

Evaluating the Sensitivity of ABMs to Agents' Decision Functions

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Abstract

This paper frames Monopoly as a controlled environment for studying decentralized decision making, bounded rationality, and adaptive behavior through agent-based simulation. The study combines an agent-based model of the board game with an evolutionary meta-model that updates behavioral parameters across repeated generations of play. The aim is not to reproduce every detail of the commercial game, but to use its rule structure as a compact laboratory in which strategic choices, liquidity constraints, and cumulative advantage can be observed under comparable conditions. The paper first discusses the evolutionary and economic foundations of agent-based modeling, then describes the game dynamics that make Monopoly suitable as a stylized economic system. After that, it presents the model architecture, the evolutionary layer, and the experimental design adopted to compare different scenario configurations.

Keywords

Agent-based modeling, evolutionary computation, computational economics, serious games, Monopoly

1. Introduction

Agent-based modelling (ABM) is a simulation methodology in which a system is modelled not by directly representing its overall dynamics, but by describing the behaviour of its individual components and their interactions [1, 2]. ABM has a long tradition of being used to study economic complex systems [3], where by economic we mean systems that assign values to limited resources in order to manage them [4]. Despite its undoubted success [5], the literature also shows that these models face several methodological challenges [3, 6].

A first issue concerns empirical validation, since the flexibility of ABMs can make it difficult to determine whether a model captures real causal mechanisms or simply reproduces observed patterns in an ad hoc way [7]. This problem is reinforced by the limited availability of strong empirical benchmarks and by the fact that different models may reproduce similar macro-level regularities [8]. A second problem is the output analysis [9]. Because ABMs are usually stochastic, researchers must deal with repeated simulations, variance stability, sensitivity analysis, and the interpretation of high-dimensional outputs [10, 11]. These tasks become even more difficult when model outcomes are spatial and temporal at the same time [12]. Another major theme is behavioral realism [13]. Many contributions argue that standard assumptions regarding individual behaviors could be too restrictive for the analysis of real economical behavior [14, 8]. ABMs are valuable precisely because they relax these assumptions, but this also makes model design more complex [15]. Also, the field still faces difficulties in standardization, replication, and comparison across models, especially because modeling choices and implementation details can strongly affect results [16, 17]. The final issue, and the one treated in this work, is explanatory underdetermination [18, 15]. Even when a model generates a target macro-pattern, this does not prove that the underlying micro-rules are the correct explanation of the real process [19]. Several different

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micro-specifications may produce the same aggregate result, and this is a strong issue when dealing with minimal model explanations [20].

In this context, a game can provide a useful experimental setting for exploring this issue and adding a novel dimension to it, as games typically involve complex decision-making processes in which agents interact within systems shaped by fixed institutional rules, stochastic events, scarce resources, and repeated strategic interactions. Games have long been used for similar purposes [21, 22].

For the purposes of this research, we decided to employ a slightly simplified version of a classic game that is simple, yet not trivial: Monopoly. Although it is a board game, its structure resembles a stylized economic system in which agents accumulate assets, manage liquidity, face asymmetric opportunities, and react to the actions of others. In our model, agents are endowed with a degree of risk sensitivity that shapes their purchasing behaviour through a corresponding decision function. A population of players is then evolved over multiple generations, with those achieving the highest overall ranks in the game being given the greatest chance to reproduce and pass on their risk preferences through a differential evolutionary process [23, 24].

Consequently, at the end of the evolutionary process, the population’s risk sensitivity converged to a pseudo-optimal value [25, 26]. In this paper, we tested the convergence of risk sensitivity under different purchasing functions. This means that the overall experiment was repeated multiple times, each time using a slightly different function to determine whether or not to buy a property. Interestingly, the final results differed substantially from one another, as different models of the same phenomenon, all based on plausible intuitions about how a Monopoly player decides whether or not to purchase an asset, produced a variety of outcomes. The main implication is the need for further caution when modelling complicated social, and especially economic, agent-based systems and when inferring general rules solely from face validation [27].

The remainder of the paper is organized as follows. Section 2 reviews the relevant background. Section 3 presents the methodology, including the model description, the evolutionary meta-model, and the experimental design. Section 4 discusses the interpretation of the results, and Section 5 concludes.

2. Methodology

2.1. Agent-Based Model Description

The model formalizes Monopoly as a discrete-time system where a defined set of agents traverse a closed-loop board, interacting through asset acquisition, capital transfers, and strategic resource management. In the standard game, players secure properties, levy rents, and reinvest in development to augment future cash flows. Over time, this process drives a cumulative dynamic where the interplay between asset ownership and immediate liquidity dictates survival and long-term success.

While we employ Monopoly as our testbed, the methodological insight, that ABM outcomes depend critically on the functional form of behavioral rules, applies broadly to any agent-based economic system where decision mechanisms are specified intuitively rather than empirically validated.

