A Three-Layer Reduction Framework for Scalable Verification of Maritime Vessel Systems under Uncertainty

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Abstract

The formal verification of complex real-world systems, such as maritime vessels operating in dense traffic, is fundamentally limited by the explosion of state space exploration. This challenge is particularly pronounced in settings involving multiple interacting entities, hybrid continuous–discrete dynamics, and uncertainty. It arises with a wide range of domains, including autonomous systems, constraint-based reasoning, and time-critical decision-making. This work proposes a three-layer reduction framework for scalable verification, based on the principle of eliminating behaviors that are irrelevant, infeasible, or negligible with respect to a given verification goal. The approach integrates: (i) dependency-driven reduction using graph-based representations of constraint systems and cone-of-influence analysis to extract only query-relevant components of the situation model; (ii) structural or geometric feasibility pruning to eliminate vessel configurations that exclude navigation situations of interest.; and (iii) probabilistic relevance filtering to discard low-impact behaviors while providing explicit bounds (confidence interval) on approximation error induced by model uncertainty. The proposed framework provides formal guarantees: dependency and feasibility reductions preserve correctness, while probabilistic pruning introduces a bounded and controllable deviation characterized by a confidence parameter. The approach is instantiated in the domain of maritime collision verification as a case study, building on prior work in geometric constraint-based analysis. As regards the future work, this research aims to establish a unified reduction principle for scalable constraint-based verification across domains, bridging logical reasoning, constraint propagation, and probabilistic analysis.

Keywords

Scalable verification, State-space explosion, Constraint-based reasoning, Dependency analysis, Probabilistic pruning, Uncertainty modeling, Maritime collision verification

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1. Introduction and Problem Description

Ensuring the safety of complex autonomous systems is increasingly challenging due to the rapid growth of possible system configurations. This difficulty becomes more pronounced in environments where multiple agents interact, decisions evolve over time, and uncertainty affects system behavior.

Autonomous maritime navigation represents a particularly demanding example of this problem. Vessels must comply with navigation rules while reacting to dynamic conditions such as traffic density, environmental disturbances, and imperfect sensor information. These factors lead to a large number of possible interaction scenarios, rendering verification based on explicit state models impractical, whereas verification using symbolic state models may remain tractable and constitutes the direction of this work.

Existing verification approaches face challenges in this context. Explicit state verification methods, including simulation-based techniques, are effective for analyzing narrowly defined scenarios; however, their scalability becomes limited when applied to property verification in broader, multi-agent settings. This research addresses these challenges by investigating how verification complexity can be managed through a structured reduction of the constraints defining the vessels' COLREG-compliant behavior space.

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2. Background and Overview of the Existing Literature

Constraint Programming (CP) [1] has been widely adopted as a mature and expressive paradigm for modeling systems involving complex constraints in a human-readable form while supporting formal reasoning and automated decision-making. Its applications span a wide range of domains, including databases [2], combinatorial optimization [3], scheduling [4], and even music theory [5].

Constraint Logic Programming (CLP) is a specialization of CP, which combines logic programming and constraint solving [1]. Over the past fifty years, several extensions have been proposed [6]. One of the earliest introduced support for linear equations and inequalities over real numbers by relying on incremental variants of Gaussian elimination and the Simplex algorithm, leading to the development of CLP($\mathscr{R}$) [7], followed by CLP($\mathscr{FD}$), which has introduced constraints over a finite domains.

Although CLP offers generic expressive power, it requires domain-specific knowledge to trim the space exploration during the constraint solving. Geometric constraints Geometric approaches have been developed to represent spatial interactions between moving entities, particularly in applications such as collision avoidance in mobile multi-robot systems. These approaches are effective in specifying movement target trajectories, but often do not directly account for uncertainty. Bertagnon and Gavanelli [8] exploited geometric information to improve constraint propagation for solving the Euclidean Traveling Salesperson Problem (ETSP). They defined new constraints to avoid crossings, eliminate aligned points, and perform symmetry breaking. They also made use of the convex hull method, which computes the smallest convex set that encloses all the points, forming a convex polygon. In Bellodi et al. [9], the authors have used a binary classifier to reduce the complexity of the constraints by retaining those that are effective.

