Surface-Similarity vs. Voxel-Similarity Based Registration of Multimodality Medical Images Using Simulated Annealing

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This paper develops a framework for research in the field of multimodality image registration. A certain formal treatment of the registration problem is proposed. We discuss some aspects of the surface and voxel-similarity measures in a context of simulated annealing based optimization.

Keywords: registration, similarity measure, simulated annealing

1 Introduction

The advent of powerful graphics workstations and new developments in tomographic scanner technology has led to an explosive growth in availability of volumetric medical data. For diagnostic and therapeutic purposes different imaging modalities are applied to the same patient. The importance of correlation and integration of such volumetric information is becoming increasingly recognized in the medicine. The process of finding a transformation that align two or more data sets according to anatomical structures is called registration. Because the information provided by different modalities is complementary, 3D medical data registration has been established as the method to improve clinical diagnosis and treatment.
In this paper a comparison of some registration methods based on surface-similarity and voxel-similarity measures is described. The goal of our research is to investigate different similarity measures as well as convergence properties of simulated annealing based registration algorithms.
The structure of this paper is as follows: we first present our formulation of the problem. It introduces also some concepts that are used throughout this paper. A definition of the three cost functions used in our work is given in section 3. A simulated annealing method is presented in section 4. In section 5 we present comparative results for the three cost functions, and we close with a discussion of the results.

2 Formulation of the registration problem

The problem is posed as an optimization problem where the objective is to minimize the dissimilarity (misregistration) function, or maximize some similarity function between the two data sets. Because the possible number of local minima can be very large, we are using simulated annealing to explore the space of possible solutions. To formulate different approaches solving the registration problem in a more formalized way, the notion of feature is introduced. A feature is a localizable property of the data set. A number of different features has been proposed in the literature. Common examples of features are points, edges/lines, ridges, surfaces, sub-volumes, or all voxels with corresponding grey-values. In order to use the features in the registration process it is first necessary to identify them. In the surface-similarity approach the feature space is a set of surface points (voxels) whose extraction is done in a segmentation step (normally non-automatic). In the voxel-similarity approach, the feature space is composed of all data grey-values.
Let us define a *model* as a finite set of features in the first data set and an *object* as a finite set of features in the second one. We assume, that the *object* is geometrically transformed in the registration process to fit the *model*. The next subsection describes this transformation more precisely.

2.1 Matching transformation

In the past a variety of transformation models have been proposed that are being applied when performing registration. When both data volumes are sampled equidistantly along x, y, and z axes (isotropic sampling) and the object to register is a rigid body, the matching transformation can be expressed as a sequence of rotations and translations in 3D. A less rigorous alternative for anisotropic sampled volumes offers the affine transformation. To obtain accurate non-rigid registration of possibly non-linear deformed objects the rigid and affine transformations are not sufficient. One must use 2^{nd} or higher order polynomial transformation.
In this work we assume, that the subject to register is a rigid body. Thus, we can define a 4×4 homogenous coordinate transformation matrix M parameterized by a vector $v = (r_x, r_y, r_z, t_x, t_y, t_z) \in \mathbb{R}^6$, where r_x, r_y, r_z, t_x, t_y, and t_z are rotations around and translations along each of the three major coordinate axes accordingly. We define the matching transformation $\mathcal{T}$ as a geometric transformation represented by the matrix M. During the registration process every *object* feature location is processed through this transformation, giving its equivalent location in the *model* volume. A transformation defined by the matrix M and the parameter vector v we denote by $\mathcal{T}_v$.
The reason to choose a rigid body transformation instead of a more accurate polynomial transformation is that in the polynomial approach it is necessary to search in more dimensional parameter space. In addition, we are not able to decompose the parameters into intuitive physical meaning transformations such as in the rigid body case.

2.2 Statement of the optimization problem

Let $C : \mathbb{R}^6 \rightarrow \mathbb{R}$ be a continuous real-valued function. Our cost functions are constructed such that favourable properties decrease the value of the cost function. Then the registration problem can be stated as follows:

$$v_{opt} = arg\ min\ \{\ C(v) \mid v \in \mathcal{M}\ \} \tag{1}$$

where $\mathcal{M} \subset \mathbb{R}^6$ and normally $\mathcal{M}$ is simply a six-dimensional interval $\mathcal{M} = \{v \in \mathbb{R}^6 : r_{xmin} \leq r_x \leq r_{xmax}, \ldots, t_{zmin} \leq t_z \leq t_{zmax}\}$. The problem is to find a 3D rigid transformation which minimizes this cost function. Global maximization problems can also be investigated in this framework, since $max\{C(v)\} = -min\{-C(v)\}$ on arbitrary $\mathcal{M}$.

