

Iterative Closest Point Algorithm in the Presence of Anisotropic Noise

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Abstract. The Iterative closest point (ICP) algorithm is a widely used method for 3D point set registration. It iteratively establishes point correspondences between two input data sets and computes a rigid transformation accordingly. From a statistical point of view, the algorithm implicitly assumes that the points are observed with isotropic Gaussian noise. In this paper, we present the first variant of the ICP which accounts for anisotropic localization uncertainty in *both* steps of the algorithm and show that in the presence of anisotropic noise, the modified ICP is better suited for both, establishment of point correspondences and transformation computation.

1 Introduction

The iterative closest point (ICP) algorithm [1] is a widely used method for 3D point set registration in the context of medical image processing. In order to find an optimal alignment between two input point clouds the algorithm iteratively (1) establishes point correspondences given the current alignment of the data and (2) and computes a rigid transformation accordingly. It can be shown that the method converges to an at least local minimum with respect to a mean-square distance metric. From a statistical point of view, the algorithm implicitly assumes that the points are observed with zero-mean, identical and isotropic Gaussian noise. In practice, however, point localization errors may be highly anisotropic. Optical tracking systems and time-of-flight (TOF) cameras, for instance, typically have a much higher localization uncertainty in the view direction of the camera. Although various ICP variants have been proposed in the literature [2], the issue of anisotropic noise has so far been given very little attention. This may be due to the fact that all known closed-form solutions for registering two point sets with known correspondences implicitly assume isotropic noise [3]. To our knowledge, Estépar [4] were the first and so far the only ones who proposed a variant of the ICP, the so-called generalized total least squares ICP algorithm (GTLS-ICP), that addresses this issue. The applied iterative method for computing a rigid transformation based on the current estimation of point correspondences accounts for anisotropic noise in both the target and the source models. However, the standard closest point operator based on the Euclidean distance is applied for establishing the point correspondences (Fig. 1(e)).

In this paper, we propose a closest point operator for handling point localization inaccuracies and show that in the presence of anisotropic noise, it is better suited for identifying corresponding points than the Euclidean distance. Furthermore, we present a variant of the ICP algorithm based on this operator and the recently introduced point registration algorithm by Balachandran and Fitzpatrick [3].

2 Materials and Methods

2.1 Closest Point Operator and ICP with Anisotropic Weighting

Let $P = \{\mathbf{p}_1, \dots, \mathbf{p}_{N_P}\}$ be a noisy source point set (e.g., a set of points acquired with a TOF camera) to be registered to a model point set $X = \{\mathbf{x}_1, \dots, \mathbf{x}_{N_X}\}$ (e.g., the set of vertices from a surface mesh extracted from computed tomography (CT) data). In the original ICP algorithm, the goal is to find the rigid body transformation that minimizes the mean squared Euclidean distance between corresponding points (Fig. 1(e)).

In order to account for anisotropic noise, we modify the original ICP algorithm as follows: In addition to the two point sets to be registered, a 3-by-3 weight matrix W_i is provided for each point \mathbf{p}_i in the source point set, reflecting the statistical noise model of the input data. Based on the definition of these weights W_i , the closest point operator is defined as follows:

$$C_{\text{new}}(\mathbf{p}, X) = \arg \min_{\mathbf{x}_i \in X} \|W_i(\mathbf{p} - \mathbf{x}_i)\|_2 \quad (1)$$

As in [3], anisotropic localization errors can be accounted for by giving less weight to the direction associated with the most noise (e.g., the axis along the direction of view of a TOF camera). In addition, outliers can be removed by setting $W_i = \mathbf{0}$. $W_i = I \forall i$ yields the standard version C_{original} of the closest point operator.

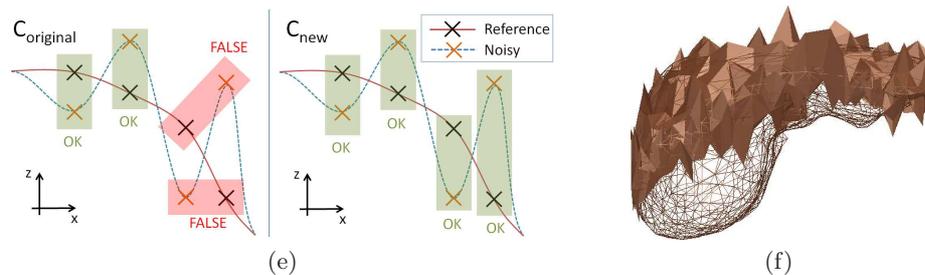


Fig. 1. (a) Schematic illustration of the establishment of point correspondences. Reference mesh (brown solid line), noisy mesh (dotted blue line) and correspondences (boxes) are shown for C_{original} (standard closest point operator) and C_{new} (new closest point operator with less weight given to the direction z). (b) Noisy submesh registered to a reference liver mesh via the proposed anisotropic ICP.