Hu et al. [10] presented a multi-objective optimization approach for trajectory planning of Autonomous Surface Vehicles (ASVs), considering Collision Avoidance and COLREG rules. The COLREG rules and physical constraints were formulated as mathematical inequalities. A multiobjective optimisation problem was then solved using particle swarm optimisation. The system consisted of a global path planner and a local path planner. The global planner generated the path offline as a sequence of waypoints, and the local, online planner was invoked if obstacles were detected between any two waypoints, generating a path from the current position to the next waypoint. Situational awareness and risk management were implemented by applying a Closest Point of Approach (CPA)-based method. The approach was validated through simulations accounting for multi-vessel scenarios as well as for non-COLREG-compliant vessels. Simulations in which practical issues such as measurement noise, disturbance, and vehicle dynamics are explicitly taken into account are mentioned to be presented in the future.

Despite these advances, current approaches face important challenges. Many methods do not scale well to scenarios involving multiple interacting agents. Others lack a unified framework that combines logical reasoning, geometric constraints, and probabilistic modeling. Furthermore, few approaches provide formal guarantees when simplifying the verification problem.

3. Goal of the Research

The goal of this research is to develop a scalable and formally grounded approach to safety verification for maritime systems operating under uncertainty. The work focuses on reducing the computational complexity of maritime verification while preserving meaningful guarantees regarding navigational safety and COLREG compliance.

A major challenge in maritime verification is the combinatorial growth of the search space generated by multi-vessel interactions, environmental conditions, navigation parameters, and probabilistic uncertainties. Exhaustive exploration of all possible configurations rapidly becomes computationally infeasible as the number of vessels and operational parameters increases. Consequently, the research investigates reduction-based verification techniques capable of eliminating infeasible or non-critical configurations early in the reasoning process. The proposed approach relies on the integration of

domain-specific constraints and generic consistency algorithms within a unified logical framework. Domain-specific constraints capture maritime knowledge such as geometric encounter conditions, collision-risk situations, navigational restrictions, and COLREG rules. Generic consistency algorithms are then used to propagate constraints and reduce the variable domains explored during verification. The objective is to minimize the explored configuration space while preserving collision-prone and safety-critical scenarios. The work is guided by the following research questions:

- How can multi-vessel maritime behavior and uncertainty be represented within a unified logical framework?
- How can the exploration of maritime encounter configurations be reduced without omitting critical collision-prone situations?
- How can domain-specific constraints and consistency algorithms be combined to improve the scalability of maritime verification?
- How can probabilistic approximations be introduced while maintaining control over their impact on verification correctness and safety guarantees?

Figure 1 illustrates the conceptual architecture of the proposed framework. The variables layer represents the maritime system state, including vessel parameters, environmental conditions, and navigational configurations. The functions and predicates layer derives geometric relations, collision-risk properties, and logical conditions from the variable domains. Bidirectional interaction between functions and predicates supports the evaluation of navigation properties and reasoning consistency. The constraints layer encodes COLREG rules, safety requirements, geometric restrictions, and probabilistic conditions used to reduce the search space and verify system properties. Bidirectional interaction between the variables and constraints layers enables continuous refinement of the explored domains through constraint propagation and consistency checking.

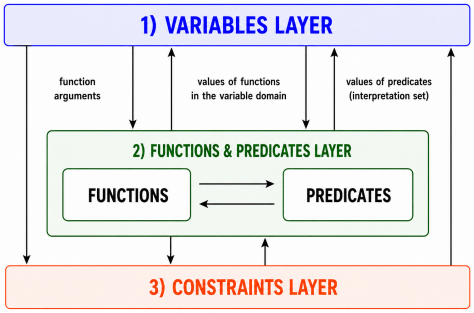


Figure 1: Conceptual layered architecture of the proposed reduction-based maritime verification framework.