3 Cost function definition

In this section we define three popular misregistration functions discussed in the literature ([1],[2]).

3.1 Surface similarity

For any vector $v = (r_x, r_y, r_z, t_x, t_y, t_z) \in \mathbb{R}^6$ we define the cost function as:

$$C(v) = \sum_{i=1}^{N} d_R^2(\mathcal{T}_v(p_i)) \tag{2}$$

where $d_R(\mathcal{T}_v(p))$ is the Euclidean distance between the position of the *object* point p after being transformed by $\mathcal{T}_v$ and the closest point of the *model*. N is the number of *object* features.

3.2 Cross-correlation

The normalized cross-correlation between the *model* and *object* is given by :

$$C(v) = -\frac{\sum_{i=1}^{N} g_R(\mathcal{T}_v(p_i)) \cdot g_O(p_i)}{\sqrt{\sum_{i=1}^{N} g_O^2(p_i)} \cdot \sqrt{\sum_{i=1}^{N} g_R^2(\mathcal{T}_v(p_i))}} \tag{3}$$

where $g_O(p)$ is a grey-value of the *object* voxel p, and $g_R(\mathcal{T}_v(p))$ is a grey-value of the *model* voxel at the position $\mathcal{T}_v(p)$.

3.3 Information entropy of the scatter-plot histogram

The value of the cost function based on the information entropy (see [2]) is calculated according to the following formula :

$$C(v) = -\sum_{i=1}^{N} P(g_O(p_i), g_R(\mathcal{T}_v(p_i))) \log(P(g_O(p_i), g_R(\mathcal{T}_v(p_i)))) \tag{4}$$

where $P(g_O(p_i), g_R(\mathcal{T}_v(p_i)))$ is a probability distribution of the scatter-plot histogram.

4 Simulated annealing

$v_0 := (r_x, r_y, r_z, t_x, t_y, t_z)$;
$k := 0$; $t_0 := t_{start}$;   (* t_{start} – initial temperature *)
REPEAT
    REPEAT
        choose random neighbour parameter vector v_1;
        $\Delta C := C(v_1) - C(v_0)$;
        IF $\Delta C < 0$ **THEN** $v_0 := v_1$; **ELSE**
            IF $random(0,1) < \exp(\frac{-\Delta C}{t_k})$
            THEN $v_0 := v_1$;   (* uphill move *)
    UNTIL *equilibrium()*;
    $t_{k+1} := \alpha_k \cdot t_k$; $k := k + 1$;
UNTIL *stop-criterion()*;   (* termination *)

Figure 1: Procedure Simulated Annealing

The simulated annealing method (see Fig. 1) has proved in many fields of industry and technology to be a promising tool for solving complex nonlinear optimization problems. Simulated annealing is a probabilistic hill-climbing technique which is based on the annealing/cooling process of metals. In contrast to the deterministic hill-climbing procedure deteriorations of the cost function can also be accepted. Starting at an arbitrary point in the search space the algorithm repeatedly generates a new solution by making small transformations to the current solution. If an improvement is reached, the new parameter configuration is accepted. In case of deterioration, acceptance is probabilistic, depending on the cost function value and on the actual parameter t_k called temperature. The number of intermediate configurations (inner loop) and the decrease of the temperature (α_k) are controlled according to a cooling schedule. The function *equilibrium* determines the length of a single run at the constant temperature level. An extensive study of the method can be found in [3].