The current framework retains these fundamental mechanics while adopting a streamlined, scalable architecture. The board is modeled as an ordered sequence of cells, representing either physical properties or structural states such as *Start* or *Jail*. Player movement is governed by stochastic transitions via dice rolls; each arrival at a cell triggers a state-dependent interaction, typically involving either an asset acquisition opportunity or a mandatory resource transfer to an opponent.

Several auxiliary features of the original game are intentionally abstracted. *Chance* and *Community Chest* events are excluded to prevent exogenous noise from hiding the underlying accumulation process. Likewise, the complexity of the bargaining and auction mechanisms is reduced by stylized interaction rules, because the primary aim is to model the main economic drivers rooted in ownership, rent extraction, and liquidity constraints.

Property valuations follow an ordered gradient that preserves the economic hierarchy between different sectors of the board. Rent levels scale with development intensity, enabling the model to simulate the compounding effects of capital investment. This architecture highlights a central trade-off:

while aggressive accumulation and development maximize returns, agents must preserve sufficient liquidity to withstand adverse stochastic outcomes.

Temporal evolution occurs in discrete steps, with each increment representing an individual turn. During a turn, an agent moves, resolves cell-specific obligations, assesses acquisition or exchange options, and concludes with development decisions. *Jail* functions as a temporary mobility constraint, while bankruptcy serves as the terminal state, removing agents from the system when liquidity fails to meet payment obligations. Through these integrated mechanisms, the model characterizes the endogenous emergence of wealth concentration, competition, and financial fragility.

Agent-Based Behavioral Specification

The model treats players as heterogeneous agents defined by their current state and specific behavioral traits. An agent's state is tracked through their liquid capital C_i , property portfolio, board position, and *jail* status. To capture diverse decision-making styles, each agent is assigned a *sensitivity* parameter α_i . This value determines how the agent weighs the trade-off between keeping cash on hand and seeking high returns. Those with a high α_i lean toward aggressive investment, while those with lower values prioritize a more conservative, liquid-heavy strategy.

Property Evaluation

A single, consistent evaluation function drives agent decisions across the simulation. For any agent i considering a property j , the attractiveness is calculated as:

$$S_{ij} = \underbrace{\left(\frac{C_i}{p_j}\right)^{\alpha_i}}_{\text{Affordability}} \cdot \underbrace{\frac{r_j}{\bar{R}}}_{\text{Relative Return}} \cdot \underbrace{\left(1 + \frac{n_{ij} + k}{n_j + k}\right)}_{\text{Completion Effect}} \quad (1)$$

Here, p_j is the purchase price, r_j is the base rent, and $\bar{R}$ represents the average rent across the entire board. The first part of the formula scales the desire for a property based on whether the agent can actually afford it. The second part compares the property's potential income to the market average, while the third accounts for the *completion effect*—the strategic value of finishing a color group. In this component, n_{ij} is the number of properties in that group already owned by the agent, n_j is the total size of the group, and $k > 0$ is a smoothing constant that keeps new, empty areas from being ignored entirely.

Purchase Decisions

The model tests three different ways agents might act on these scores. The first is a straightforward deterministic rule: an agent only buys a property if its score meets or exceeds a fixed benchmark θ :

$$\text{Buy}_{ij} = \begin{cases} 1 & \text{if } S_{ij} \geq \theta \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

This reflects a rigid, heuristic-based approach. The second method introduces a relative comparison, where the agent compares the current property against the average score of all other available properties on the board:

$$\bar{S}_i = \frac{1}{|\mathcal{U}|} \sum_{j \in \mathcal{U}} S_{ij} \quad (3)$$

The likelihood of a purchase then becomes probabilistic:

$$p_{ij} = \frac{S_{ij}}{S_{ij} + \bar{S}_i} \quad (4)$$

This better mirrors real-world behavior, where an opportunity is judged based on what else is currently on the market.

The third option uses a saturating function to model the decision. Here, the score is first dampened using a natural log:

$$\tilde{S}_{ij} = \ln(1 + S_{ij}) \quad (5)$$

And then converted into a probability:

$$p_{ij} = 1 - e^{-\lambda \tilde{S}_{ij}} \quad (6)$$

In this case, $\lambda > 0$ controls how sensitive the agent is to the score. This version allows for plenty of variety (stochasticity) but ensures that once a property is "good enough," further increases in its score have a diminishing impact on the final decision.

Table 1

Behavioral parameters governing purchase decisions

Parameter	Range	Interpretation
α_i	[1, 12]	Investment propensity: controls sensitivity to affordability in evaluation.
λ	[0.01, 0.1]	Decision sensitivity: controls steepness of probabilistic purchase response.