The central objective is therefore to establish a reduction-based verification methodology capable of combining symbolic reasoning, geometric optimization, probabilistic analysis, and domain-specific constraints in order to support scalable safety verification for autonomous maritime systems.

4. Current Status of the Research

To support the integration of geometric reasoning, probabilistic analysis, and constraint-based verification, a domain-specific verification architecture, referred to as the *Collision Verification Tool (CVT)*, has been designed (see Figure 2). The framework follows a layered architecture separating application, verification, and execution concerns. At the application layer, the framework manages the specification of maritime scenarios, including vessel parameters, environmental conditions, and navigational constraints. This layer also supports interaction with external simulation environments through a middleware interface. The verification layer constitutes the core of the framework. The *Explorer* module generates candidate vessel encounter scenarios using constraint logic programming techniques over

bounded domains. A *Geometric Optimization* component introduces analytically derived constraints to eliminate infeasible configurations early in the exploration process, thereby reducing the search space before verification. The *Solver* evaluates the remaining configurations by applying logical inference and constraint propagation to verify compliance with collision avoidance requirements. In addition, the middleware interface enables communication with external simulators for higher-fidelity evaluation of selected scenarios. At the execution layer, the framework relies on a CLP(FD)-based computational engine that supports efficient constraint solving and scalable scenario exploration.

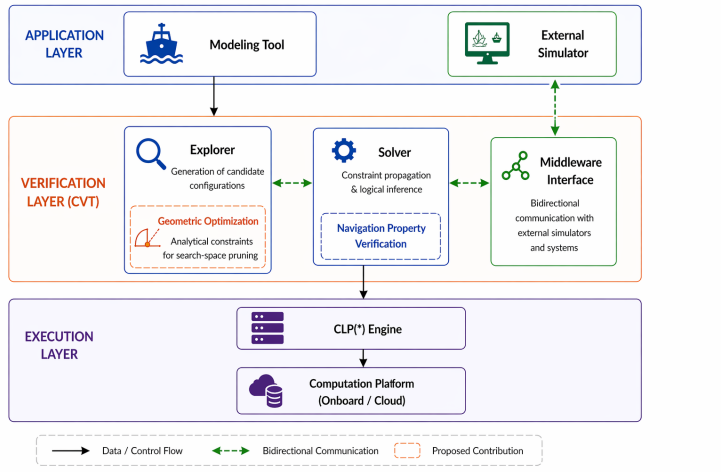


Figure 2: Layered architecture of the Collision Verification Tool (CVT).

Figure 2 illustrates the overall organization of the proposed framework and highlights the responsibilities of each component within the verification process. Research has progressed toward the development of a structured methodology for managing the verification complexity of CLP models describing maritime situations. Initial efforts have focused on modeling vessel interactions using constraint-based representations and incorporating uncertainty into decision-making processes. A key direction that has emerged is the organization of verification as a sequence of reductions, where infeasible or unlikely behaviors are systematically excluded from analysis. The objective is to establish a unified framework connecting logical reasoning, geometric modeling, probabilistic analysis, and simulation-based refinement. Work is currently ongoing to integrate these components into a coherent probabilistic logic programming framework while maintaining both expressiveness and computational efficiency. One of the main challenges is incorporating different probability density functions into the reasoning mechanism, particularly asymmetric distributions with varying skewness parameters, as these can significantly influence the reasoning outcomes.

5. Preliminary Results Accomplished

5.1. Geometric Optimization for Collision Verification

The first stage of the research focused on improving the efficiency of collision verification for Maritime Autonomous Surface Ships (MASS) through geometric reasoning and constraint-based analysis. The developed approach extends previous collision verification models by considering variations in relative vessel speeds, allowing the study of more realistic encounter situations.