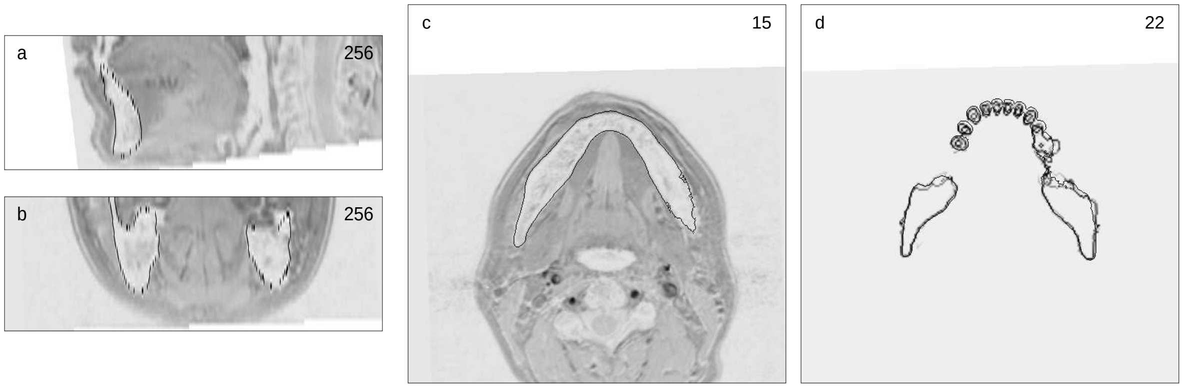

Figure 2: Matched MR and CT data of the lower jaw.

5 Results and discussion

We have tested the above similarity measures on several multi-modal and single-modal medical data. In the case of surface-similarity based registration a segmentation step was performed to extract anatomically relevant surfaces in both data. In our investigations, thresholding and 3D mathematical morphology was sufficient. To accelerate the distance computation in this approach, we have applied the chamfer distance transform method [4]. In the voxel-similarity registration all voxels were taken into account, with an automatic pre-processing step normalizing the grey-values of both data sets.
In the Fig. 2 the quality of the match we have found using simulated annealing is presented (a–c: CT contours on the MR slices, d: superimposed CT and MR contours, black and grey accordingly).

During our work we found out that the surface-similarity measure is the better choice for majority of the tested registration problems. In this case the simulated annealing exhibits a superior convergence behaviour. In the second case, if the initial starting values are close to the optimum (and the set $\mathcal{M}$ is relatively small), the simulated annealing process converges to the optimum well. If these initial values are not near global optimum (or $\mathcal{M}$ is large), the method has poor convergence properties and may never get near it. Figure 3 shows plots of the tested cost functions (a. surface-similarity, b. cross-correlation, c. information entropy) generated from two identical MR images around the global optimum. This idealized situation let us imagine the topography of those cost functions in 6-dimensional search space. The plots show the importance of the initial position and the size of the set $\mathcal{M}$ in the voxel-similarity based registration. The surface-similarity landscapes seem to be the best class of cost function landscapes containing the features that are most relevant to the performance of the simulated annealing method.

6 Conclusions

We have investigated practical properties of selected similarity measures used for the accurate combination of different MR, CT and PET/SPECT images (3D). The general

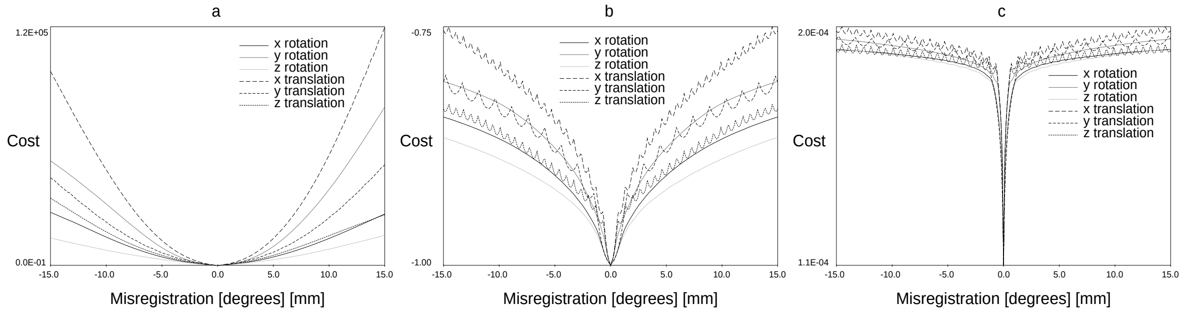

Figure 3: Similarity measure plots

registration problem requires measures that provide a robust assessment of the similarity between *model* and *object*. The surface-similarity based methods are superior to voxel-based methods. The main reason is, that the segmentation process makes use of the operator's anatomical knowledge. Although, the actual voxel based methods for solving registration problems are not entirely satisfactory, since they do not appear to take full advantage of the constraints inherent in the problems, and since the computation of the cost function is exhaustive and time consuming, they should be preferred as a finishing method to speed up the final stages of convergence of another optimization method such as for instance genetic algorithms.

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