Decidability of Expressive Description Logics with Role Compositions (extended abstract)

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Well-known Description Logics with decidable basic inference problems are $\mathcal{ALC}$, which allows for Boolean propositional constructors on concepts and (universal and existential) value restrictions on atomic roles, and its extension $\mathcal{ALCN}$ introducing (non-qualified) *number restrictions* on atomic roles. However, in order to better fulfill requirements of real-world application domains, more expressive extensions of the basic concept languages have been investigated. One direction along which useful extensions have been sought is the introduction of *complex* roles under number restrictions. In fact, expressive extensions of $\mathcal{ALCN}$ have been defined as $\mathcal{ALCN}(M)$ with the adoption of role constructors $M \subseteq \{\circ, ^-, \sqcup, \sqcap\}$ [1]. By allowing (different kinds of) complex roles also in value restrictions, different families of Logics can also be defined: for example $\mathcal{ALC}_+\mathcal{N}$ (or $\mathcal{ALC}_{\mathrm{reg}}\mathcal{N}$) allows the transitive closure of atomic roles (or regular roles) under value restrictions [1]. Also Logics $\mathcal{ALC}\bar{\mathcal{N}}(M)$, allowing for the same role constructors either in value *and* in number restrictions, can be considered [3]. Further extensions involve the introduction of *qualified* number restrictions [4] on complex roles, giving rise to $\mathcal{ALCQ}(M)$ languages.

Our ongoing research is aimed at improving the (un)decidability results presented by Baader and Sattler in [1] for $\mathcal{ALCN}$ extensions including composition of roles ($\circ$): they proved that concept satisfiability in $\mathcal{ALCN}(\circ, \sqcap)$ and $\mathcal{ALCN}(\circ, ^-, \sqcup)$ is undecidable via reduction of a domino problem, and provided a sound and complete Tableau algorithm for deciding satisfiability of $\mathcal{ALCN}(\circ)$-concepts. In particular, our current investigations are focused on two conjectures, partially supported by some preliminary results:

1. **Undecidability of $\mathcal{ALCN}(\circ)$ extended with inverse roles** (which is equivalent to $\mathcal{ALC}\bar{\mathcal{N}}(\circ, ^-)$).

2. **Decidability of $\mathcal{ALCN}(\circ)$ extended with qualified number restrictions** (i.e. $\mathcal{ALCQ}(\circ)$).

To show undecidability of $\mathcal{ALC}\bar{\mathcal{N}}(\circ, ^-)$-concept satisfiability and prove (1), we are working on a reduction of the unrestricted domino problem. In this reduction, we are trying to base the reachability of all the grid points that tile the plane on a concept similar to $\exists^{=1}(U \circ R) \circ (U \circ R)^{-} \sqcap \forall U.(\exists R \sqcap \forall R.(\exists^{\geq n} R^{-} \sqcap \forall R^{-}.\exists U^{-}))$, a starting individual s should be instance of. The first conjunct is used to ensure that s is the unique $(U \circ R)$-predecessor of any point that can be reached via $U \circ R$. The second conjunct ensures that there are at least n of such successors, that will play the role of grid points in the construction. Horizontal and vertical successor relations on the grid would be encoded by means of $R^{-} \circ R$ chains; grid closure and tiling constraints would be enforced with the introduction of nine different types of grid points defining a complete $(R^{-} \circ R)$-neighborhood on the plane. We are currently working on fixing details of the encoding for the tiling system and on the correctness proofs.

To prove (2), we are working on a translation of $\mathcal{ALCQ}(\circ)$-concept expressions into semantically equivalent concepts in $\mathcal{ALCN}(\circ)$ extended with number restrictions on intersections of role chains with the same length, that has been proved decidable by Baader and Sattler [1]. Furthermore, we are also working on an extension with qualified number restrictions of their Tableau algorithm for $\mathcal{ALCN}(\circ)$, in order to possibly come out with an effective decision procedure. The extension is mainly based on a modification of the completion rules dealing with number restrictions on role chains to cope with qualified restrictions and on the introduction of a *choose-rule* similar to the one used in $\mathcal{ALCQ}$ Tableaux [4] (also the definition of *clashes* needs some modification). In other words, the intended extension is similar to the one that upgrades the $\mathcal{ALCN}$ Tableau into the $\mathcal{ALCQ}$ Tableau (in their "non-optimized" version [2]). Termination of the algorithm should be proved, as it was for $\mathcal{ALCN}(\circ)$, based on the still valid property that every individual in the ABox the algorithm builds is connected to the root node by role chains of uniquely defined length. We are currently working on proof details.

In conclusion, although we have not completed our work with sound and complete proofs yet, we are confident that all the details can be properly settled and conjectures (1) and (2) can become theorems.

References

[1] F. Baader and U. Sattler. Expressive Number Restrictions in Description Logics. *Journal of Logic and Computation*, 9(3):319–350, 1999.

[2] F. Baader and U. Sattler. An Overview of Tableau Algorithms for Description Logics. *Studia Logica*, 69:5–40, 2001.

[3] F. Grandi. On Expressive Number Restrictions in Description Logics. In *Proc. of Intl' Workshop on Description Logics (DL'01)*, 2001.

[4] B. Hollunder and F. Baader. Qualifying Number Restrictions in Concept Languages. In *Proc. of Intl' Conf. on the Principles of Knowledge Representation and Reasoning (KR'91)*, 1991.