To support this objective, geometric relations associated with collision-risk situations were reformulated and integrated into a Constraint Logic Programming (CLP) environment. These geometric constraints enable the early elimination of infeasible encounter configurations before the verification stage, thereby reducing the overall search space explored by the solver. The proposed method operates on bounded two-dimensional maritime scenarios and supports efficient exploration of vessel interactions while maintaining acceptable computational complexity. By eliminating non-relevant configurations

prior to verification, the framework improves the efficiency of collision-risk analysis while preserving the correctness of the verification process. Experimental observations showed that collision-prone configurations can be identified within short execution times, demonstrating the suitability of the approach for real-time navigation assistance and warning systems.

This work resulted in a published conference paper and under review journal paper: (1) *Autonomous Vessels Collision Verification: Geometric Optimization* (AIAI 2025) [11], which proposes a geometric constraint-based optimization approach for reducing infeasible vessel encounter configurations during collision verification. (2) An extended journal version entitled *Autonomous Vessel Collision Avoidance Verification Using Geometric Optimization* is currently under review at *Integrated Computer-Aided Engineering (ICAE)* [12]. The extended work further develops the geometric optimization framework by incorporating relative speed variations between vessels and refining collision-risk geometry analysis under more realistic navigation conditions. Additional analytical constraints were introduced to further reduce the search space explored during verification while preserving efficient reasoning complexity for collision-prone situation analysis.

5.2. Two-phase Path Planning for Fuel-Efficient Safe Navigation

The second stage of the research investigated the integration of symbolic reasoning with simulation-based refinement for safe and fuel-efficient maritime navigation. The objective was to improve both the realism and the operational applicability of generated navigation paths while preserving compliance with COLREG requirements.

The proposed solution follows a two-phase path planning strategy. In the first phase, an abstract navigation path is generated using Constraint Logic Programming (CLP), considering navigational constraints, obstacle locations, vessel interactions, and environmental conditions. In the second phase, the generated path is refined through simulation feedback in order to account for vessel maneuverability, dynamic behavior, and environmental disturbances such as leeway effects. To improve the scalability of the planning process, geometric reduction techniques were incorporated to eliminate infeasible navigation configurations during path exploration. This reduction strategy significantly decreased the size of the explored search space while preserving safety properties and COLREG compliance. Experimental evaluation showed substantial reductions in verification complexity together with improved path planning efficiency. The work also explored the interaction between the Collision Verification Tool (CVT) and a Vessel Simulation Framework (VSF), enabling generated navigation paths to be evaluated under realistic operational conditions and vessel dynamics. This integration supports higher-fidelity analysis and allows symbolic path planning solutions to be refined using simulation-based feedback. These contributions demonstrate that both geometric reasoning and simulation-supported planning can be used to simplify the verification problem while improving the realism of maritime navigation analysis. They also provide the foundation for the ongoing development of a broader reduction-based verification methodology integrating geometric reasoning, simulation, and probabilistic analysis for autonomous maritime systems.

This work resulted in the publication: *Two-phase Path Planning for Fuel-Efficient Safe Navigation* (MARESEC 2025)[13], which introduces a hybrid framework combining constraint-based path planning, geometric optimization, and simulation-driven refinement for maritime navigation.

6. Open Issues and Expected Achievements

Several challenges remain open and define the next phase of research. A primary challenge is the integration of probabilistic logical reasoning, geometric constraints, and probabilistic modeling within a single framework. Ensuring that this integration remains computationally efficient is a key concern. Another important issue is the scalability of the approach to scenarios involving many interacting agents. Techniques for reducing the number of configurations must be carefully designed to avoid excluding relevant behaviors. Further work is also required to formalize the impact of probabilistic simplifications of multi-parametric asymmetric probability density functions and to establish bounds on

the resulting approximation. The expected result of this research is a general methodology for reducing verification complexity in systems that combine structured rules and uncertainty. The approach is intended to be extended beyond maritime navigation, including other autonomous and safety-critical domains.

7. Declaration on Generative AI

During the preparation of this work, the authors used ChatGPT (OpenAI) and Grammarly in order to perform grammar and spelling checks, improve language quality, and enhance the clarity and readability of the manuscript. No generative AI tools were used to generate the scientific content.

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