Negotiation and Exchange

Agents may reallocate properties through bilateral exchanges, utilizing an evaluation logic consistent with their initial purchase decisions:

$$V_{ij} = \left(\frac{C_i}{p_j} \right)^{\alpha_i} \cdot \left(1 + \frac{n_{ij} + k}{n_j + k} \right) \quad (7)$$

This symmetry ensures that agents maintain a coherent strategic profile across different decision domains. The monetary transfers required to settle these exchanges are defined as:

$$A_{ij} = p_j \cdot (1 + \ln \alpha_i) \quad (8)$$

This formula captures a heterogeneous willingness to pay while incorporating diminishing marginal effects. While higher α_i values prompt agents to offer larger premiums, the logarithmic transformation prevents transfer values from escalating to unrealistic levels. This helps maintain numerical stability and prevents the simulation's wealth dynamics from becoming distorted by extreme outliers.

Development Decisions

Once movement and potential exchanges are resolved, agents shift their focus to capital investment. Out of all feasible development options, the agent selects the action that maximizes:

$$S_{ij}^{build} = \left(\frac{C_i}{K_j} \right)^{\alpha_i} \cdot \frac{\Delta r_j}{\Delta R} \quad (9)$$

In this context, k_j represents the construction cost and Δr_j denotes the expected marginal increase in rent. This ensures that development is a function of both immediate financial capacity and the relative profitability of the upgrade compared to other board opportunities.

Wealth Dynamics

The relationship between acquisition, rent collection, negotiation, and development creates a tight feedback loop between asset ownership and liquidity. While aggressive strategies can accelerate wealth accumulation, they simultaneously increase an agent's exposure to financial shocks. Bankruptcy occurs endogenously when an agent's liquidity is insufficient to meet their obligations, naturally driving the system toward asset concentration. So, rather than relying on complex, forward-looking optimization, the model is built on boundedly rationality and state-dependent rules. By comparing different purchase mechanisms, it is possible to isolate exactly how various behavioral assumptions influence emergent phenomena like monopoly formation and overall system stability.

Parameter Space

The parameter α_i , referred to as sensitivity, is the primary driver of agent behavior. It dictates how affordability (the ratio of cash c_i to price p_j) is perceived and converted into a strategic choice. It is the parameter that would be studied in the analyzing of different purchases functions.

At $\alpha_i = 1$, agents respond to affordability in a linear fashion; proportional changes in liquidity lead to direct, proportional changes in property evaluation. When $\alpha_i > 1$, this relationship becomes convex, amplifying the attractiveness of highly affordable assets while further penalizing those out of reach. This leads to more selective and aggressive investment patterns. Initial testing indicated that values exceeding $\alpha_i \approx 10$ result in unstable dynamics characterized by systematic over-investment. Consequently, the study restricts the range to $\alpha_i \in [1, 12]$ to maintain behavioral variety without hitting numerical saturation.

The parameter λ functions as a global decision intensity setting for the environment. It regulates the speed at which a high evaluation score is translated into a definitive purchase action. While λ is formally any value greater than zero, its effective range is governed by the scale of the transformed score:

$$\tilde{S}_{ij} = \ln(1 + S_{ij}) \quad (10)$$

Since this score typically falls between 0.03 and 0.08, increasing λ causes the exponential function to saturate quickly. This means that even with moderate evaluation scores, higher λ values will push the purchase probability toward one, creating a more aggressive market environment.

2.2. Evolutionary Meta-model Description

The agent-based framework relies on fixed behavioral rules, meaning agents do not undergo any intra-simulation learning. While this ensures a better understanding for the decision-making setting, it limits the agents' ability to adapt to competitive pressures. To study how the behavior could adapt without introducing explicit learning, the simulation is embedded within an evolutionary meta-model driven by a genetic algorithm. In this architecture, behavioral adaptation occurs across generations rather than during a single match, allowing strategies to shift through selection, recombination, and mutation.

The evolutionary population consists of N_{AG} individuals. Each agent i is defined by a single continuous parameter:

$$\alpha_i \in [\alpha_{\min}, \alpha_{\max}] \quad (11)$$

which represents their sensitivity to liquidity risk. This parameter determines the weight of affordability in the evaluation of investment opportunities. High values of α_i trigger aggressive acquisition and a higher tolerance for liquidity shocks, whereas lower values favor a more defensive, cash-heavy stance.

The evolutionary process unfolds in discrete generational cycles. In each cycle, the population is sampled to form groups for the underlying matches. For every run, n individuals are drawn without replacement to act as players. This sampling process is repeated N_{runs} times per generation to ensure that the performance of each behavioral trait is tested across diverse competitive settings.

