecture Notes in Computer Science*, Springer, 2009, pp. 282–296. doi:10.1007/978-3-642-04985-9_26.

[13] M. Palmirani, G. Governatori, A. Rotolo, S. Tabet, H. Boley, A. Paschke, Legalruleml: Xml-based rules and norms, in: Rule-Based Modeling and Computing on the Semantic Web, volume 7018 of *Lecture Notes in Computer Science*, Springer, 2011, pp. 298–312. doi:10.1007/978-3-642-24908-2_30.

[14] H.-P. Lam, M. Hashmi, B. Scofield, Enabling reasoning with legalruleml, in: Rules and Reasoning, volume 9718 of *Lecture Notes in Computer Science*, Springer, 2016, pp. 184–199. doi:10.1007/978-3-319-42019-6_16.

[15] J. L. Rodríguez, L. van der Torre, Deontic Logic and Legal Systems, Cambridge University Press, 2014. doi:10.1017/CBO9781139565413.

[16] M. P. Singh, Agent communication languages: Rethinking the principles, IEEE Computer 31 (1998) 40–47. doi:10.1109/2.735854.

[17] M. P. Singh, An ontology for commitments in multiagent systems, Artificial Intelligence and Law 7 (1999) 97–113. doi:10.1023/A:1008317430146.

[18] P. Yolum, M. P. Singh, Reasoning about commitments in the event calculus, Annals of Mathematics and Artificial Intelligence 42 (2004) 227–253. doi:10.1023/B:AMAI.0000034528.55456.d9.

[19] M. Alberti, F. Chesani, M. Gavanelli, E. Lamma, P. Mello, P. Torroni, Verifiable agent interaction in abductive logic programming: The sciff framework, ACM Transactions on Computational Logic 9 (2008). doi:10.1145/1462179.1462184.

[20] J. Gelati, G. Governatori, A. Rotolo, G. Sartor, Normative autonomy and normative co-ordination, Artificial Intelligence and Law 12 (2004) 53–81. doi:10.1023/B:ARTI.0000045995.49470.11.

[21] G. Governatori, A. Rotolo, A computational framework for institutional agency, Artificial Intelligence and Law 16 (2008) 25 – 52. URL: https://www.scopus.com/inward/record.uri?eid=2-s2.0-43449093134&doi=10.1007%2fs10506-007-9056-y&partnerID=40&md5=cd2ff460b05da9d6db8c03f30e1ebb87. doi:10.1007/s10506-007-9056-y, cited by: 32.

[22] J. Plaza, Logics of public communications, in: Proceedings of the 4th International Symposium on Methodologies for Intelligent Systems, 1989, pp. 201–216.

[23] H. van Ditmarsch, W. van der Hoek, B. Kooi, Dynamic Epistemic Logic, Springer, 2007. doi:10.1007/978-1-4020-5839-4.

[24] Y. U. Ryu, R. M. Lee, Defeasible deontic reasoning and its applications to normative systems, Decision Support Systems 14 (1995) 59–73. URL: https://www.sciencedirect.com/science/article/pii/016792369400002A. doi:https://doi.org/10.1016/0167-9236(94)00002-A.

[25] G. Antoniou, D. Billington, G. Governatori, M. Maher, Representation results for defeasible logic, ACM Transactions on Computational Logic 2 (2001) 255 – 287. URL: https://www.scopus.com/inward/record.uri?eid=2-s2.0-84867763616&doi=10.1145%2f371316.371517&partnerID=40&md5=855444f8613de5710756a84ec615c89f. doi:10.1145/371316.371517, cited by: 302; All Open Access, Green Open Access.

[26] D. Nute, Defeasible logic, in: Lecture Notes in Artificial Intelligence (Subseries of Lecture Notes in Computer Science), volume 2543, 2003, p. 151 – 169. URL: https://www.scopus.com/inward/record.uri?eid=2-s2.0-23144443824&doi=10.1007%2f3-540-36524-9_13&partnerID=40&md5=6157956c6dfa299ca84cf4b472364a07. doi:10.1007/3-540-36524-9_13, cited by: 66.

[27] E. Kontopoulos, T. Zetta, N. Bassiliades, Semantically-enhanced authoring of defeasible logic rule bases in the semantic web, in: Proceedings of the 2nd International Conference on Web Intelligence, Mining and Semantics, WIMS '12, Association for Computing Machinery, New York, NY, USA, 2012. URL: https://doi.org/10.1145/2254129.2254199. doi:10.1145/2254129.2254199.

[28] H. Bhuiyan, G. Governatori, A. Bond, S. Demmel, M. B. Islam, A. Rakotonirainy, Traffic rules encoding sing defeasible deontic logic, Frontiers in Artificial Intelligence and Applications 334 (2020) 3 – 12. URL: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85098668250&doi=10.3233%2fFAIA200844&partnerID=40&md5=e692f1cd6b180072e1a9e6bbfa377900. doi:10.3233/FAIA200844, cited by: 13; All Open Access, Green Open Access, Hybrid Gold Open Access.

[29] F. Olivieri, G. Governatori, M. Cristani, A. Rotolo, A. Sattar, Deontic meta-rules, Journal of Logic and Computation 34 (2024) 261 – 314. URL: https://www.scopus.com/inward/record.uri?eid=2-s2.0-85186728514&doi=10.1093%2flogcom%2fexac081&partnerID=40&md5=e26d63be75ea9d51f6118aa1f75f171f. doi:10.1093/logcom/exac081, cited by: 6; All Open Access, Green Open Access.