Given the weights W_i for each point in the noisy input data set, the aim of the anisotropic ICP algorithm is to find a rotation matrix R and a translation vector \mathbf{t} such that the following error metric is minimized

$$e(R, \mathbf{t}) = \frac{1}{N_P} \sum_{i=1}^{N_P} \|W_i (R\mathbf{y}_i + \mathbf{t} - \mathbf{p}_i)\|_2^2 \quad (2)$$

where $\mathbf{y}_i = C_{\text{new}}(\mathbf{p}_i, \hat{X})$ with $\hat{X} := \{R\mathbf{x}_j + \mathbf{t}\}, j = 1, \dots, N_X$. As in [3], we assume, that the localization error in each point is normally distributed with zero-mean and can be represented by three uncorrelated components along a set of orthogonal principal axes \mathbf{x} , \mathbf{y} and \mathbf{z} . We set $W_i = \text{diag}(\sigma_{ix}^{-1}, \sigma_{iy}^{-1}, \sigma_{iz}^{-1})$ with $\sigma_{ix}, \sigma_{iy}, \sigma_{iz}$ denoting the standard deviations associated with point \mathbf{p}_i .

The modified ICP works as follows:

1. Initialize variables: $k := 1; X^0 := X; e^0 := \infty; R_{\text{global}} := I; \mathbf{t}_{\text{global}} := \mathbf{0}$
2. Compute the current corresponding points $Y^k = \{\mathbf{y}_i^k\}, i = 1, \dots, N_P$ with $\mathbf{y}_i^k := C_{\text{new}}(\mathbf{p}_i, X^{k-1})$,
3. Compute the rotation matrix R^k ; the translation vector \mathbf{t}^k and the registration error e^k for mapping the point set Y^k onto the point set P using [3]

$$(R^k, \mathbf{t}^k) = \arg \min_{R, \mathbf{t}} \frac{1}{N_P} \sum_{i=1}^{N_P} \|W_i (R\mathbf{y}_i^k + \mathbf{t} - \mathbf{p}_i)\|_2^2 \quad (3)$$

$$e^k = \frac{1}{N_P} \sum_{i=1}^{N_P} \|W_i (R^k \mathbf{y}_i^k + \mathbf{t}^k - \mathbf{p}_i)\|_2^2 \quad (4)$$

4. Apply the rigid body transformation computed in the previous step to X^{k-1} to obtain the transformed model point set X^k .
5. Update the global rigid transformation

$$R_{\text{global}} = R^k R_{\text{global}}; \mathbf{t}_{\text{global}} = R^k \mathbf{t}_{\text{global}} + \mathbf{t}^k \quad (5)$$

6. if $|e^k - e^{k-1}| < \epsilon$ or the maximal number of iterations has been reached, terminate. Otherwise, set $k := k + 1$ and go to step 2.

Note that the algorithm maps the model point set to the noisy source point set because this allows us to perform all calculations in the coordinate system in which the noise model is given. The inverse transformation is given by the rotation matrix R_{global}^{-1} and the translation vector $-R_{\text{global}}^{-1} \mathbf{t}_{\text{global}}$.

We now show, that the presented algorithm converges to an at least local minimum with respect to the error metric e by proving the following theorem:

Provided that the iterative registration algorithm proposed in [3] yields the optimal rigid transformation with respect eq. 4, the registration error for mapping corresponding points decreases in each iteration, i.e., $e^k \leq e^{k-1} \forall k > 0$.

Proof by contradiction: Let us assume that $\exists k$ with $e^k > e^{k-1}$. Hence

$$\sum_{i=1}^{N_P} \|W_i (R^k \mathbf{y}_i^k + \mathbf{t}^k - \mathbf{p}_i)\|_2^2 > \sum_{i=1}^{N_P} \|W_i (R^{k-1} \mathbf{y}_i^{k-1} + \mathbf{t}^{k-1} - \mathbf{p}_i)\|_2^2 \quad (6)$$

Due to the definition of the closest neighbour (1), the following equation holds for each point in the noisy data set \mathbf{p}_i (note: $(R^{k-1}\mathbf{y}_i^{k-1} + \mathbf{t}^{k-1}) \in X^{k-1}$)

$$\|W_i(\mathbf{y}_i^k - \mathbf{p}_i)\|_2^2 \leq \|W_i(R^{k-1}\mathbf{y}_i^{k-1} + \mathbf{t}^{k-1} - \mathbf{p}_i)\|_2^2 \quad (7)$$