After each match k , every participant i is assigned a match-specific fitness score. Rather than focusing on a binary win-loss outcome, the fitness metric balances competitive standing and economic efficiency. The fitness for individual i in match k is defined as:

$$F_i^{(k)} = w_R W_i^{(k)} + w_V V_i^{(k)}, \quad \text{where } w_R + w_V = 1 \quad (12)$$

The first component, $W_i^{(k)}$, is a ranking score. Given a final rank $\pi_i^{(k)}$ (where 0 is the winner and $n - 1$ is the last player), the score is normalized as:

$$W_i^{(k)} = \frac{n - 1 - \pi_i^{(k)}}{n - 1} \quad (13)$$

This assigns a value of 1 to the victor and 0 to the last-place finisher, while the second component, $V_i^{(k)}$, measures relative economic performance. If $x_i^{(k)}$ represents the maximum wealth attained by an agent during the match, the value is normalized within that specific game:

$$V_i^{(k)} = \frac{x_i^{(k)} - x_{\min}^{(k)}}{x_{\max}^{(k)} - x_{\min}^{(k)}} \quad (14)$$

where $x_{\min}^{(k)}$ and $x_{\max}^{(k)}$ are the wealth boundaries observed among all players in that run. This choice reflects the competitive nature of the system: what matters is outperforming the immediate opposition rather than reaching an absolute wealth threshold; since individuals are sampled randomly, the number of matches played (K_i) can vary between agents. To prevent participation frequency from biasing the results, the algorithm uses an average fitness per appearance:

$$\bar{F}_i = \frac{1}{K_i} \sum_{k=1}^{K_i} F_i^{(k)} \quad (15)$$

Mean performance drives selection pressure in the genetic algorithm. Each generation, the population is updated via elitism, fitness-proportional selection, crossover, and mutation. A fraction e of top individuals carries over unchanged. The remaining population is filled via roulette-wheel selection, where reproduction probability is proportional to mean fitness $\bar{F}_i$. Offspring are created through recombination with probability c . During crossover, the child's α value is sampled uniformly between the parents; otherwise, the child inherits a parent's value. Finally, mutation rate m introduces a random perturbation δ , with values clipped to $[\alpha_{\min}, \alpha_{\max}]$ to maintain model bounds. Importantly, the genetic algorithm does not select only winners, instead, it uses fitness-proportional (roulette wheel) selection where all agents compete based on their composite fitness across multiple matches. An agent finishing second or third in many games may have higher overall fitness than a rare winner with lower wealth accumulation.

2.3. Experimental Design

The experimental analysis focuses on three specific areas. First, it tracks how the sensitivity parameter α evolves when subjected to competitive selection. Second, it repeats the evolutionary meta-model to test how these outcomes change across different economic and structural setups of the Monopoly board. Finally, it compares how the three specific purchase rules influence both individual agent behavior and the resulting macro-level dynamics of the system. The design centers on three dimensions:

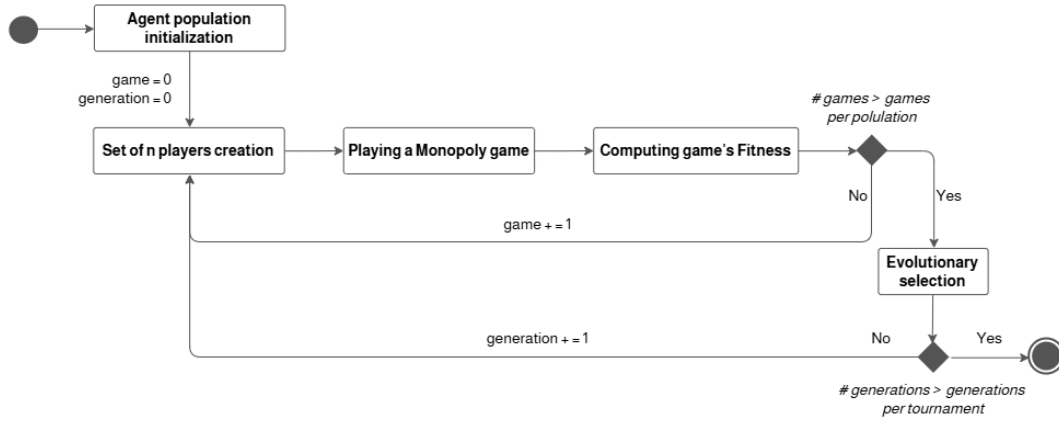
1. Behavioral parameter evolution

The genetic algorithm operates on $\alpha_i \in [1, 12]$, representing an agent's tolerance for liquidity risk. The initial population is sampled uniformly to ensure the dynamics can explore the full range of behavioral regimes, from extreme caution to high-stakes aggression.

Table 2

Parameters and hyperparameters of the evolutionary meta-model

Name	Description	Allowed values
N_{AG}	Number of individuals in the evolutionary population	$N_{AG} \in [n, \infty)$
N_{runs}	Number of matches per generation	$N_{runs} \in [1, \infty)$
N_{gen}	Number of generations	$N_{gen} \in [1, \infty)$
α_i	Individual liquidity risk sensitivity	$\alpha_i \in [\alpha_{min}, \alpha_{max}]$
w_R	Weight of ranking performance in match fitness	$w_R \in [0, 1]$
w_V	Weight of economic performance in match fitness	$w_V \in [0, 1], w_R + w_V = 1$
e	Elitism rate	$e \in [0, 1]$
c	Crossover rate	$c \in [0, 1]$
m	Mutation rate	$m \in [0, 1]$
δ	Mutation scale	$\delta > 0$
T_{max}	Maximum number of time steps per match	$T_{max} \in [1, \infty)$

**Figure 1:** Process of the evolutionary meta model employed for the experiments

2. Environmental scenarios

The Monopoly environment varies across several scenarios to test the model under different fiscal and structural pressures. The parameters for these configurations are listed in Table 3. These setups create a spectrum of environments, from *cash-strapped* systems requiring strict liquidity management to *capital-heavy* markets where asset competition is the primary driver.

Table 3

Monopoly environment scenarios

Parameter	Description	Values
n	Number of players	$\{2, 4, 8\}$
c_0	Initial Cash reserves	$\{750, 1500, 3000, 5000\}$
n_{cells}	Total board cells	40
n_{areas}	Count of color groups	12
p_j	Range of property prices	$[50, 300]$
r_j	Range of base rents	$[10, 50]$
k_j	Construction costs	50

3. Decision rules

The study evaluates the three purchase mechanisms independently: the deterministic threshold, the relative probabilistic rule, and the saturating probabilistic rule. Each rule is tested under identical environmental conditions to isolate its impact.

Simulation Environment

The architecture is built on a dual-layer implementation: the agent-based logic is executed in NetLogo 7.0.3, while the evolutionary meta-model is orchestrated via Python 3.12.1 by pynetlogo library.

Table 4
Experimental Design and Simulation Configuration

Dimension	Component	Specification / Value
Experimental Factors	Behavioral Evolution	Genetic Algorithm on $\alpha_i \in [1, 12]$
	Scenarios	Variations in n, c_0, p_j, r_j
	Decision Rules	deterministic, Relative, Saturating Probabilistic
Software Stack	Agent-Based Core	NetLogo 7.0.3
	Meta-Model Layer	Python 3.12
	Key Libraries	numpy, pynetlogo, multiprocessing, pandas
Evolutionary Config	Generations (N_{gen})	30
	Population Size (N_{AG})	100
	Runs per Generation (N_{runs})	300
Match Dynamics	Time Limit (T_{max})	750 steps
	Termination	Agent bankruptcy or T_{max}
Analytical Focus	Micro-level	Distribution of α (mean, dispersion)
	Macro-level	Wealth concentration, survival patterns

The analysis operates on two scales. Micro-level tracking of α over generations reveals how dominant strategic regimes emerge, while macro-level metrics—such as wealth concentration and survival rates—map these outcomes back to the underlying decision rules, and by linking parameter *sensitivity* to fitness, the model isolates the selective pressures driving evolution under environmental constraints.

3. Results Discussion

The experimental framework is defined by several distinct configurations designed to test the sensitivity of the evolutionary process to initial conditions. Table 5 summarizes the specific parameters for each scenario, including the number of participating players, price boundaries, and varying levels of starting capital. These scenarios are grouped into three primary cohorts (a, b, and c) to facilitate a comparative analysis of scaling effects and resource distribution.

Table 5
Experimental scenarios considered in the evolutionary simulations.

Scenario ID	Players	Price Min	Price Max	Initial Cash	Cells
a1	2	50	300	750	40
a2	2	50	300	1500	40
a3	2	50	300	3000	40
a4	2	50	300	5000	40
b1	4	50	300	750	40
b2	4	50	300	1500	40
b3	4	50	300	3000	40
b4	4	50	300	5000	40
c1	8	50	300	750	40
c2	8	50	300	1500	40
c3	8	50	300	3000	40
c4	8	50	300	5000	40

The property prices follow a fixed schedule derived from standard Monopoly, here all 28 standard property squares (excluding utilities and railroads) are always present on the board, with their prices determined by the established game structure and their prices do not change throughout the evolutionary runs.

The evolutionary simulations demonstrate that the liquidity risk sensitivity parameter, α , does not converge to a universal equilibrium. Instead, its long-run trajectory is systematically shaped by the interplay between purchase rules, competitive density, and initial environmental liquidity. This finding is pivotal: it suggests that even under identical market conditions and evolutionary pressures, subtle variations in behavioral decision-making can precipitate markedly different adaptive outcomes.

3.1. Scenario [a] analysis: $n_{\text{players}} = 2$

Three distinct regimes emerge here (Figure 2, Table 6). The relative rule is aggressively expansionary, pushing α upward regardless of liquidity and hitting high values once $c_0 \geq 1500$. The deterministic rule, however, hinges on the environment: it stays low under scarcity ($c_0 = 750$) but climbs to meet the relative rule as liquidity improves, becoming nearly identical in richer settings. Meanwhile, the saturating rule remains the outlier; it stays conservative during scarcity and scales only slightly as c_0 grows, never approaching the levels of its counterparts.