In consequence (due to $a_i \leq b_i \forall i \Rightarrow \sum a_i \leq \sum b_i$)

$$\sum_{i=1}^{N_P} \|W_i(\mathbf{y}_i^k - \mathbf{p}_i)\|_2^2 \leq \sum_{i=1}^{N_P} \|W_i(R^{k-1}\mathbf{y}_i^{k-1} + \mathbf{t}^{k-1} - \mathbf{p}_i)\|_2^2 \quad (8)$$

This, however, would mean that $\hat{R}^k := I$ and $\hat{\mathbf{t}} = 0$ would yield a better registration result than R^k and \mathbf{t}^k in step k of the algorithm (eq. 6), which is in contradiction to the presupposition of the theorem. Hence, $e^k \leq e^{k-1} \forall k$, q.e.d.

2.2 Experiments

To evaluate the proposed variant of the ICP algorithm, five liver meshes were extracted from human CT data (approx. 1000 vertices per mesh). For each liver, four submeshes were extracted representing a cranial, caudal, dorsal and ventral view on the liver respectively. Next, zero-mean, identical Gaussian noise with covariance matrices $\text{diag}(1, 1, \sigma^2)$ (cranial/caudal) or $\text{diag}(1, \sigma^2, 1)$ (dorsal/ventral) was added to the meshes, with $\sigma^2 \in \{1^2, 3^2, 5^2, 7^2, 9^2\}$ mm. Both, C_{original} and C_{new} were then applied to establish point correspondences between the two meshes, and the mean percentage of correct correspondences was determined. Next, both versions of the ICP algorithm were used to register the two point sets (Fig. 1(f)). After convergence, the percentage of correct point correspondences was computed again, and the translation error and the rotation

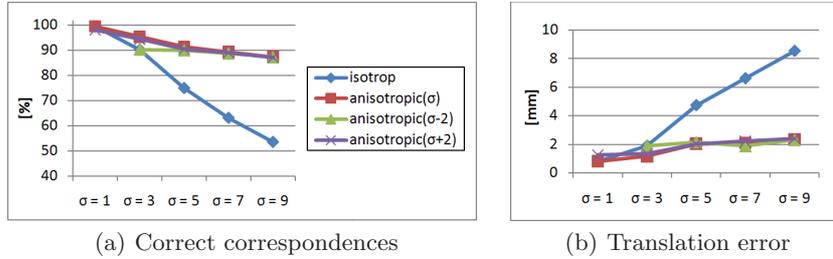


Fig. 2. Mean percentage of correct point correspondences before application of the ICP (a) and mean translation error after running the ICP (b) as a function of the standard deviation σ corresponding to the direction of the most noise (both averaged over 20 submeshes). *Isotropic* represents the standard closest point operator C_{original} and the standard ICP respectively. When C_{new} was applied, the component of the weight matrix corresponding to the most noise was set to $1/\sigma$, $1/(\sigma - 2)$ and $1/(\sigma + 2)$ for a correct noise estimation (anisotropic(σ)), underestimated noise (anisotropic($\sigma-2$)) and overestimated noise (anisotropic($\sigma+2$)) respectively, while the weights corresponding to the remaining two components was set to 1 (all values in mm).

error were determined as in [4], with the null vector and the identity matrix serving as ground truth.

3 Results

As shown in Fig. 2, the proposed closest point operator C_{new} performs considerably better than the standard operator in the presence of anisotropic noise. Even for a standard deviation of 9 mm along the view direction of the camera, the method yields a rate of correct correspondences of $87 \pm 5\%$ compared to $54 \pm 6\%$ with the standard method. Similar improvements were obtained for the translation error (2.3 ± 1.9 mm vs. 8.5 ± 7.9 mm) the percentage of correct correspondences *after* ICP convergence ($87 \pm 4\%$ vs. $52 \pm 6\%$) as well as for the rotation error ($0.2 \pm 0.1^\circ$ vs. $0.6 \pm 0.4^\circ$). Furthermore, the results indicate that the noise distribution must not be known exactly.

4 Discussion

In this paper, we presented the first variant of the ICP which accounts for anisotropic noise in *both* steps of the iterative algorithm: Computation of point correspondences and transformation computation. The presented results indicate that the proposed method is better suited for 3D point registration than the standard approach in the presence of anisotropic noise, even when the noise distribution is not exactly known. To establish the new method as a standard procedure, the following work remains to be done: (1) Efficient implementation for run-time optimization, (2) assessment of convergence speed and accuracy with random initial positions, (3) comparison with other variants of the ICP as well as with other methods for point registration in the presence of anisotropic noise (e.g. [4]), and (4) evaluation of the effects of anisotropic noise in the reference data set.

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