Table 6
Final Mean Alpha Matrix per Scenario [a] and Buy Rule

Initial Cash	relative	saturating	deterministic
750	11.75	3.17	2.11
1500	12.00	3.89	8.85
3000	11.99	5.31	11.83
5000	11.99	8.35	11.99

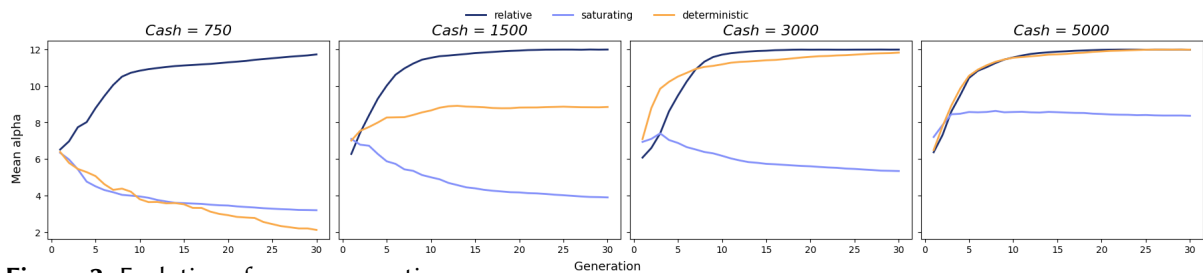


Figure 2: Evolution of α over generations

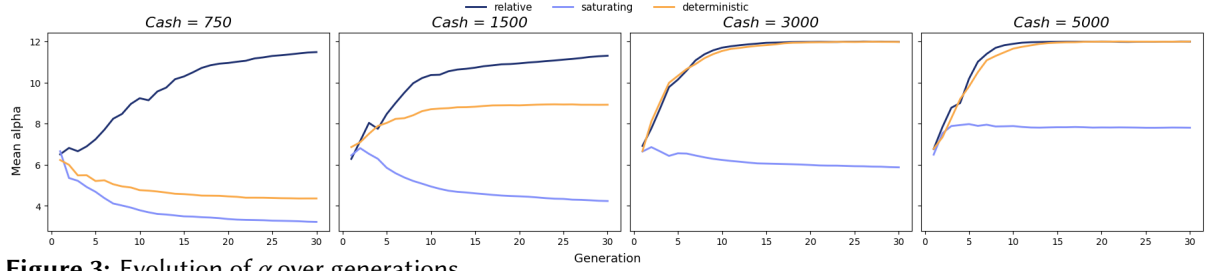
3.2. Scenario [b] analysis: $n_{\text{players}} = 4$

The $n = 4$ results (Figure 3, Table 7) sharpen the established hierarchy. The relative rule still favors high α , though convergence slows when liquidity is tight. Interestingly, the deterministic rule acts as a pivot: it diverges under scarcity but effectively merges with the relative rule's trajectory once $c_0 \geq 3000$, suggesting these two strategies yield identical evolutionary outcomes in favorable environments. While the saturating rule stays conservative, its equilibrium shifts upward compared to the $n = 2$ case; more competition raises its baseline, even if its response remains fundamentally bounded.

Table 7

Final Mean Alpha Matrix per Scenario [b] and Buy Rule

Initial Cash	relative	saturating	deterministic
750	11.52	3.21	4.34
1500	11.34	4.22	8.92
3000	11.99	5.86	11.97
5000	11.99	7.79	11.99

**Figure 3:** Evolution of α over generations

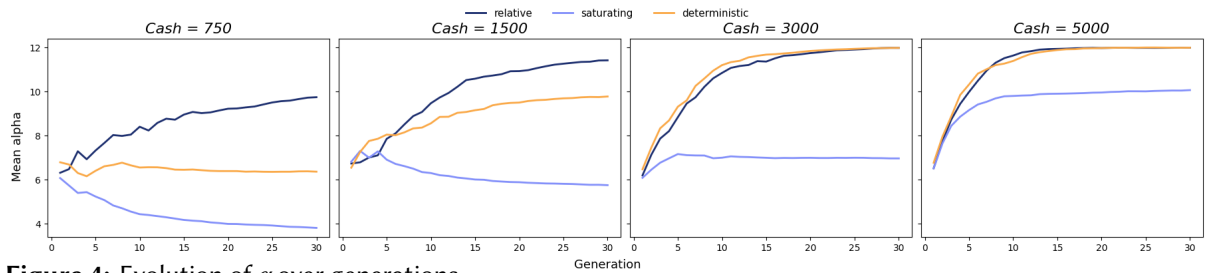
3.3. Scenario [c] analysis: $n_{\text{players}} = 8$

At $n = 8$ (Figure 4 and Table 8), heightened competition crowds the outcome space. The relative rule stays expansionary, though it finally meets its match in low-liquidity settings, stabilizing lower than in wealthier environments. The deterministic rule essentially mirrors this behavior when liquidity is high—trajectories at $c_0 = 3000$ and 5000 are nearly identical. Even the saturating rule, though still the most conservative, shifts its pace; it now scales smoothly with liquidity, reaching higher peaks than in the $n = 2$ or $n = 4$ cases while maintaining its characteristic bounded response.

Table 8

Final Mean Alpha Matrix per Scenario [c] and Buy Rule

Initial Cash	relative	saturating	deterministic
750	9.79	3.78	6.36
1500	11.43	5.72	9.77
3000	11.98	6.94	11.97
5000	11.98	10.08	11.99

**Figure 4:** Evolution of α over generations

Collectively, these results define three distinct yet partially overlapping evolutionary regimes. The relative and deterministic rules do not function as independent behavioral classes; instead, they converge toward similar outcomes when the environment permits aggressive strategies, only diverging as constraints tighten. In contrast, the saturating rule represents a structurally different regime, defined by bounded responsiveness and a lower sensitivity to both liquidity and competition.

Combining trajectory analysis with final distributions is vital, as trajectories reveal the underlying path to these outcomes, clearly identifying where apparent convergence actually masks ongoing dynamics or prolonged transitional phases.

3.4. Speed of Convergence

Beyond the equilibrium levels, the speed and character of convergence reveal much about the underlying evolutionary efficiency. Figure 5 maps the generation where each scenario hits a stationary regime, providing a clearer look at how these populations actually stabilize.

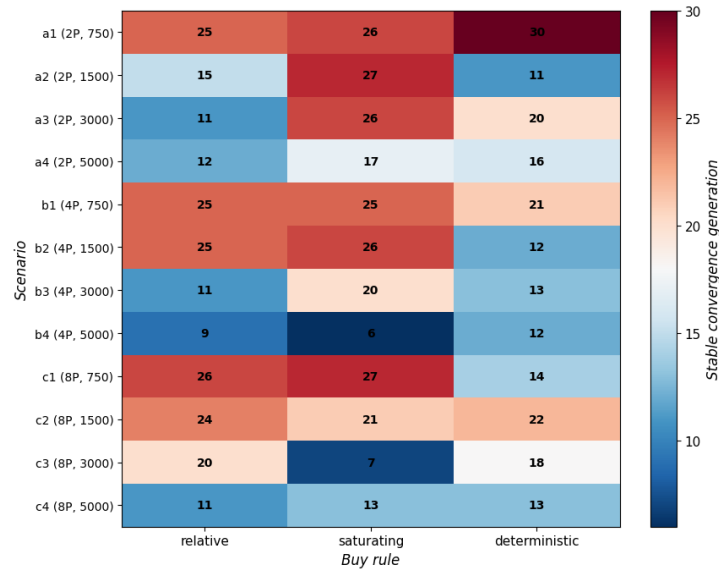


Figure 5: Stable convergence generation of α across scenarios and Buy rules.

Here (Figure 5) lower values indicate that the evolutionary dynamics stabilize earlier, whereas higher values denote slower adjustment. The figure shows that convergence speed is not uniform across decision rules, revealing a more nuanced picture than equilibrium values alone suggest. The deterministic rule consistently stabilizes faster across most scenarios, reaching a stationary regime within 7-20 generations. In contrast, the relative rule shows high variability in convergence speed (9-37 generations), and the saturating rule exhibits the slowest convergence, typically requiring 13-27 generations. This speed differential reflects the underlying selection pressure exerted by each decision rule's functional form. A closer look at the trajectories suggests that "stable" is a relative term; under the relative rule, α often maintains a persistent upward drift. This is especially true when liquidity is scarce or competition is fierce, indicating that the population remains under active evolutionary pressure long after the initial settlement phase. This drift is far less pronounced for the deterministic rule and nearly absent for the saturating rule, which levels off with much more certainty.

This distinction matters because a system can appear stable in its values while still undergoing meaningful directional adaptation. From this perspective, convergence speed is less a property of the algorithm and more a reflection of how a specific decision rule carves out the selection landscape.

Even as more players enter the fold and intensify these dynamics, the underlying structure holds. Competition ratchets up selection pressure, but that impact is always filtered through the rule in play: it fuels the upward drift for the relative rule, acts situationally on the deterministic rule, and only slightly nudges the ceiling for the saturating rule. Ultimately, competition does not push the system in a vacuum; it interacts directly with the logic of the decision mechanism itself.

From an empirical standpoint, three primary patterns emerge regarding the nature of these rules. First, the relative rule is systematically expansionary, selecting the highest observed values of α in most environments. This is directly linked to its analytical form where, as α increases, affordable properties receive disproportionately higher scores, granting agents with higher sensitivity a selective advantage. Second, the deterministic rule is highly context-dependent rather than inherently aggressive. Its discontinuous structure implies that the impact of α depends on the distribution of opportunities relative to the decision boundary; consequently, it only mirrors the relative rule's amplification when

liquidity is sufficiently abundant. Third, the saturating rule defines a structurally distinct regime by mapping scores into bounded purchase probabilities. This compresses differences across opportunities and reduces the marginal benefit of increasing α , which dampens the advantage of aggressive strategies and prevents the system from converging toward a high-sensitivity regime.

4. Discussion and conclusions

The main meta-result of this study is methodological. The three purchase functions generate deeply different evolutionary outcomes, even though they are all plausible representations of the same buying decision. In the meta-model evolution, the relative rule systematically drives α toward very high values, the saturating rule keeps α much lower and more bounded, and the deterministic rule shifts from conservative to aggressive depending on liquidity conditions. This means that the long-run behavior of the system is not determined only by the game environment, but also by the analytical form used to model individual choice. In particular, rules that appear similar at face value can produce different convergence levels, different speeds of stabilization, and different responses to competition. As a result, model outcomes cannot be interpreted independently from the behavioral function embedded in the agents.

This issue is particularly problematic because the differences across purchase rules do not concern only the absolute equilibrium values of α , but also the relative effect of the environmental parameters on its evolution.

Acknowledging the predictable impact of decision functions on outcomes in evolutionary optimization generally, this work makes a more specific contribution by demonstrating that plausibly equivalent decision rules diverge in their evolutionary dynamics. These are not trivial variations, as they represent different strategic regimes that cannot be discovered through theoretical analysis alone. This finding challenges the common modeling practice of selecting behavioral functions based on face validity without systematic comparison.

This raises a broader methodological question. In the agent-based modeling literature, much attention has been devoted to testing the robustness of results under parameter variation, but far less work has examined the robustness of results with respect to the functional form of the underlying decision rules. This is a serious limitation, especially when those functions are introduced mainly on intuitive grounds rather than being supported by data, calibration, or preliminary behavioral analysis. In such cases, the function is often treated as a reasonable modeling choice, while its theoretical and empirical implications remain underexplored. However, different plausible functions may encode different causal assumptions and therefore generate different system dynamics. For this reason, assessing the quality of the underlying behavioral functions should be considered an essential part of model validation, especially in economical settings.

Emphasizing the sensitivity of agent-based models does not mean denying their value. On the contrary, these models remain a powerful way to explore generative hypotheses, especially in economic contexts where aggregate patterns may emerge from decentralized interactions. Their strength lies precisely in the possibility of making behavioral assumptions explicit and testing their systemic consequences. At the same time, this strength comes with clear open issues, such as the dependence of results on modeling choices that are not always empirically grounded.

The main limitation of this work is at the same time one of its main strengths: the results are generated in a game-based environment rather than in a real economic system. This clearly reduces direct empirical relevance, but it also makes it possible to isolate the effects of decision rules within a controlled and comparable setting. Our results suggest a general principle: when modeling bounded rational agents, the functional form of decision rules shapes not only equilibrium outcomes but also the evolutionary stability and convergence speed of behavioral traits. This applies to economic simulations, organizational models, and any system where agent behavior emerges from plausible but empirically

unvalidated choice mechanisms.

Our findings support recent calls in the ABM literature for explicit model validation protocols. Rather than presenting a single best model, research should systematically explore the sensitivity of conclusions to modeling choices: especially behavioral specifications that lack strong empirical grounding.

A possible future development would be the definition of modeling best practices aimed at reducing this problem in a systematic way. In particular, such practices could help make explicit how behavioral functions are chosen, compared, and justified during model design. This would also provide readers and reviewers with clearer evidence that the issue has been considered rather than implicitly ignored. A promising direction for future research is to use this setting to test Batterman's idea of minimal models in combination with LLM-based agents. Monopoly is especially suitable for this purpose because most large language models already possess prior knowledge of the game, which makes it possible to study their behavior without building the full decision structure from scratch. This would allow not only the observation of how LLMs play under controlled conditions, but also the comparison of different interpretive models used to explain their choices. Extensions of this work should investigate whether the functional-form sensitivity identified here holds under more realistic behavioral models that include opponent modeling, strategic negotiation, and multi-step planning. A particularly interesting question is whether more sophisticated agents would show convergence to similar strategies across different decision functions (suggesting the simplified model is adequate) or divergence (suggesting that function form effects compound with behavioral complexity).

Declaration on Generative AI

During the preparation of this work, the authors used OpenAI ChatGPT and Anthropic Claude for code assistance, grammar and spelling checks, rewording, and formatting assistance. The authors reviewed and edited the outputs as needed and take full responsibility for the publication's content.

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